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THE CONVERGENCE INVESTIGATION OF MESHLESS FINITE BLOCK METHOD AND FINITE ELEMENT METHOD

Zixuan Wang[1], Ji Zhao[1], Tianbiao Yu[1], Wen P.H.[2]

[1]School of Mechanical Engineering and Automation, Northeastern University, Shenyang, PR China

[2]School of Engineering and Materials Science, Queen Mary, University of London, London, UK

Abstract. *The finite element method is one of the most widely used numerical method in engineering analysis, however, the bad convergence and the complexity of meshing reduce the reliability of the simulated results. Therefore, in this work, a meshless finite block method was applied on heat transfer analysis and elastic deformation analysis. It combines the ideas of finite element and boundary element. A better convergence of meshless finite block method than finite element method was proved.*

Introduction. To address the shortcomings of finite element method (FEM), some more advanced numerical methods were proposed, such as boundary element method [1] and meshless method [2]. The meshless finite block method (FBM) is developed from meshless method firstly by Li and Wen [3]. The physical domain is divided into several blocks like elements in FEM, and the adjacent blocks are connected by continuous conditions. The Lagrange interpolation constructs the differential matrices in normalized domain with nodes following Chebyshev's distribution. The infinite element can be introduced by a block of quadratic types, which contributes to reduce the nodes used. The FBM is considered to have higher precision than other meshless methods. The present study compares the convergence of FBM and FEM (ABAQUS) to prove the higher accuracy of FBM.

The convergence comparisons in terms of heat transfer and elastic deformation. The judgment of the convergence is based on the similarity of simulated results with different node density. A model with three blocks used in this study is shown in Fig.1. Taking the node density of block II as the standard, the parameter l indicates the half length of heat source or force source.

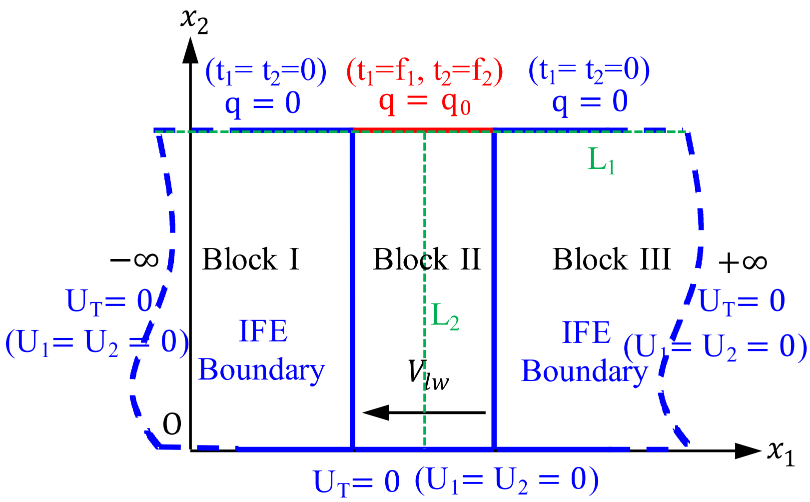

Figure 1 – The block partition and boundary conditions for workpiece

For heat transfer process, an even-distributed moving heat source is applied on the workpiece. The two-dimensional transient heat transfer governing equation for isotropic and continuously homogeneous media Ω can be seen as:

$$\kappa\left(\frac{\partial^2 u}{\partial x_1^2}+\frac{\partial^2 u}{\partial x_2^2}\right)-\rho c V_w \frac{\partial u}{\partial x_1}=\rho c \frac{\partial u}{\partial t}, \qquad \mathbf{x}\in\Omega,\ t>0 \tag{1}$$

The surface temperature results (along L_1) are shown in Fig. 2. The normalized temperature is defined as $U_T=\pi\kappa V_w\left(u-u_0\right)/2\alpha q_0$. It is obvious that the convergence of FBM for heat transfer is better than that of FEM. Only a slight difference occurred in the case of 81 nodes. While for FEM, the difference between the cases with 81 nodes and 841 nodes is remarkable. And the curve with 81 nodes produces oscillation.

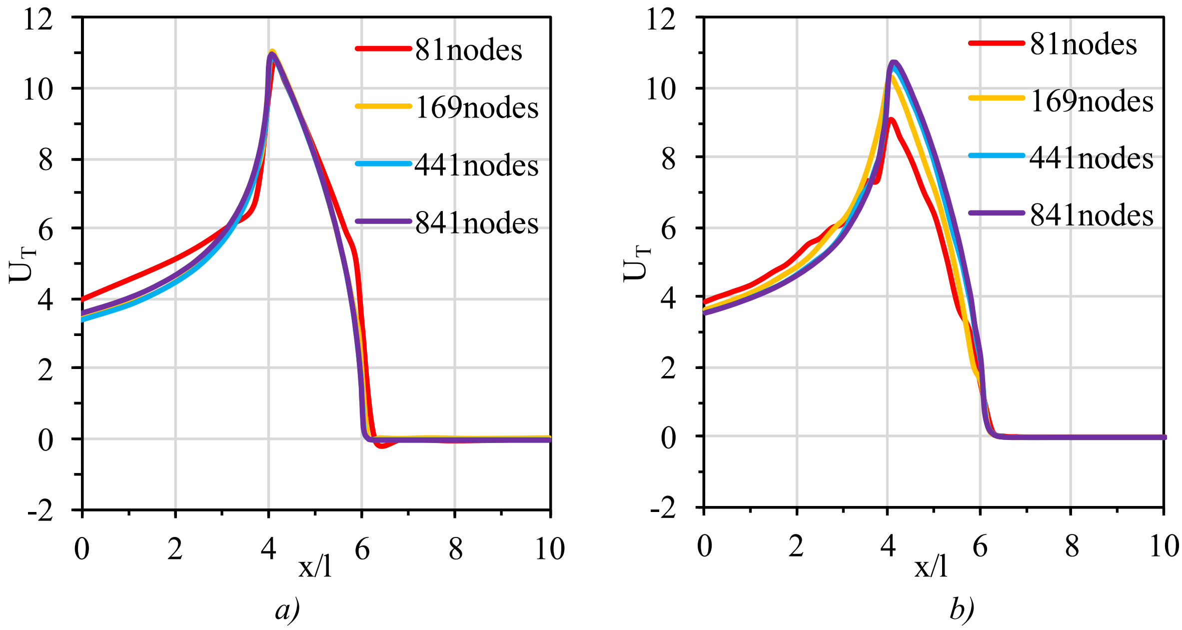

Figure 2 – The convergence comparisons for temperature on line L_1 between *(a)* FBM and *(b)* FEM

For elastic deformation process, the pressure and traction are applied on the top surface of block II. The equilibrium equations of plane stress in 2D homogeneous media Ω are as follows:

$$\frac{\partial\sigma_{11}}{\partial x_1}+\frac{\partial\sigma_{12}}{\partial x_2}=b_1, \frac{\partial\sigma_{12}}{\partial x_1}+\frac{\partial\sigma_{22}}{\partial x_2}=b_2, \quad \left(x_1,x_2\right)\in\Omega \tag{2}$$

The normalized normal stress ψ is defined as σ/E. The surface stresses (along L_1) and the stresses along middle line (along L_2) are evaluated as shown in Fig. 3. It is hard to see difference between the FBM results with 169 nodes and 841 nodes, and a small difference occurs for the curve with 49 nodes. As a contrast, With the decreasing of node density, the accuracy of FEM results is getting worse. The curve with 49 nodes along L_2 has a serious distortion.

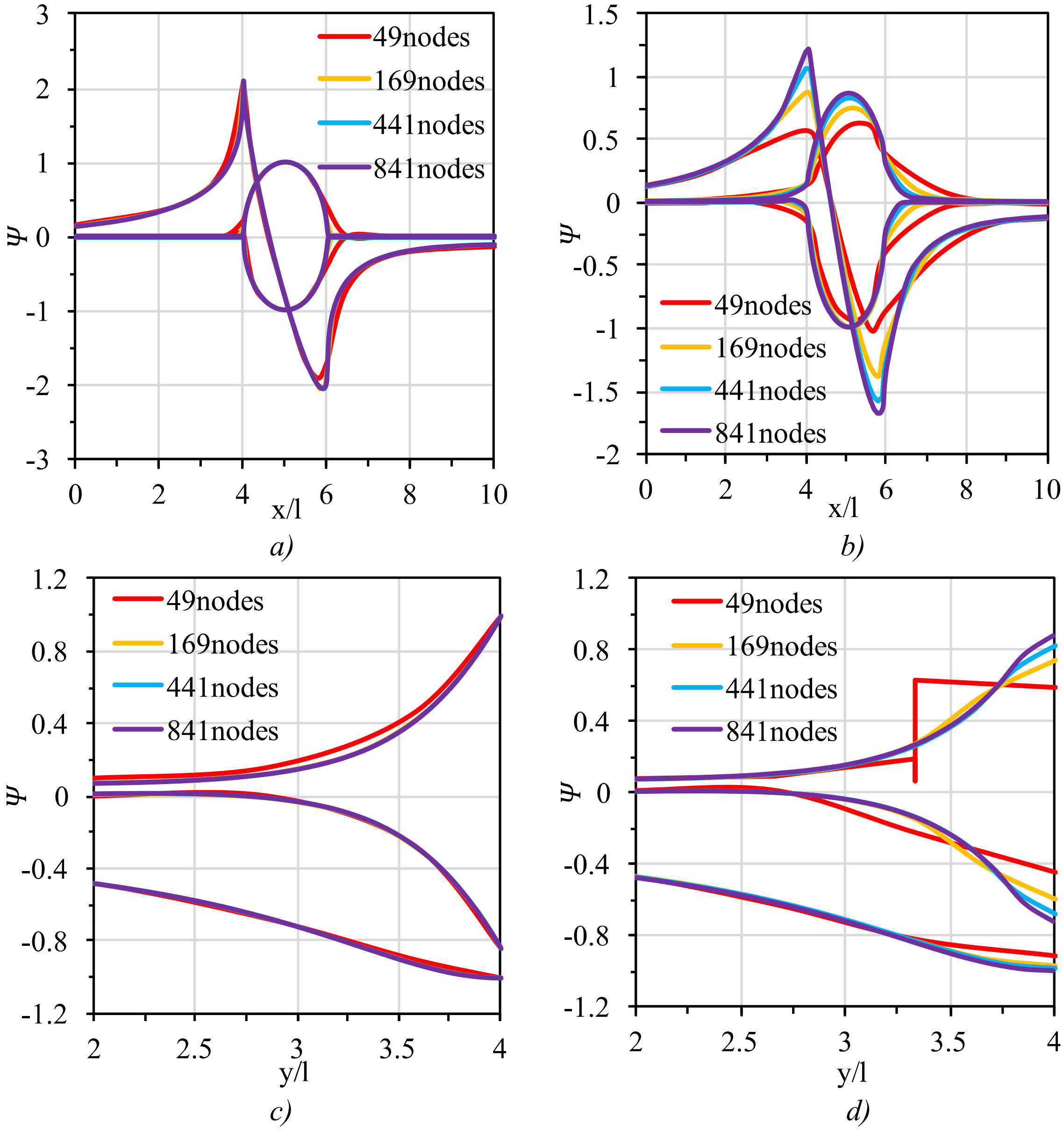

Figure 3 – The convergence comparisons for stress on line L_1
between *(a)* FBM and *(b)* FEM and on line L_2 between *(c)* FBM and *(d)* FEM

Conclusion. The convergence comparisons between meshless finite block method and finite element method (ABAQUS) show that the FBM has a better convergence than FEM in terms of heat transfer and elastic deformation process. The FEM (ABAQUS) is more sensitive to node density, and it might bring larger error. Therefore, for FBM, less nodes will be used to save computational expense without loss of accuracy.

References

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