Кафедра «Технологическое оборудование»

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ТЕОРИЯ РЕЗАНИЯ

THEORY OF METAL CUTTING

Пособие

для студентов, обучающихся на английском языке, по специальности 1-36 01 01 «Технология машиностроения»

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Целью данного пособия является обеспечение понимания студентами традиционных процессов резания материалов. Основное внимание уделяется определению элементов режима резания, движений, а также геометрии инструмента. Изложены основные положения процесса образования стружки, динамики, теплообразования, трибологии и взаимодействия между обрабатываемым материалом и инструментом.

The objective of this handbook is to provide a student understanding of conventional cutting processes applied to metallic workpieces. The definition of machining regime elements, movements, and tool geometry are considered. The mechanics of chip formation, dynamics, temperature generation, tribology and interactions between workpiece material and cutting tool are described.

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CONTENTS

INTRODUCTION: SCOPE AND SIGNIFICANCE	
OF "THEORY OF METAL CUTTING"	4
1 KINEMATICS OF MACHINING	7
1. KINEMATICS OF MACHINING	······/ 7
1.1. Dasic motion	
1.3. Workpiece surfaces for principal machining processes	9
1.4. Types of Cutting	10
1.5. Reference systems	12
1.6. The machining regime	13
1.7. Uncut (undeformed) chip thickness	15
2 CUTTING TOOL GEOMETRY	17
2.1. Cutting tool geometry standards	
2.2. Tool elements and surfaces	
2.3. Reference plane of single-point turning tool	
in the static coordinate system	19
2.4. Tool angles for single-point turning tool	
in the static coordinate system	20
2.5. Tool angles in the kinematic coordinate system	
3. THEORY OF CHIP FORMATION	
IN METAL MACHINING	
3.1. Mechanism of chip formation	25
3.2. Types of chip formation in metal cutting	27
3.3. Built-up-Edge (BUE) formation	29
A MECHANICAL ANALYSIS OF THE MACHINING	
PROCESS STATIC AND DYNAMIC CUTTING FORCES	31
5. CUTTING TEMPERATURE	34
6. TOOL WEAR OF SINGLE-POINT TURNING TOOLS	37
REFERENCES	41
СЛОВАРЬ	42

INTRODUCTION: SCOPE AND SIGNIFICANCE OF "THEORY OF METAL CUTTING"

What is "Theory of metal cutting"?

The study of the theory of metal cutting is fundamental to mechanical engineering. This course is an introduction to the concept of material removal processes and the basic principle of machining.

In the mechanical engineering industry, it is used terms: machining, material removal processes, metal cutting. What are the differences?

Material removal processes are a family of shaping operations, the common feature of which is a removal of material from a starting workpiece so the remaining part has the desired geometry.

We may use the term *Material removal processes*. It covers all kinds of materials (metals, alloys, nonmetals, glass, plastic) and such processes as:

- Machining – material removal by a sharp cutting tool, e. g., turning, milling, drilling.

In the mechanical engineering industry, the term *machining* is used to cover chip-forming operations. Most machining today is carried out to shape metals and alloys. The term *metal cutting* is used here because research has shown certain characteristic features of the behavior of metals during cutting which dominate the process.

- Abrasive processes – material removal by hard, abrasive particles, e. g., grinding.

There is a great similarity between the operations of cutting and grinding. Our ancestors ground stone tools before metals were discovered and later used the same process for sharpening metal tools and weapons. The grinding wheel does much the same job as the file, which can be classified as a cutting tool, but has a much larger number of cutting edges, randomly shaped and oriented. Each edge removes a much smaller fragment of metal than is normal in cutting. It is largely because of this difference in size we mustn't be applied investigations into metal cutting to the grinding.

- Nontraditional processes – other material removal processes are known as non-traditional processes because they use lasers, electron beams, chemical erosion, electric discharges, and electrochemical energy to remove material rather than cutting or grinding tools.



Fig. 1. The simplest case of machining (orthogonal free cutting)

Machining is a metal removal process in which a wedge-shaped tool engages a workpiece to remove a layer of material in the form of a chip.

Mechanism of machining: Metal ahead of the cutting tool is compressed. This results in the deformation or elongation of the crystal structure – resulting in shearing of the metal. The metal flowing into space above the tool in the form of a chip.

The three principal machining processes are classified as *turning*, *drilling*, and *milling*.

Why Machining is important?

Machining is the most versatile of all manufacturing processes in its capability to produce a diversity of part geometries and geometric features with high precision and accuracy.

The machining of metals and alloys plays a crucial role in a range of manufacturing activities. Metal cutting is commonly associated with big industries (automotive, aerospace, home appliance, etc.).

Advantages of machining:

1. *Variety of work materials*. Machining can be applied to a wide variety of work materials. Virtually all solid metals can be machined. Plastics and plastic composites can also be cut by machining. Ceramics pose difficulties because of their high hardness and brittleness; however, most

ceramics can be successfully cut by the abrasive machining processes discussed in Chapter 25.

2. Variety of part shapes and geometric features. Machining can be used to create any regular geometries, such as flat planes, round holes, and cylinders. By introducing variations in tool shapes and tool paths, irregular geometries can be created, such as screw threads and T-slots. By combining several machining operations in sequence, shapes of almost unlimited complexity and variety can be produced. Casting can also produce a variety of shapes but it lacks the precision and accuracy of machining. Casting, forging, bar drawing create the general shape of the starting work part. Machining provides the final shape, dimensions, finish, and special geometric details that other processes cannot create.

3. *Dimensional accuracy.* Machining can produce dimensions to very close tolerances. Some machining processes can achieve tolerances of 0,025 mm (0,001 in), much more accurate than most other processes.

4. *Good surface finishes*. Machining is capable of creating very smooth surface finishes. Roughness values less than 0,4 microns can be achieved in conventional machining operations. Some abrasive processes can achieve even better finishes.

On the other hand, certain **disadvantages** are associated with machining and other material removal processes:

- *Wasteful of material*. Machining is inherently wasteful of material. The chips generated in a machining operation are wasted material. Although these chips can usually be recycled, they represent waste in terms of the unit operation.

- *Time consuming*. A machining operation generally takes more time to shape a given part than alternative shaping processes such as casting or forging.

"Theory of metal cutting" provides knowledge about:

- the mechanics of chip generation;
- the geometry of cutting tools;
- the force and velocity analysis in the cutting process;
- the principle of tribological analysis of surfaces in contact;
- thermal aspects of machining;
- tool wear models and tool life models;

- the problems in the economy of machining.

1. KINEMATICS OF MACHINING

1.1. Basic motion

To perform machining operations, relative motion is required between the tool and the workpiece. This relative motion is achieved in most machining operations and is a combined motion consisting of several elementary motions, such as the primary motion and the secondary motion.

1.Primary motion (Dr) – the main motion provided by a machine tool or manually, to cause relative motion between the tool and workpiece so that the face of the tool approaches the workpiece material.

The primary motion is only able to cause chip removal for more than one revolution or stroke only if there is a feed motion. Usually, the primary motion absorbs most of the total power required to perform a machining operation.

Cutting speed (V) – instantaneous velocity of the primary motion of the selected point on the cutting edge relative to the workpiece.

2. Feed motion (Ds) – the motion provided by a machine tool or manually, to cause an additional relative motion between the tool and workpiece,

which, when added to the primary motion, leads to repeated or continuous chip removal and the creation of a machined surface with the desired geometric characteristics.

This motion usually absorbs a small proportion of the total power required to perform a machining operation.

Feed speed (Vs) – the instantaneous velocity of the feed motion of the selected point on the cutting edge relative to the workpiece.

When the feed is intermittent, for example in the case of a planing operation, the feed speed is not defined.



Fig. 2. Basic motions: 1 – work surface; 2 – transient surface; 3 – machined surface

3. Resultant cutting motion. The motion resulting from simultaneous primary motion and feed motion.

Resultant cutting speed (Ve) – the instantaneous velocity of the resultant cutting motion of the selected point on the cutting edge relative to the workpiece.

1.2. Type of machining

The three principal machining processes are classified as *turning*, *drilling*, and *milling*.

Turning is used to generate a cylindrical shape (fig. 3). In this process, the workpiece is rotated and the cutting tool removes the unwanted material in the form of chips. The cutting tool has a single cutting edge. The speed motion is provided by the rotating workpiece, and the feed motion is achieved by the cutting tool moving slowly in a direction parallel to the axis of rotation of the workpiece.



Fig. 3. Turning operation

Drilling is used to creating a round hole (fig. 4). In this process, the cutting tool is rotated and feed against the workpiece fixed in a holding device. The cutting tool typically has two or more cutting edges. The tool is fed in a direction parallel to its axis of rotation into the workpiece to form the round hole.

Milling is used to removing a layer of material from the work surface (fig. 5). It is also used to produce a cavity in the work surface. In the first case, it is known as slab-milling and in the second case, it is known as end-milling. Basically, the milling process is used to produce a plane or

straight surface. The cutting tool has multiple cutting edges. The speed motion is provided by the rotating milling cutter. The direction of the feed motion is perpendicular to the tool's axis of rotation.



1.3. Workpiece surfaces for principal machining processes

The three basic surfaces of the workpiece are normally considered (fig. 6):

1 – the *work surface* is the surface of the workpiece to be removed by machining;

2 - the *transient surface* is the surface being cut by the major cutting edge;

3 – the *machined surface* is the surface produced after the cutting tool passes.



Fig. 6. Workpiece surfaces for principal machining processes

Note that the transient surface is always found between the work surface and the machined surface. In most machining operations, the cutting edge does not form the machined surface.

1.4. Types of Cutting

The geometry of practical machining operations (turning, drilling, milling) is somewhat complex. The simplified models of machining are available that neglect many of the geometric complexities, yet describe the mechanics of the process quite well. They are called the orthogonal cutting model and oblique cutting model, fig. 7.



Fig. 7. Orthogonal (a) and oblique (b) cutting

Proper definitions and illustrations of these items are important for the comprehension of the basic and advanced concept of the tool geometry.

Fig. 7 illustrates the difference between orthogonal and oblique cutting.

Orthogonal cutting is the type of cutting where the straight cutting edge of the wedge-shaped cutting tool is at right angle to the direction of cutting as shown in fig. 7, a.

Plane strain condition is the case, i. e., a single "slice" (by a plane perpendicular to the cutting edge) of the orthogonal cutting model can be considered in the analysis of the chip formation model.

Oblique cutting is the type of cutting where the straight cutting edge of the wedge-shaped cutting tool has not at right angle to the direction of cutting fig. 7, *b*.

In orthogonal cutting (fig. 7, *a*), the cutting edge is perpendicular to the direction of primary motion while in oblique cutting (fig. 7, *b*) it is not. The angle which the straight cutting edge makes with the direction of the cutting speed is known as the cutting edge inclination angle λ s. The plastic deformation of the layer being removed in oblique cutting is more complicated than that in orthogonal cutting [4]. Therefore, this type of cutting cannot be represented by a 2D model.

Free cutting is that type of orthogonal or oblique cutting when only one cutting edge is engaged in cutting. Although this definition is widely used in the literature on metal cutting [4–5], it does not provide the proper explanation for the idea of free cutting. For example, if a cutting edge is not straight, it does not perform free cutting. In contrast, a number of cutting edges can be simultaneously engaged in cutting in surface broaching but each edge is engaged in free cutting. In the definition, «free» means that the elementary chip flow vectors from each point of the cutting edge are parallel to each other and do not intersect any other chip flow vectors. An example of free cutting is shown in fig. 8, *a*. If more that one adjacent cutting edges are involved in cutting edge is not straight (fig. 8, c), the chip flow formed at different cutting edges or at different points of the same cutting edges cross each other causing greater chip deformation and thus a greater cutting force than in free cutting.



Fig. 8. Model showing: (*a*) free and (*c*), (*d*) non-free cutting

Non-free cutting is that type of cutting where more than one cutting edge is engaged in cutting so that the chip flows from the engaged cutting edges interact with each other (fig. 8, b, c).

Fig. 9 shows a comparison of chips obtained in orthogonal cutting (a) and that obtained in non-free cutting (turning) (b). For a turning test tool with a nose radius similar to that shown in fig. 7, b was used and the uncut chip thickness was chosen to be small so the chip almost conforms to the shape of the major cutting edge, nose radius, and the minor cutting edge. As seen, non-free cutting causes non-uniform chip deformation.



Fig. 9. Shapes of chips obtained in (a) free and (b) non-free cutting

1.5. Reference systems

Reference systems of planes are necessary for defining and specifying the angles of a cutting tool. There are three coordinate systems (the Systems of Consideration of Tool Geometry):

- the tool coordinate system (fig. 10, *a*);
- the static coordinate system (fig. 10, b);
- the kinematic coordinate system (fig. 10, c).

The tool coordinate system (the tool-in-hand system) is used in the manufacture and control of tools. It is the angles defined in the tool co-ordinate system that is specified in the working drawings of the tools.

The static coordinate system (the tool-in-machine system) is the most commonly used in practice. Used to approximate cutting wedge angles during cutting and to account for changes in these angles when the tool is mounted on a machine tool. The kinematic coordinate system (the tool-in-use system) is used to determine the actual (working) angles of the wedge that occur directly during cutting. It is required to define the orientation of a cutting tool with respect to the machine tool.



Fig. 10. Types of reference systems

The angles of the cutting tool are defined in reference planes. There are 4 main reference planes in *the coordinate system* (fig. 10):

1. Tool reference plane Pr is perpendicular to the assumed direction of primary motion and parallel to a plane of the tool for its orienting.

2. Assumed working plane Pf is perpendicular to the reference plane Pr and containing the assumed direction of feed motion.

3. Tool cutting edge plane Ps is perpendicular to Pr, and contains the major cutting edge.

4. Orthogonal plane Po is perpendicular to the projection of the cutting edge into the reference plane. When the cutting edge is not straight, there are an infinite number of orthogonal planes defined for each point of the curved cutting edge. For a given point of the curved cutting edge, the orthogonal plane is defined as the plane which is perpendicular to the tangent to the projection of the cutting edge into the reference plane edge at the point of consideration.

1.6. The machining regime

The cutting speed, cutting feed, feed rate, depth of cut, and material removal rate are considered with practical examples of calculations.

Fig. 11 illustrates the basic components of the machining regime in the turning.

The cutting speed (V) is the rate at which the uncut surface of the work passes the cutting edge of the tool, usually expressed in units of $m/\min \text{ or } m \cdot \min^{-1}$:

$$V = \frac{\pi D_1 n}{1000}$$

where $\pi = 3, 14;$

 D_1 – diameter of a workpiece (mm);

n – rotational speed (rpm) or rev/min.



Fig. 11. Component of the machining regime in turning: I – work surface; 2 – transient surface; 3 – machined surface

The cutting feed (S) is the distance in the direction of feed motion at which the cutting tool advances into the workpiece per one revolution, thus the feed is measured in millimeters per revolution (mm/rev).

The feed rate (S_M) is the velocity of the tool in the feed direction. It is measured in millimeters per minute (mm/min) and is calculated as

$$S_{\rm M} = S \cdot n.$$

14

The depth of cut (t) is the thickness of metal removed from the bar (workpiece), measured in a radial direction (mm):

$$t = \frac{D_1 - D}{2}.$$

The product of these three gives **the rate of metal removal**, a parameter often used in measuring the efficiency of a cutting operation.

The material removal rate, known as **MRR**, in mm³/min in turning and boring is given by:

$$MRR = 1000 \cdot S \cdot V \cdot t,$$

where V – cutting speed, m/min;

S – cutting feed, mm/rev;

t – depth of cut, mm.

1.7. Uncut (undeformed) chip thickness

Uncut (undeformed) chip thickness (known also as the chip load) is one of the most important characteristics in any metal cutting process as it defines many other important parameters, such as, for example, contact stresses on the tool–chip interface, amount of plastic deformation of the layer being removed, tool life, cutting force and power.



Fig. 12. Sense of the uncut (undeformed) chip thickness

It follows from fig. 12 that the uncut chip **thickness** in turning is calculated as:

$$a = S \cdot \sin \varphi$$

Uncut (undeformed) chip width:

$$b = \frac{t}{\sin \phi}$$

Uncut (undeformed) chip cross-sectional area:

$$A = a \cdot b$$
.

Substituting Eqs, one can obtain:

 $A = t \cdot S$.

The tool cutting edge angle φ is probably the most important angle of the tool geometry as it has a multi-faced influence on practically all aspects of the metal cutting process and greatly affects the outcomes of a turning operation.

2. CUTTING TOOL GEOMETRY

2.1. Cutting tool geometry standards

Understanding tool geometry is key to improving the efficiency of practically all machining operations.

There are two established tool geometry standards, namely ISO Standard (ISO 3002-1) and GOST Standard (GOST 25762-83) in Belarussia. A simple comparison of these standards shows that GOST Standard is based on the notions and definitions of the ISO Standard. Both of them contain clear and functional definitions, one – in the Russian language and others in English.

This chapter presents the definitions of the tool geometry in the static coordinate system (fig. 10, b).

The definitions of ISO 3002-1 are grouped into three clauses:

- 1. certain elements and surfaces of the tool;
- 2. the main reference plane is used to define the geometry of the tool;
- 3. angles are required to define the effective geometry.

A step-by-step approach makes this complicated question simple as considerations begin with the simplest geometry of a single-point cutting tool and finish with the geometry of a multiple-point cutting tool.

2.2. Tool elements and surfaces

The design components of the cutting tool are defined as follows (fig. 13):

Shank is the part of the tool by which it is held.

Head is the functional part of parts of the tool each comprised of chip producing elements; the cutting edges, face and flank are therefore elements of the cutting part.

Wedge is the portion of the cutting part enclosed between the face and the flank. It can be associated with either the major or minor cutting edge.

Face is the surface or surfaces over which the chip flows. The face intersects the flank to form the major cutting edge and the minor cutting edge.

Flank is the tool surface or surfaces over which the surface produced on the workpiece passes. Where it is necessary to distinguish the flanks associated with the major and minor cutting edges, that part of the flank which intersects the face to form the major cutting edge is called *the major flank*, and that part of the flank which intersects the face to form the minor cutting edge is called *the minor flank*, for example, major first flank, minor first flank, etc.



Fig. 13. Surfaces and elements of right-hand cutting tool

Cutting edge is intended to perform cutting. This is a theoretical line of intersection of the face and the flank.

Tool major cutting edge (fig. 13). That entire part of the cutting edge which commences at the point where the tool cutting edge angle φ is zero (fig. 14) and of which at least a portion is intended to produce the transient surface on the workpiece.

Tool minor cutting edge (fig. 13). The remainder of the cutting edge, if any, and where present commences at the point on the cutting edge where φ is zero (fig. 14) but extends from this point in a direction away from the tool major cutting edge. It is not intended to produce any of the transient surface on the workpiece. Some tools may have more than one tool minor cutting edge as, for example, in the case of a cutting off tool.

Active cutting edge (minor, major) (fig. 14). That portion of the working cutting edge which is actually engaged in cutting at a particular instant generating both the transient and machined surfaces on the workpiece.



Fig. 14. Cutting edges of right-hand cutting tool

2.3. Reference plane of single-point turning tool in the static coordinate system



Fig. 15. Reference plane of the right-hand single point turning tool: *I* - work surface; 2 - transient surface; 3 - machined surface; *4* - tool cutting edge plane Ps; 5 - tool reference plane Pr; *6* - orthogonal plane Po; 7 - assumed working plane Pf; *9* - direction of the feed motions at facing and straight turning respectively

Tool reference plane Pr 5 is a plane through the selected point on the cutting edge, so chosen as to be either parallel or perpendicular to a

plane or axis of the tool convenient for locating or orienting the tool for its manufacture, sharpening and measurement.

The plane must be chosen and defined for each individual type of cutting tool so that it meets the prescribed above and is generally oriented perpendicular to the assumed direction of primary motion. For ordinary lathe tools, it is a plane parallel to the base of the tool.

Assumed working plane Pf 7 is a plane through the selected point on the cutting edge and perpendicular to the tool reference plane Pr and so chosen as to be either parallel or perpendicular to a plane or an axis of the tool convenient for locating or orienting the tool for its manufacture, sharpening and measurement.

The plane must be chosen and defined for each individual type of cutting tool so that it meets the conditions prescribed above and is generally oriented parallel to the assumed direction of feed motion. For ordinary lathe tools, it is a plane perpendicular to the tool axis.

Tool cutting edge plane Ps 4 is a plane tangential to the cutting edge at the selected point and perpendicular to the tool reference plane Pr.

Tool orthogonal plane Po 6 is a plane through the selected point on the cutting edge and perpendicular both to the tool reference plane Pr and to the tool cutting edge plane Ps.

2.4. Tool angles for single-point turning tool in the static coordinate system

The geometry of a cutting element is defined by certain basic tool angles and thus precise definitions of these angles are essential.

According to the ISO 3002-1 rake, wedge, and clearance (flank angle) angles are designated by γ , β , and α , respectively, and these are further identified by the subscript of the plane of consideration (fig. 16).

The definitions of basic tool angles *in orthogonal plane Po* are as follows:

 $-\gamma$ - the rake angle is the angle between the reference plane (the trace of which in the considered plane of measurement appears as the normal to the direction of primary motion) and the tool face.

 $-\alpha$ – the clearance (flank angle) is the angle between the tool cutting edge plane Ps and the tool flank plane.

 $-\beta$ – the wedge angle is the angle between the face and flank.



For all cases, the sum of the rake, wedge and clearance angles is 90°, i. e. $\gamma + \beta + \alpha = 90^{\circ}$.

Fig. 16. Reference plane of the right-hand single point turning tool

The definitions of basic tool angles *in the reference plane Pr* are as follows:

 $-\phi$ - tool cutting edge angle, as the acute angle between the projection of the main cutting edge into the reference plane and the assumed working plane Pf (or the assumed direction of feed motion).

 $-\phi_1$ – tool minor (end) cutting edge angle, as the acute angle between the projection of the minor (end) cutting edge into the reference plane and the assumed working plane Pf (or the assumed direction of feed motion).

 $-\varepsilon$ – tool angle between the projection of the main cutting edge and the projection of the minor (end) cutting edge into the reference plane.

For all cases, the sum of the φ , φ_1 , and ε angles is 180°, i. e. $\varphi + \varphi_1 + \varepsilon = 180^\circ$.

The definition of inclination angle (fig. 17) in the plane tangential to the cutting edge at the selected point and perpendicular to the tool reference plane Pr are as follows:

 $-\lambda$ – inclination angle is the angle between the cutting edge and the reference plane. Numbers 1 and 2 (fig. 17) designate the ends of the cutting edge. As such, if the tool tip 1 locates below point 2, then the inclination angle λ is positive; if points 1 and 2 are at the same level, then $\lambda = 0$; and when the tool tip 1 locates above point 2, then the inclination angle λ is negative.



Fig. 17. Sense of the sign of the inclination angle

The sign of the inclination angle defines the chip flow direction as shown in fig. 18. When λ is positive, the chip flows to the right and can potentially damage the machined surface. When λ is negative the chip flows to the left. When $\lambda = 0$, the chip flow direction in the reference plane is entirely determined by the tool cutting edge φ .



Fig. 18. Influence of the sign of the inclination angle on the direction of chip flow

2.5. Tool angles in the kinematic coordinate system

The kinematic coordinate system (the tool-in-use system) considers the geometry of the cutting tool accounting for machining kinematics. When the cutting tool is being used, the actual direction of the primary motion and the feed motion may differ from the assumed directions in the static coordinate system.

As the parameters of the tool geometry are affected by the actual resultant motion of the cutting tool relative to the workpiece, a new system, referred to as kinematic coordinate system should be considered and the corresponding tool angles, referred to as the working angles, should be established in this new system.

Reference Planes in the kinematic coordinate system:

1. Working reference plane Pre is a plane through the selected point on the cutting edge and perpendicular to the *resultant cutting direction*.

2. Working plane Pfe is a plane through the selected point on the cutting edge and containing both the direction of primary motion and the direction of feed motion. This plane is thus perpendicular to the working reference plane Pre.

3. Working cutting edge plane Pse is a plane tangential to the cutting edge at the selected point and perpendicular to the working reference plane Pre. This plane thus contains the resultant cutting direction.

4. Working orthogonal plane Poe is a plane through the selected point on the cutting edge and perpendicular both to the working reference plane Pre and to the working cutting edge plane Pse.

The angles of the cutting tool in the kinematic coordinate system are defined in these planes in the same manner as in the static coordinate system.

The foregoing analysis implies that the basis of the kinematic coordinate system is the proper determination of the reference plane **Pre**. To do that, the direction of the resultant motion should be identified. As discussed previously, this direction, defined by the directional vector **Ve** is the vectorial sum of the directional vector of prime motion **V** and the directional vector of the resultant feed motion V_s , i. e.,

$$V_e = V + V_S$$

This directional vector is always tangential to the trajectory of the resultant tool motion.



Fig. 19. Sense of kinematic rake and flank angles in the general case

As follows from this model, the rake and the clearance (flank) angles in the kinematic coordinate system in the working plane Pf as shown in fig. 19 is calculated as:

$$\gamma_e = \gamma + \eta_S; \quad \alpha_e = \alpha - \eta_S,$$

where angle η_s calculates as

$$\tan \eta_S = \frac{V}{V_s} = \frac{S}{\pi \cdot D},$$

where S – feed per revolution, mm/rev;

D – diameter of the workpiece, mm.

3. THEORY OF CHIP FORMATION IN METAL MACHINING

The first scientific studies of metal cutting were conducted between 1850 and 1885 [1–2]. Many of the early investigations concerned the mechanism of chip formation. Time [2] studied chip formation in metals and wood and apparently coined the term "chip". Tresca [3, 4] and Mallock [5] also studied chip formation and identified it as a shearing process. Other researchers quantified power requirements in machining.

3.1. Mechanism of chip formation

Machining is a process of gradual removal of excess material from the preformed blanks in the form of chips.

The form of the chips is an important index of machining because it directly or indirectly indicates:

- Nature and behavior of the work material under machining condition.

- Specific energy requirement (amount of energy required to remove a unit volume of work material) in machining work.

- Nature and degree of interaction at the chip-tool interfaces.

Knowledge of basic mechanism(s) of chip formation helps to understand the characteristics of chips and to attain favorable chip forms.

The geometry of most practical machining operations is somewhat complex. A simplified model of machining is available that neglects many of the geometric complexities, yet describes the mechanics of the process quite well. It is called the orthogonal cutting model (fig. 20, a). Although an actual machining process is three-dimensional, the orthogonal model has only two dimensions that play active roles in the analysis.

As the tool is forced into the material, the chip is formed by shear deformation along a plane called the *shear plane*, which is oriented at an angle $\boldsymbol{\varphi}$ with the surface of the work. Only at the sharp cutting edge of the tool does failure of the material occur, resulting in separation of the chip from the parent material. Along the shear plane, where the bulk of the mechanical energy is consumed in machining, the material is plastically deformed.



Fig. 20. Orthogonal cutting in the side view (*a*) and shear strain during chip formation (*b*)

During cutting, the cutting edge of the tool is positioned a certain distance below the original work surface. This corresponds to the thickness of the chip prior to chip formation, t_1 . As the chip is formed along the shear plane, its thickness increases to t_2 . The ratio of t_1 to t_2 is called *the chip thickness ratio* (or simply **the chip ratio**) *r*:

$$r = \frac{t_1}{t_2}.$$

Since the chip thickness after cutting is always greater than the corresponding thickness before cutting, the chip ratio will always be less than 1.0.

The reason can be attributed to:

- compression of the chip ahead of the tool;

- frictional resistance to chip flow;

lamellar sliding according.

The geometry of the orthogonal cutting model allows us to establish an important relationship between the chip thickness ratio, the rake angle, and *the shear plane angle*:

$$\tan \varphi = \frac{r \cdot \cos \gamma}{1 - r \cdot \sin \gamma}.$$

The shear strain that occurs along the shear plane can be estimated by examining fig. 20, *b* shows shear deformation approximated by a series of parallel plates sliding against one another to form the chip. Consistent with our definition of shear strain, each plate experiences the shear strain:

$$\varepsilon = \tan(\varphi - \gamma) + \cot\varphi$$

The pattern and extent of total deformation of the chips due to the primary and the secondary shear deformations of the chips ahead and along the tool face, depend upon:

- work material;

- tool (material and geometry);

- the machining speed (V) and feed (S);

- cutting fluid application.

3.2. Types of chip formation in metal cutting

We should note that there are differences between the orthogonal model and an actual machining process.

First, the shear deformation process does not occur along a plane, but within a zone. If shearing were to take place across a plane of zero thickness, it would imply that the shearing action must occur instantaneously as it passes through the plane, rather than over some finite (although brief) time period. For the material to behave in a realistic way, the shear deformation must occur within a thin shear zone. This more realistic model of the shear deformation process in machining is illustrated in fig. 21. Metal-cutting experiments have indicated that the thickness of the shear zone is only a few thousandths of millimeters. Since the shear zone is so thin, there is not a great loss of accuracy in most cases by referring to it as a plane.



Fig. 21. Realistic view of chip formation

Second, in addition to shear deformation that occurs in the shear zone, another shearing action occurs in the chip after it has been formed. This additional shear is referred to as secondary shear to distinguish it from primary shear. Secondary shear results from friction between the chip and the tool as the chip slides along the rake face of the tool. Its effect increases with increased friction between the tool and the chip. The primary and secondary shear zones can be seen in fig. 21.

Third, the formation of the chip depends on the type of material being machined and the cutting conditions of the operation. Three basic types of the chip can be distinguished, illustrated in fig. 22:

Continuous chip. When ductile work materials are cut at high speeds and relatively small feeds and depths, long continuous chips are formed. A good surface finish typically results when this chip type is formed. A sharp cutting edge on the tool and low tool–chip friction encourage the formation of continuous chips. Long, continuous chips (as in turning) can cause problems with regard to chip disposal and/or tangling about the tool. To solve these problems, turning tools are often equipped with chip breakers.



Fig. 22. Types of chip formation in metal cutting: a – continuous; b – serrated; c – discontinuous

The following condition favors the formation of continuous chips:

- work material ductile;
- cutting velocity high;
- feed low;
- rake angle positive and large;
- cutting fluid both cooling and lubricating.

Serrated chips. These chips are semi-continuous in the sense that they possess a saw-tooth appearance that is produced by a cyclical chip formation of alternating high shear strain followed by low shear strain. This type of chip is most closely associated with certain difficult-tomachine metals such as titanium alloys, nickel-base superalloys, and austenitic stainless steels when they are machined at higher cutting speeds. However, the phenomenon is also found with more common work metals (e. g., steels) when they are cut at high speeds.

The following condition favors the formation of serrated chips:

- work material - ductile;

– cutting velocity – low;

- feed - high;

- rake angle - negative and small;

- cutting fluid - both cooling and lubricating.

Discontinuous chip. When relatively brittle materials (e. g., cast irons) are machined at low cutting speeds, the chips often form into separate segments (sometimes the segments are loosely attached). This tends to impart an irregular texture to the machined surface. High tool–chip friction and large feed and depth of cut promote the formation of this chip type.

The following condition favors the formation of discontinuous chips:

- of irregular size and shape: - work material - brittle like grey cast iron;

- of regular size and shape: - work material ductile but hard and work hardenable;

- feed rate - large;

tool rake – negative;

- cutting fluid - absent or inadequate.

3.3. Built-up-Edge (BUE) formation

When machining ductile materials at low-to-medium cutting speeds, friction between tool and chip tends to cause portions of the work material to adhere to the rake face of the tool near the cutting edge. This formation is called a *built-up edge* (BUE) (fig. 23). The formation of a BUE is cyclical; it forms and grows, then becomes unstable and breaks off. Much of the detached BUE is carried away with the chip, sometimes taking portions of the tool rake face with it, which reduces the life of the cutting tool. Portions of the detached BUE that are not carried off with

the chip become imbedded in the newly created work surface, causing the surface to become rough.



Fig. 23. Scheme of built-up-edge formation

The following condition favors the formation of continuous chips with BUE chips:

- work material ductile;
- cutting velocity low (~0,5 m/s);
- small or negative rake angles;
- feed medium or large;
- cutting fluid inadequate or absent.

With the growth of the BUE, the force, F (shown in fig. 23) also gradually increases due to wedging action of the tool tip along with the BUE formed on it. Whenever the force, F exceeds the bonding force of the BUE, the BUE is broken or sheared off and taken away by the flowing chip. Then again BUE starts forming and growing. This goes on repeatedly.

Formation of BUE causes several harmful effects, such as:

- It unfavorably changes the rake angle at the tool tip causing an increase in cutting forces and power consumption.

- Repeated formation and dislodgement of the BUE causes fluctuation in cutting forces and thus induces vibration which is harmful to the tool, job and the machine tool.

- Surface finish gets deteriorated.

- May reduce tool life by accelerating tool-wear at its rake surface by adhesion and flaking occasionally, the formation of thin flat type stable BUE may reduce tool wear at the rake face.

4. MECHANICAL ANALYSIS OF THE MACHINING PROCESS. STATIC AND DYNAMIC CUTTING FORCES

The aspects of the cutting forces concerned:

Magnitude of the cutting forces and their components.

- Directions and locations of action of those forces.

- Pattern of the forces: static and / or dynamic.

Knowing or determination of the cutting forces facilitates or is required for:

- Estimation of cutting power consumption, which also enables selection of the power source(s) during the design of the machine tools;

- Structural design of the machine - fixture - tool system;

- Evaluation of the role of the various machining parameters (process – V, S, t, tool – material and geometry, environment – cutting fluid) on cutting forces;

- Study of behavior and machinability characterization of the work materials;

- Condition monitoring of cutting tools and machine tools.

Several forces can be defined relative to the orthogonal cutting model. Based on these forces, shear stress, coefficient of friction, and certain other relationships can be defined.



Fig. 24. Forces in metal cutting: a -forces acting on the chip in orthogonal cutting; b -forces on the tool that can be measured

Consider the forces acting on the chip during orthogonal cutting in figure. The forces applied against the chip by the tool can be separated into two mutually perpendicular components: *friction force* and *normal*

force to friction (fig. 24). *The friction force* F is the frictional force resisting the flow of the chip along the rake face of the tool. *The normal force to friction* N is perpendicular to the friction force. These two components can be used to define the coefficient of friction between the tool and the chip:

$$\mu = \frac{F}{N}.$$

The friction force and its normal force can be added vectorially to form a resultant force R, which is oriented at an angle b, called the friction angle. The friction angle is related to the coefficient of friction as

In addition to the tool forces acting on the chip, there are two force components applied by the workpiece on the chip: *shear force* and *nor-mal force to shear*. *The shear force Fs* is the force that causes shear deformation to occur in the shear plane, and *the normal force to shear Fn* is perpendicular to the shear force. Based on the shear force, we can define the shear stress that acts along the shear plane between the work and the chip:

$$\tau = \frac{F_S}{A_S}$$

where A_S = area of the shear plane.

Vector addition of the two force components F_S and Fn yields the resultant force R'. In order for the forces acting on the chip to be in balance, this resultant R' must be equal in magnitude, opposite in direction, and collinear with the resultant R.

None of the four force components F, N, F_S , and F_n can be directly measured in a machining operation because the directions in which they are applied vary with different tool geometries and cutting conditions. However, it is possible for the cutting tool to be instrumented using a force measuring device called a dynamometer, so that two additional force components acting against the tool can be directly measured: cutting forces F_x , F_y , F_z .



Fig. 25 -Cutting force and its components: *a* – as applied to the workpiece, *b* – as applied to the tool

The resultant cutting force, R is resolved as,

and

 $\mathbf{P}_{\mathbf{X}\mathbf{Y}} = \mathbf{P}_{\mathbf{X}} + \mathbf{P}_{\mathbf{Y}},$

 $\mathbf{R} = \mathbf{P}_{\mathbf{Z}} + \mathbf{P}_{\mathbf{X}\mathbf{Y}}$

where,

 $P_X = P_{XY} \sin \varphi$

and

 $\mathbf{P}_{\mathbf{Y}} = \mathbf{P}_{\mathbf{X}\mathbf{Y}} \cos \varphi,$

 P_Z – tangential component taken in the direction of Z axis.

 P_X – axial component is taken in the direction of longitudinal feed or X axis.

 P_Y – radial or transverse component is taken along Y axis.

Significance of P_Z , P_X and P_Y :

 P_Z : Called the main or major component as it is the largest in magnitude. It is also called power component as it being acting along and being multiplied by V_C decides cutting power (P_Z, V_C) consumption.

 P_{Y} : May not be that large in magnitude but is responsible for causing dimensional inaccuracy and vibration.

 P_X : It, even if larger than P_Y , is least harmful and hence least significant.

33

5. CUTTING TEMPERATURE

During metal cutting, in particular, there are several temperature effects that need to be considered.

Of the total energy consumed in machining, nearly all of it (98 %) is converted into heat. This heat can cause temperatures to be very high at the tool–chip interface – over 600 °C is not unusual. The remaining energy (2 %) is retained as elastic energy in the chip.

Cutting temperatures are important because high temperatures:

1) reduce tool lif;

2) produce hot chips that pose safety hazards to the machine operator;

3) can cause inaccuracies in workpart dimensions due to thermal expansion of the work material.



Fig. 26. The heat sources (a) and the distribution of heat in machining (b)

An orthogonal single-point cutting operation is schematically illustrated, indicating the distribution of heat and heat sources (fig. 26):

1) the heat sources within the three zones :

 Q_d – the plastic deformation is the major source of heat generation. The heat generated in the main 'body' within the cutting region via both the primary and secondary zones is a result here, of the workpiece's plastic deformation.

 $Q_{\text{fr.rake}}$ – still more intensive heat is generated at the tool–chip interface – along the rake face, where pronounced friction takes place.

 $Q_{\text{fr.flank}}$ – the minor source of heat generation is friction at the tool – workpiece interface – along the flank face.

2) the distribution of **heat** in machining:

 $Q_{\rm chip}$ – the majority of heat being swept away with the chips.

 Q_{tool} – the remainder of heat is either conducted through the tool or conducted/convected into the workpiece Q_{work} .

 Q_{env} – if no coolant application is present in the machining operation, then any heat loss to the ambient air (environment) becomes insignificant.

An equation has been developed that governs the temperature distribution in machining, this being an 'energy-based equation' as follows:

$$Q_{\rm d} + Q_{\rm fr.rake} + Q_{\rm fr.flank} = Q_{\rm chip} + Q_{\rm tool} + Q_{\rm work} + Q_{\rm env}$$

When metal is cut, energy is expended in deforming the chip and in overcoming the friction between the tool and the workpiece. Almost all of this energy is converted to heat Q_d , $Q_{fr.rake}$, $Q_{fr.face}$.

The heat producing high temperatures in the deformation zones and surrounding regions of the chip Q_{chip} , tool Q_{tool} , and workpiece Q_{work} .

The generated heat distribution when cutting depends upon the thermal properties of the tool, workpiece and chip. Therefore, **the thermal diffusivity** (ω , m²/sec), will determine the rate at which heat transfers through the material, while also controlling the penetration depth of the surface temperature. While **the thermal conductivity** (λ , W/m · C°) is a measure of material ability to conduct heat and determines the quantity of heat being absorbed by a given mass of material.

Cutting temperatures are of interest because they affect machining performance. Temperatures in the primary deformation zone, where the bulk of the deformation involved in chip formation occurs, influence the mechanical properties of the work material and thus the cutting forces.

Temperatures on the rake face of the tool have a strong influence on tool life.

As temperatures in this area increase, the tool softens and either wears more rapidly through abrasion or deforms plastically itself. In some cases constituents of the tool material diffuse into the chip or react chemically with the cutting fluid or chip, leading ultimately to tool failure.

Since cutting temperatures increase with the cutting speed, temperature-activated tool wear mechanisms limit maximum cutting speeds for many tool–work material combinations. An understanding of temperatures in this region therefore provides insight into the requirements for tool materials and coatings. Finally, temperatures on the flank (relief) face of the tool affects the finish and metallurgical state of the machined surface. Moderate levels of these temperatures induce residual stresses in the machined surface due to differential thermal contraction, while high levels may leave a hardened layer on the machined part.

The relationship between temperature and cutting conditions:

$$T = K \cdot V^m$$

where T – measured tool–chip interface temperature;

V- cutting speed.



Fig. 27. Diagram showing typical temperature distribution in tools as a function of cutting speed

The parameters K and m depend on cutting conditions (other than V) and work material. Fig. 27 plots temperature versus cutting speed for several work materials, with equations determined for each material. A similar relationship exists between cutting temperature and feed; however, the effect of feed on temperature is not as strong as cutting speed.

6. TOOL WEAR OF SINGLE-POINT TURNING TOOLS

As a single-point turning tool works, its cutting portion wears and tool wear leads to tool failure. Normally, tool wear is a progressive process. In roughing operations, a tool failure criterion is set to limit the amount of tool wear, and thus prevent its breakage. In a finishing operation, a tool wear criterion is commonly based upon the maximum allowable surface roughness of the machined surface and/or the dimensional accuracy of the machined parts while other criteria of surface integrity [4] can also be used.

Three possible modes by which a cutting tool can fail in machining:

1. Fracture failure. This mode of failure occurs when the cutting force at the tool point becomes excessive, causing it to fail suddenly by brittle fracture.

2. Temperature failure. This failure occurs when the cutting temperature is too high for the tool material, causing the material at the tool point to soften, which leads to plastic deformation and loss of the sharp edge.

3. Gradual wear. The gradual wearing of the cutting edge causes loss of tool shape, reduction in cutting efficiency, an acceleration of wearing as the tool becomes heavily worn, and finally tool failure in a manner similar to a temperature failure.

The mechanisms that cause wear at the tool-chip and tool-work interfaces in machining:

- Abrasion. This is a mechanical wearing action caused by hard particles in the work material gouging and removing small portions of the tool. This abrasive action occurs in both flank wear and crater wear; it is a significant cause of flank wear.

- Adhesion. When two metals are forced into contact under high pressure and temperature, adhesion or welding occurs between them. These conditions are present between the chip and the rake face of the tool. As the chip flows across the tool, small particles of the tool are broken away from the surface, resulting in attrition of the surface.

- **Diffusion**. This is a process in which an exchange of atoms takes place across a close contact boundary between two materials. In the case of tool wear, diffusion occurs at the tool–chip boundary, causing the tool surface to become depleted of the atoms responsible for its hardness. As this process continues, the tool surface becomes more susceptible to abrasion and adhesion. Diffusion is believed to be a principal mechanism of crater wear.

- Chemical reactions. The high temperatures and clean surfaces at the tool-chip interface in machining at high speeds can result in chemical reactions, in particular, oxidation, on the rake face of the tool. The oxidized layer, being softer than the parent tool material, is sheared away, exposing new material to sustain the reaction process.

– **Plastic deformation**. Another mechanism that contributes to tool wear is the plastic deformation of the cutting edge. The cutting forces acting on the cutting edge at high temperatures cause the edge to deform plastically, making it more vulnerable to abrasion of the tool surface. Plastic deformation contributes mainly to flank wear.

Standard ISO 3685:1993 Tool-life testing with single-point turning tools defines tool wear on the flank and rake faces of the cutting tool. Fig. 28 shows the basic characteristics of tool wear occurring on the rake and flank faces.



Fig. 28. Types of tool wear according to Standard ISO 3685:1993

The wear of the rake face is known as crater wear. The chip flows across the rake face, resulting in severe friction between the chip and rake face, and leaves a scar on the rake face which usually parallels the major cutting edge. The crater wear can increase the working rake angle and reduce the cutting force, but it will also weaken the strength of the cutting edge. The parameters used to measure the crater wear are shown in fig. 28. The crater depth KT is the most commonly used parameter in evaluating the rake face wear.

Wear on the flank (relief) face is called flank wear and results in the formation of a wear land. Wear land formation is not always uniform along the major and minor cutting edges of the tool. That is why, for wear measurements, the major cutting edge is considered to be divided into four regions, as shown in fig. 28:

1) region C is the curved part of the cutting edge at the tool corner;

2) region B is the remaining straight part of the cutting edge in zone C;

3) region N is the quarter of the worn cutting edge length b farthest away from the tool corner.

Tool wear curves illustrate the relationship between the amount of flank (rake) wear and the cutting time or the overall length of the cutting path. These curves are represented in linear coordinate systems using the results of cutting tests, where flank wear is measured after certain time periods (fig. 29).

Normally, there are three distinct regions that can be observed on such curves.



Fig. 29. Tool wear as a function of cutting time

The first region is the region of preliminary or **rapid initial wear**. Relatively high wear rate (an increase of tool wear per unit time or length of the cutting path) in this region is explained by accelerated wear of the tool layers damaged during its manufacturing or re-sharpening.

The second region is the region of **steady-state wear**. This is the normal operating region for the cutting tool.

The third region is known as the tertiary or **accelerated wear region**. Accelerated tool wear in this region is usually accompanied by high cutting forces, temperatures and severe tool vibrations. Normally, the tool should not be used in this region.

Tool wear depends not only on the cutting time or the length of the cutting path but also on the parameters of the tool geometry (rake, flank, inclination angles, radius of the cutting edge, etc.), cutting regimes (cutting speed, feed, depth of cut), properties of the work material (hardness, toughness, structure, etc.), presence and properties of the cutting fluid and many other parameters of the machining system. In practice, however, the cutting speed is of prime concern in the consideration of tool wear. As such, tool wear curves are constructed for different cutting speeds while keeping other machining parameters invariable. When the amount of wear reaches the permissible tool wear VBBc (fig. 28), the tool is said to be worn out.

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СЛОВАРЬ

Machining – обработка резанием Workpiece – обрабатываемая деталь, изделие Cutting – резание Cutting tool – режущий инструмент Primary motion – главное движение Cutting speed – скорость резания Feed motion – движение подачи Feed speed – скорость подачи Turning – точение **Drilling** – сверление Milling – фрезерование Work surface – обрабатываемая поверхность Transient surface – поверхность резания Machined surface – обработанная поверхность Orthogonal cutting – прямоугольное резание Oblique cutting – косоугольное резание Free cutting – свободное резание Non-free cutting – несвободное резание Tool coordinate system – инструментальная система координат Static coordinate system – статическая система координат Kinematic coordinate system – кинематическая система координат Tool reference plane – основная плоскость Assumed working plane – рабочая плоскость Tool cutting edge plane – плоскость резания Orthogonal plane – главная секущая плоскость Wedge – режущий клин Face (rake face) – предняя поверхность Flank (relief face) – задняя поврехность Major flank – главная задняя поврехность Minor flank – вспомогательная задняя поврехность Cutting edge – режущая кромка Tool major cutting edge – главная режущая кромка Tool minor cutting edge – вспомогательная режущая кромка Active cutting edge – активная часть режущей кромки Rake angle – передний угол Clearance (flank angle) – задний угол

Wedge angle – угол заострения

Tool cutting edge angle – главный угол в плане

Tool minor (end) cutting edge angle – вспомогательный угол в плане

Inclination angle – угол наклона главной режущей кромки Shear plane – плоскость сдвига Chip – стружка Chip thickness ratio – коэффициент усадки стружки Shear plane angle – угол сдвига Cutting force – сила резания Normal force – нормальная сила Friction force – сила трения Thermal diffusivity – коэффициент температуропроводности

Thermal conductivity – коэффициент теплопроводности

Built-up edge – нарост

Wear – износ

Crater wear – луночный износ

Учебное издание

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ТЕОРИЯ РЕЗАНИЯ

THEORY OF METAL CUTTING

Пособие

для студентов, обучающихся на английском языке, по специальности 1-36 01 01 «Технология машиностроения»

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