Algorithm and mathematical model for geometric positioning of segments on aspherical composite mirror

Conquet B.[1], Zambrano L.F.[2,1] Artyukhina N.K.[2], Fiodortsev R.V.[2], Silie A.R.[2,1]

[1]National Center of Optical Technologies,
Los Proceres Ave, sector La Pedregosa, housing 4, Merida 5101, Venezuela

[2] Belarusian National Technical University,
Nezavisimosty Ave., 65, Minsk 220013, Belarus

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Abstract

In recent years, the largest terrestrial and orbital telescopes operating in a wide spectral range of wavelengths use the technology of segmented composite elements to form the main mirror. This approach allows: to expand the spectral operating range from 0.2 to 11.0 µm and to increase the diameter of the entrance pupil of the receiving optical system, while maintaining the optimal value of the exponent m_S – mass per unit area.

Two variants of adjusting the position of mirror segments are considered when forming an aspherical surface of the second order, with respect to the base surface of the nearest sphere, including geometrical and opto-technical positioning.

The purpose of the research was to develop an algorithm for solving the problem of geometric positioning of hexagonal segments of a mirror telescope, constructing an optimal circuit for traversing elements when aligning to the nearest radius to an aspherical surface, and also to program the output calculation parameters to verify the adequacy of the results obtained.

Various methods for forming arrays from regular hexagonal segments with equal air gaps between them are considered. The variant of construction of arrays through concentric rings of an equal step is offered.

A sequential three-step method for distributing mosaic segments is presented when performing calculations for aligning the aspherical surface: multipath linear; multipath point; block trapezoidal.

In the course of mathematical modeling an algorithm was developed to solve the problem of geometric positioning of flat hexagonal segments of a mirror telescope. In the *Python* programming language, program loops are designed to form the data array necessary to construct a specular reflective surface of a given aperture. In the software package *Zemax*, the convergence of optical beams from flat hexagonal elements to the central region of the aspherical surface is verified.

Keywords: hexagonal, hexagonal segment, geometric and opto-technical positioning, composite mirror, algorithm, model.

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Адрес для переписки:
Конкет Б.
Национальный центр оптических технологий,
пр-т Лос Процерес, сектор Ла Педрегоса, корпус 4,
г. Мерида 5101, Венесуэла
e-mail: conquetber@gmail.com

Address for correspondence:
Conquet B.
National Center of Optical Technologies,
Los Proceres Ave, sector La Pedregosa, housing 4,
Merida 5101, Venezuela
e-mail: conquetber@gmail.com

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Алгоритм и математическая модель геометрического позиционирования асферического составного зеркала

Конкет Б.[1], Самбрано Л.Ф.[2,1], Артюхина Н.К.[2], Фёдорцев Р.В.[2], Силие А.Р.[2,1]

[1]*Национальный центр оптических технологий,*
пр-т Лос Процерес, сектор Ла Педрегоса, корпус 4, г. Мерида 5101, Венесуэла

[2]*Белорусский национальный технический университет,*
пр-т Независимости, 65, г. Минск 220013, Беларусь

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В последние годы крупнейшие наземные и орбитальные телескопы, работающие в широком спектральном диапазоне длин волн, при формировании главного зеркала используют технологию сегментированных составных элементов. Такой подход позволяет: расширить спектральный рабочий диапазон от 0,2 до 11,0 мкм и увеличить диаметр входного зрачка приемной оптической системы, при сохранении оптимального значения показателя m_S – масса на единицу площади. Цель исследований заключалась в разработке алгоритма для решения задачи геометрического позиционирования гексагональных сегментов зеркального телескопа, построения оптимальной схемы «обхода» элементов при юстировке на ближайший радиус к асферической поверхности, а также программной апробации выходных расчетных параметров с целью проверки адекватности полученных результатов.

Рассмотрены два варианта юстировки положения зеркальных сегментов при формировании асферической поверхности второго порядка, относительно базовой поверхности ближайшей сферы, включающие геометрическое и оптотехническое позиционирование.

Рассмотрены различные методики формирования массивов из регулярных шестиугольных сегментов с равными воздушными промежутками между ними. Предложен вариант построения массивов через концентрические кольца равного шага.

Представлена последовательная трехступенчатая методика распределения сегментов мозаики при выполнении расчетов по юстировке асферической поверхности: многолучевая линейная; многолучевая точечная; блочная трапецеидальная.

В ходе проведения математического моделирования разработан алгоритм для решения задачи геометрического позиционирования плоских гексагональных сегментов зеркального телескопа. На языке программирования *Python* составлены циклы программы для формирования массива данных необходимых для построения зеркальной отражающей поверхности заданной апертуры. В программном пакете *Zemax* выполнена проверка сходимости оптических лучей от плоских гексагональных элементов в центральную область асферической поверхности.

Ключевые слова: гексагональный, шестиугольный сегмент, геометрическое и оптотехническое позиционирование, составное зеркало, алгоритм, модель.

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Адрес для переписки:
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пр-т Лос Процерес, сектор Ла Педрегоса, корпус 4,
г. Мерида 5101, Венесуэла
e-mail: conquetber@gmail.com

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Introduction

In recent years, the largest terrestrial telescopes operating in a wide spectral range of wavelengths use the technology of segmented composite mirrors [1–4]. A typical representative of this class is the Keck Observatory of two telescopes (Keck Telescope) Mauna Kea, Hawaii (1996), in which two 10 meter mirrors consist of 36 segments.

The most well-known method of aligning segmented mirrors is described in the works of Mast and Nelson [5] and found practical application in the alignment of Keck telescopes. According to this method, each individual segment is described as a local curve of an aspherical surface, and all together they form a common curve of the aspherical surface. Segments have different geometric dimensions and a distorted hexagonal shape, i. e. elongate in the radial direction to ensure minimization of the intersegment gap area.

The exact mutual position of the mirror segments relative to the base surface is established in two steps [6].

The first stage involves geometrical positioning, during which the plane segments are displaced along three linear directions (along the coordinate axes *OX*, *OY* and *OZ* in the base coordinate system) and the maximum reduction of all optical rays in the central region close to the center of the curvature of the mirror is achieved.

The second stage of opto-technical positioning is carried out in three angular directions with respect to the top of the mirror segment (two slopes with respect to the optical axis and rotation around it) and minimization of the wave front difference (aberrations) at the working wavelength of the telescope (Figure 1). The error in positioning and relative positioning of individual segments should not exceed the dimensions of the working wavelength [7].

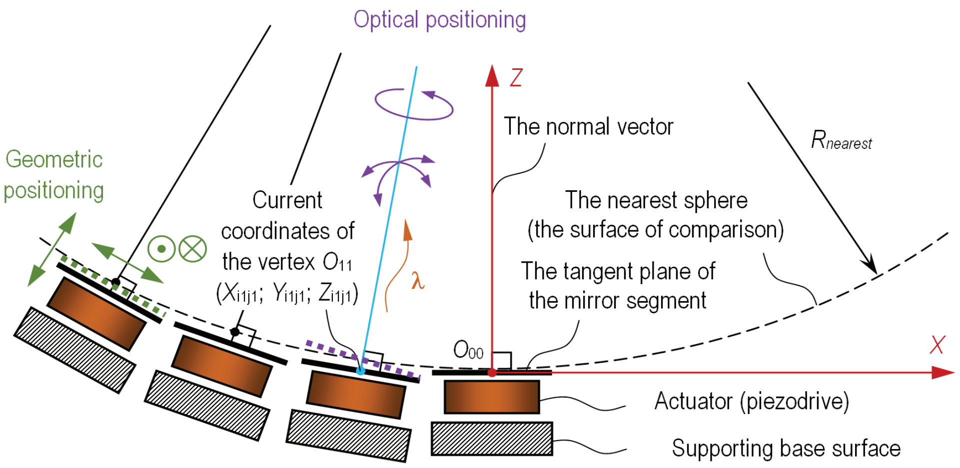


Figure 1 – Diagram of geometric and opto-technical positioning of mirror segments

The comparison method involves the process of positioning each individual mirror segment relative to a common reference surface by means of actuators (eg piezo drives).

The correct geometrical positioning of the mirror segment is performed under the condition that the normal vectors constructed from the center of the flat surface of each segment must intersect at one calculated point on the axis *OZ* coinciding with the center of curvature of the base spherical surface. The tangent plane in space is determined by three Cartesian or spherical coordinates (Figure 1) [8].

However, in practice it is impossible to realize completely identical mirror segments, and their normals do not converge at the point of the double focal length of the mirror spheroid, but in some region of this point. The optimization problem usually reduces to minimizing this region of convergence, as well as to reducing the difference between the real and calculated wavefronts, both for the entire composite mirror, and for each segment separately [9].

In [6], the efficiency of using the method of geometric computer positioning of 20 controllable hexagonal mirror segments forming a 500 mm composite mirror for 1 hour with an error of not more than 0.01 mm is shown in [6]. In the classical scheme of alignment of similar elements using an autocollimator, it takes about 20 hours. It should be noted that the complexity of the alignment increases exponentially, so it took 1 year to set up a composite telescope mirror Gran Telescopio CANARIAS with a diameter of 10.4 m (73 m^2).

The purpose of the research was to develop an algorithm for solving the problem of geometric positioning of hexagonal segments of a mirror telescope, constructing an optimal circuit for traversing elements when aligning to the nearest radius to an aspherical surface, and also to program the output calculation parameters to verify the adequacy of the results obtained.

Determining the geometric parameters of segments

When developing an imitative mathematical model of a mirror and facilitating the task of its subsequent calculation, a number of constructive assumptions and evaluation criteria are introduced.

In particular, we will assume that the segments and intersegment intervals of the mirror are located at equal and regular intervals over the entire composite surface area.

1) Tangent plane and position of segments.

The surface of each segment is determined by its curvature and the normal to it. For this reason, the position of one segment in space is equivalent to the position of the tangent plane constructed with respect to the center of symmetry of this segment in the chosen coordinate system. The tangent plane in space is given by three coordinates in a Cartesian or spherical coordinate system. The geometric position of each mirror segment is described along three linear directions in the Cartesian coordinate system.

2) Regular hexagon as a segment pattern.

Figure 2 shows a regular hexagon with six symmetries of reflection (symmetry of six lines) and six rotational symmetries (rotational symmetry of the sixth order). In general, the hexagon is the tallest regular polygon that allows regular segment mosaics to be performed.

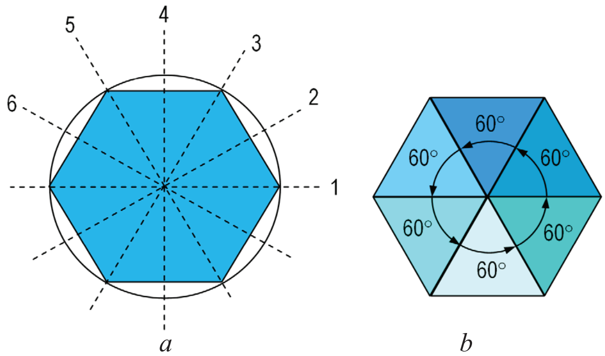


Figure 2 – Symmetry variants of a regular hexagon: *a* – reflex; *b* – rotational

Segmented mirrors with regular hexagons can significantly reduce the cost of manufacturing. The hexagonal shape of the segments has significant advantages from the technological point of view, since it facilitates the process of grinding and polishing the reflecting working surface of the mirror. In addition, when mounting segments on supports, convenient placement of actuators and sensors at the edge points is provided for their subsequent positioning and optimal position control [10].

3) The projection of segments and the coordinate system *OXYZ*.

The segmented mirror is described in such a way that when projecting onto the *XOY* plane, the individual segments are a circular array of regular and evenly spaced hexagons relative to the main optical axis collinear with the geometric axis *OZ* [11].

In the plane of the optical axis, two parameters determine the geometry of the projection of segments: the length of the side of the segment and the intersegment gap. The length of the side of the segment is determined by the distance between two adjacent vertices of the projected segment, while the intersegment gap is determined by the distance between the two sides of the adjacent projected segments.

4) Intersegment gaps and constraints imposed.

Intersegment gaps allow avoiding contact between adjacent segments and are assigned taking into account processing tolerances, in addition they allow compensation of gravitational and thermal deformations of the mirror cell [12]. The geometric size of the gap should be as small and uniform as possible across the entire width [13].

5) Segmentation approach.

In order to minimize the distortion of segments caused by the curvature of the aspherical mirror surface, two possible approaches to segmentation are proposed:

a) Consider the case of a mosaic with regular hexagons and equal gaps across the surface. According to Dan Curley's method [11], we make the assumption that all segments are identical flat hexagons, with the best relative position relative to the common aspherical surface. In the first approximation on a flat surface we create an array of identical regular hexagons separated by homogeneous gaps.

b) An alternative technique was proposed by Mast and Nelson for the TMT (Thirteen-meter telescope) [13], according to which hexagonal segments are not separated by spaces. An array of regular hexagonal segments of the same size forms a local surface curve and is placed in the *XOY* plane.

In this paper, the development of the algorithm for the geometric positioning of segmented mirrors was performed for the first segmentation version with divided air gaps.

6) Concentric rings.

Taking as a basis the symmetry of a regular hexagon, the configuration of each segment is performed through concentric rings from a central hexagon located on the entire surface (Figure 3). The simplest case is a segmented mirror of six hexagonal mirrors attached to the central hexagon. However, in the case of a large segmented mirror, it is necessary to add n concentric rings (i_1 ... i_n).

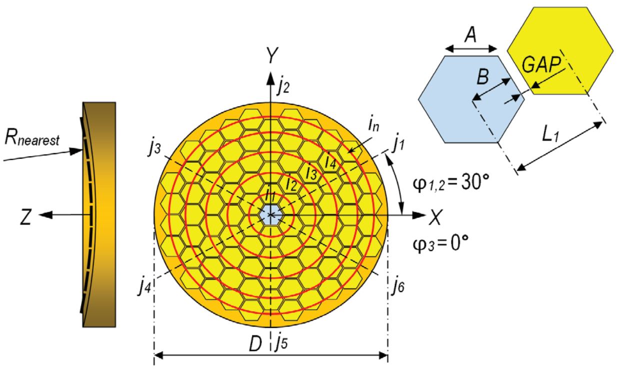


Figure 3 – Concentric rings of a hexagon in the *XOY* plane

The radius of the first ring R_1 is defined as the distance L_1 between the centers of the first hexagon and its adjacent hexagon, according to the following formula:

$$R_1 = \sqrt{3}A + GAP.$$

The length of the side of the hexagon as a function of apophema (apothem) is determined from the expression:

$$A = \frac{2\sqrt{3}}{3}B.$$

Then

$$R_1 = 2B + GAP,$$

where A and B are, respectively, the length of the side and the height of the hexagon; GAP is the air gap between two adjacent hexagons; R_{nearest} – the nearest radius to the aspherical surface. The angle φ is bounded by a ray from the origin of coordinates to the center of each segment and the *OX* axis in the plane *XOY*. The angle φ for the main diagonals for the first and second mosaic is φ = 30°, 90°, 150°, 210°, 270° and 330°. We use the index j_d to denote the basic diagonals of the arrangement. On the other hand, the is index represents each ring from the center to the last.

Thus, the radius of any subsequent ring n can be described by the following expression:

$$R_n = i_n(2B + GAP).$$

Figure 4 shows circles r_c and a circle of radius r_i of a hexagon, as well as diagonals. The radii are represented by the following formulas:

$$r_c = A,$$

$$r_i = \frac{\sqrt{3}}{2}A \quad \text{or} \quad r_i = \frac{\sqrt{3}}{2}r_c.$$

The diagonals are represented by the following formulas:

$$d_m = 2A \text{ or } d_m = 2r_c;$$

$$d_s = \sqrt{3}A \text{ or } d_s = 2r_i.$$

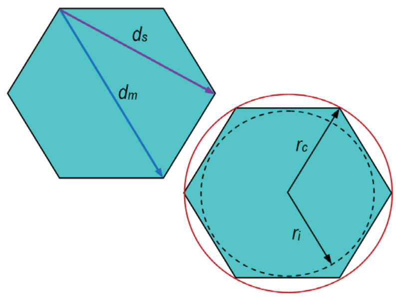


Figure 4 – Radii and diagonals of the hexagon

The longest diagonal d_m of a regular hexagon, connecting diametrically opposite vertices, is twice as long as one side. The short diagonal d_s is the line between the two vertices.

Equations for positioning a segmented mirror with an aspherical surface

The operation of the algorithm is reduced to checking the condition for ensuring the convergence of the current (actual) values of the center position of each segment of the mirror and its calculated values for the occurrence within the specified tolerance (Δ_X, Δ_Y, Δ_Z).

The trajectory of the «circumvention» of segments in the calculation is carried out according to the following schemes, shown in Figure 5 – three basic versions of the mosaic are shown with the distribution of segments over the surface of the mirror, taken into account in the calculation.

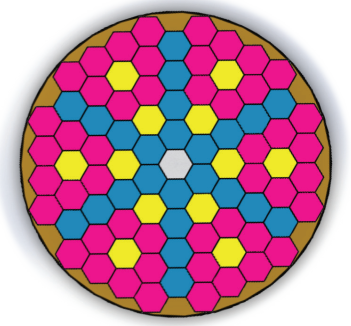


Figure 5 – Sequence distribution of segments of the mosaic when performing calculations for alignment of the aspheric surface: blue – multi-beam linear; yellow – multi-point; red – block trapezoidal

Taking into account the above considerations, the three general equations determine the geometry of the mosaics from the segmented mirrors, as described below.

The first multi-beam linear mosaic from segments

To calculate the coordinates of the mosaic segments from the first ring and the main (long) diagonals $j_1 \dots j_6$, we use the following equations:

$$X = i_{m\ (m=1\dots n)}\, L_1 \cos(\varphi);$$

$$Y = i_{m\ (m=1\dots n)}\, L_1 \sin(\varphi);$$

$$Z = (X^2+Y^2)/(2R_{\text{nearest}}),$$

where i_m – current value of the segment in the ray j_m.

The second multi-beam point mosaic from segments

To calculate the coordinates of the point diagonal mosaic segments shifted by 30 degrees relative to the first multipath circuit, we use the following equations:

$$X = i_{m(m=1\dots n)} \sqrt{3} L_1 \sin(\varphi);$$

$$Y = i_{m(m=1\dots n)} \sqrt{3} L_1 \cos(\varphi);$$

$$Z = (X^2+Y^2)/(2R_{\text{nearest}}).$$

For the two mosaic variants considered above, the calculation of the current values of coordinates in the cycle occurs until the following conditions are met:

$$T_i + A \le R_{\text{nearest}},$$

where R_{nearest} – radius of curvature of the nearest sphere to a parabolic surface; T_i – the distance, determining the central position of the current hexagonal segment, is determined by the formula:

$$T = \sqrt{X^2 + Y^2}.$$

Third block trapezoidal mosaic made of segments

To mosaic the remaining segments, we use the following equations:

$$X = kl\frac{\sqrt{3}}{2} L_1 \cos(\varphi) - T_1 \sin(\varphi);$$

$$Y = kl\frac{\sqrt{3}}{2} L_1 \sin(\varphi) - T_1 \cos(\varphi);$$

$$T_1 = (i + 1.5l)\, L_1;$$

$$Z = (X^2+Y^2)/(2R_{\text{nearest}}),$$

where k – the index, that transfers the negative current value of the X and Y coordinates to a positive value; $i = 1\dots S + A$ – range of calculated values of the current position of the segment; $l = 1\dots S$ – the current value of the segment in the trapezoidal block. The angle φ is set from the middle of the trapezoidal block within 0°…360° in 60°.

To implement the third mosaic, the following conditions must be met:

$$-1 \le k \le 1; \quad 1 \le i < S + A;$$

$$1 \le l < S; \quad T \le (R_{\text{nearest}}\, d_{prom}/d_m).$$

where d_{prom} – is the average value between longer and shorter diagonals.

The parameters entering into the expression are determined by the formulas:

$$S = R_{\text{nearest}}/L_1.$$

Simulation of a segmented aspherical mirror

Figure 6 shows a block diagram of the algorithm of the program module for forming a mirror working surface of a circular array of segmented hexagonal elements along the nearest sphere of the comparison surface.

The numerical values of the parameters used to simulate the composite segmented mirror are presented in Table.

Table

Design parameters of an aspherical composite mirror

The form	Hexagonal
Deformation value σ_s	−1
Radius of curvature of the nearest sphere R_{nearest}	9000 mm
Number of segments n	85 pieces
Intersegment gap *GAP*	10 mm
Length of a segment A	500 mm
Apothem B	433 mm
Mirror diameter D	9000 mm

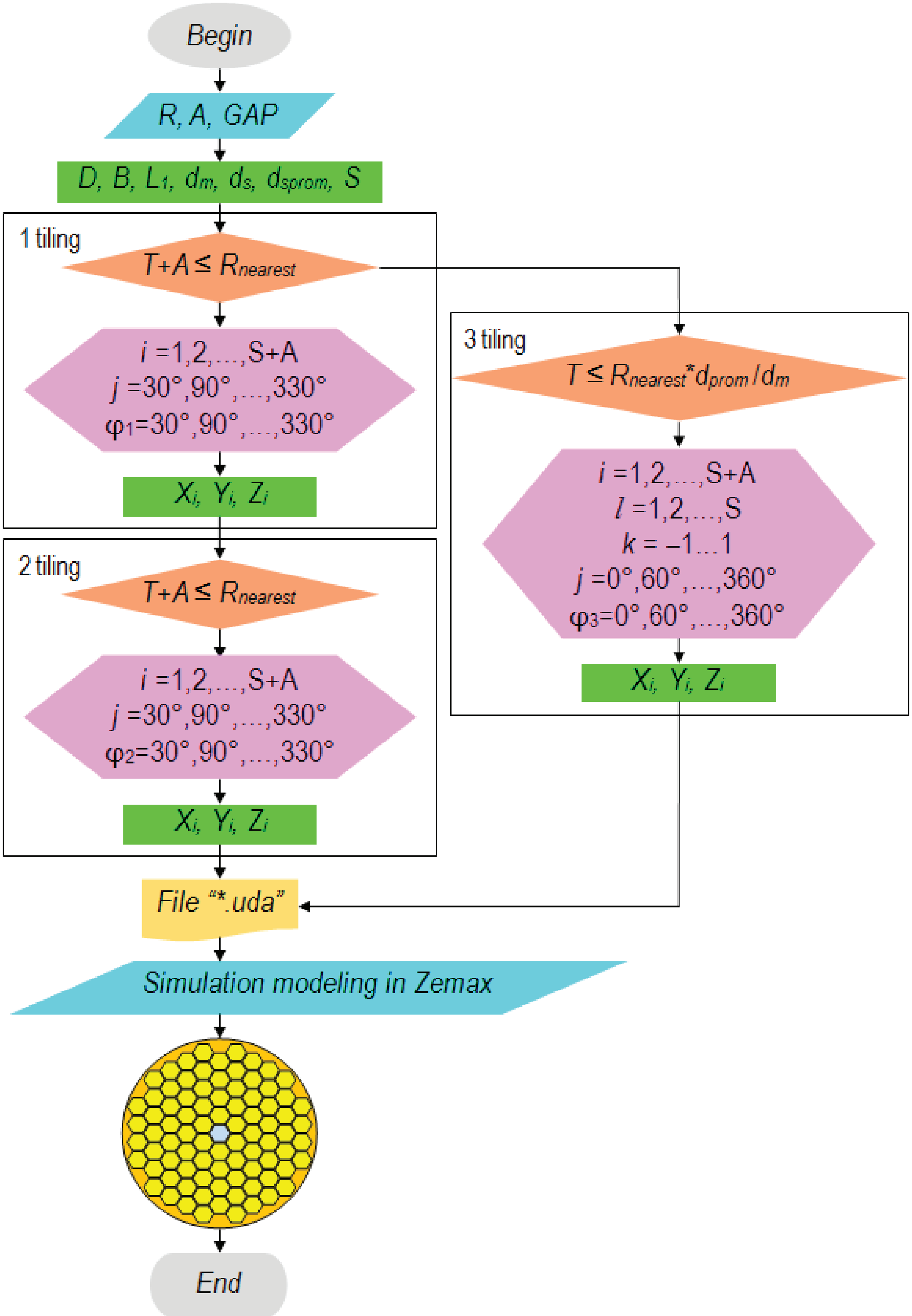
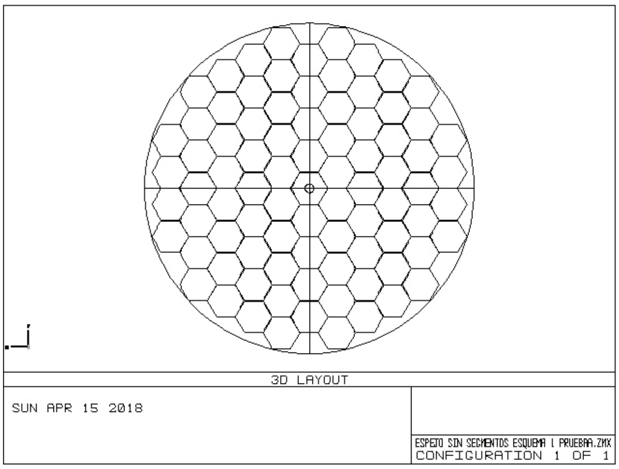
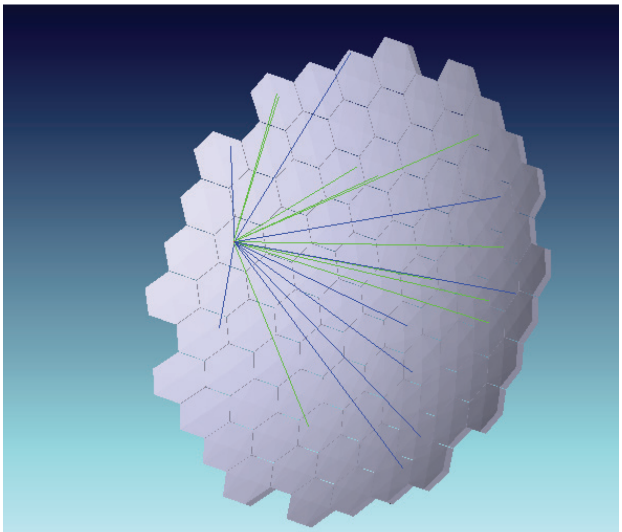


Figure 6 – Flowchart algorithm for geometric positioning of segmented mirrors

Starting with the mathematical model, the computer algorithm was developed in the *Python* programming language for geometric positioning of segments. Using the algorithm, we got one of the types of files with the «*.uda» extension, which shows a list with the positions of each segment in the *OXYZ* coordinate system. We used the resulting file in the *Zemax RFP* to simulate a composite mirror. Figure 7*a* shows the results of simulation of a segmented mirror with a parabolic surface in the *Zemax RFP*.



a



b

Figure 7 – Simulation modeling of a composite parabolic mirror: *a* – is the main projection; *b* – the spatial position of the segments in the correction of the wave front

Zemax allows you to simulate a mirror with an adaptive optical system in both sequential / mixed mode and in a purely non-sequential mode with the optimal position of the sloping segments to minimize aberrations. For the analysis, we introduce random aberrations on the input wavefront representing atmospheric effects, and optimize the slopes and *z*-positions of the segments to minimize aberrations in the image plane (Figure 7*b*).

Conclusion

Formulas for describing the coordinates of the centers of hexagonal segments are presented and boundary conditions are defined. A variant of a sequential «bypass» of segments of a mosaic of an array of elements is presented: multibeam linear; multi-point and block trapezoidal. An algorithm is developed to solve the problem of geometric positioning of flat hexagonal segments of a mirror telescope, which provides a minimum alignment time for the aspherical surface.

In the *Python* programming language, program loops are designed to form the data array necessary to construct a specular reflective surface of a given aperture. The work of the developed algorithm of geometrical positioning is checked. A circular array of 85 hexagonal elements is formed, equivalent to the diameter of a mirror with an aperture of 9000 mm.

In the software package *Zemax*, the convergence of optical beams from flat hexagonal segments to the central region of the aspherical surface is verified.

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