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MATHEMATICAL MODELLING OF THE UNMANNED AERIAL VEHICLE DYNAMICS

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The article gives a classification of the main components of unmanned aerial vehicle (UAV) systems, gives the areas in which the application of UAVs is actual in practice today. Further, the UAV is considered in more detail from the point of view of its flight dynamics analysis, the equation necessary for creating a mathematical model, as well as the model of an ordinary dynamic system as a non-stationary nonlinear controlled object, is given. Next, a description of the developed software for modeling and a description of program algorithm are given. Finally, a conclusion describes the necessary directions for further scientific researches.

Keywords: unmanned *aerial vehicle, mathematical modeling, stochastic model of system state.*

Introduction

Unmanned aerial vehicle – a fairly new direction for research and are of interest not only for practical civil application like in Urban Search and Rescue Operations [1], but also for military operations with a high degree of success [2].

According to the classification of Global Hawk RYAN aeronautical center USA, the schematic structure of unmanned aerial vehicle is (Fig. 1):

There are unmanned aerial vehicles for various purposes, a variety of aerodynamic schemes and with a variety of tactical and technical characteristics.

Theory analysis

In general, the UAV can be used in a wide variety of human activities, such as:

– archeology (search under a layer of sand);

– architecture (autonomously survey the terrain and create 2D- and 3D-maps and terrain models);

– aerial photography (UAVs allow you to create digital maps with virtually any resolution, ranging from a few centimeters to a point);

– safety monitoring (for example, during construction works);

– urban infrastructure (search for unauthorized dumps, detection of illegal buildings, quality control of the road surface, taking air samples, measuring radio emission levels);

– Forestry (fighting poachers, identifying fires, smoke, monitoring obstacles, monitoring animals);

– meteorology (search and / or study of hurricanes and other natural phenomena), etc.

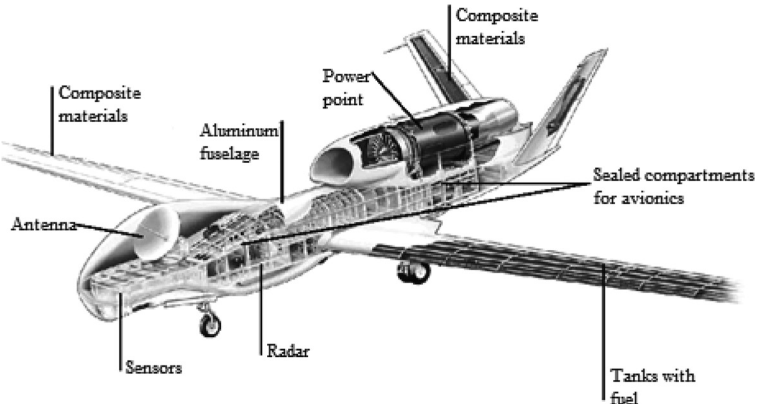


Fig. 1. Schematic structure of unmanned aerial vehicle

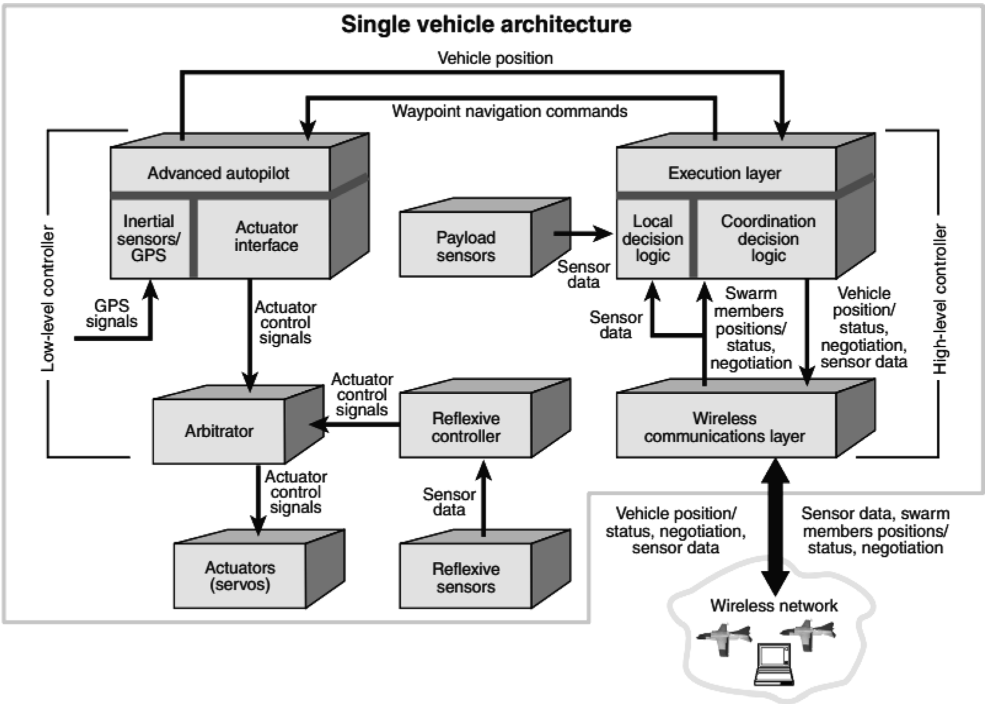


Fig. 2. Advanced structure of the UAV

Law enforcement officers and rescuers are also thinking about using UAVs in everyday practice.

An unmanned aerial vehicle can, for example, provide search for accidents (accidents) of technical equipment and missing groups of people. The search is conducted according to a pre-set flight task or by an operative route of the flight. Usually, the UAV is equipped with guidance systems, airborne radar systems, sensors and video cameras. Advanced structure of the UAV systems [3] is shown in Figure 2.

The success of the UAV is primarily due to the rapid development of microprocessor-based computers, control systems, navigation, information transfer and even artificial intelligence elements. Achievements in these areas make it possible to fly in automatic (semi-automatic) mode from take-off to landing, to solve a very wide range of tasks [4].

Consider the UAV in more detail from the point of view of the analysis of its flight dynamics.

Unmanned aerial vehicle, like other aircraft, uses its power point and aerodynamic forces for its flight in the atmosphere.

In general, the UAV movement is represented by two aspects: the forward movement of its center of mass and the rotational motion around its center of mass.

To simulate an unmanned aerial vehicle, the necessary requirements, according to V. S. Moiseev, is: «... except for their adequacy and sufficient for practical application of accuracy, we will consider the simplicity and intelligibility of models to the specialists in managing UAVs...

... The fulfillment of this requirement is conditioned by the necessity of their active participation in the development on the basis of these models of effective control laws for UAVs.

In addition, the simplicity of the applied UAV motion models implies, as practice has shown, the relatively low laboriousness of mathematical methods and algorithms used in the formation of such laws...» [5, pp. 35–37].

The model chosen should correspond to the tasks in the simulation and adequately reflect the relationship between them.

The functioning of a complex system occurs when input random signals (influences, functions, processes) are affected, as well as various random disturbances (interference).

The mathematical model of a stochastic system is described on the basis of information interaction between its constituent parts. Such a model can be represented physically by various mathematical models, depending on the completeness and degree of detail of the processes and the final research task.

In engineering practice and scientific research to describe systems from the point of view of the completeness of the description, the object is described in terms of the state space. The state of an object is understood to mean the totality of the values of xi, which completely determine its position at a given instant of time.

The most common model of dynamic objects are differential equations. It is most convenient to consider objects that are described by ordinary differential equations. The order of the system of differential equations describing the model of the object is not directly determined by the number of inputs and outputs, but depends on the operators that convert the input signals to the output [6].

Considering the above, we will describe the equation of an unmanned aerial vehicle, which is necessary for creating a mathematical model:

The state of systems for which initial states and input effects are known can be described by the following equation:

$$x(t) = f[t,\ t_0,\ x(t_0),\ \xi(t_0,\ t)], \qquad (1)$$

where $x(t)$ – a set of values of the state vector that describes the region of possible states of the system; f[¼] – transfer function of the system state; $\xi(\ldots)$ – function describing the pattern of the input effects, according to the type of solving problem.

In the modeling of dynamic systems it is important to consider the system structure and interaction of the system with the environment. $\xi(\ldots)$ parameters must be taken into account [7].

The totality of the effects of the environment to the object can be divided into two groups, according to the type of the influence of the environment to the state variables (phase coordinates) of the object.

The first group includes influences that additively change the state variables at the application point.

This means that signals proportional to these influences are summed with corresponding state variables.

The second group of environmental influences changes the state variables of the object indirectly, usually not additively.

Those effects change the operator of the object (system). This means the transformation law of input effects into output variables of the object [6].

An example of a model of an ordinary dynamic system as a nonstationary nonlinear controlled object with perturbations ξ (...) can be as follows:

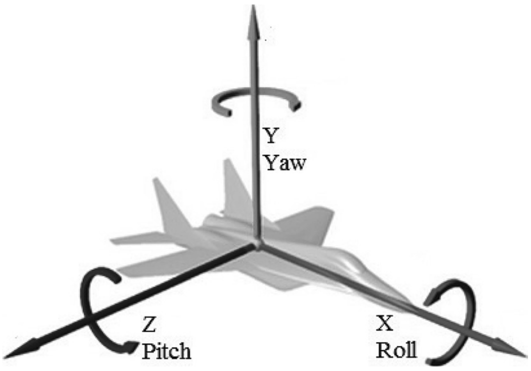


Fig. 3. Projection of the axes of the trajectory coordinate system on the UAV [8]

$$\frac{d}{dt}x(t) = f\left(t, x(t), u(t), \xi(\tau)\right), \qquad y(t) = A(t, x(t), u(t)), \qquad (2)$$

where $u(t)$ – the control vector or input variables; $y(t)$ – vector of output variables of systems.

To obtain a correct stochastic model of the state of a system with continuous time, it is necessary to use stochastic differential equations. Consequently, after appropriate transformations, the stochastic differential equation for the consideration model will have the following form [7]:

$$dx = A(x,t)dt + B(x,t)d\xi,, \qquad (3)$$

Further, in order to simulate an unmanned aerial vehicle, it is necessary to determine the coordinate system. As a basis, take the trajectory coordinate system, the projection of which axes on the UAV looks as follows (Figure 3).

As mentioned in the beginning, the UAV motion can be simplified as a motion of its center of mass, which will serve as a reference point in the trajectory reference system (*Ot*) [9].

Denote the axes of the trajectory system *Xt*, *Yt* and *Zt*, respectively; the roll angle from the axis *Xt* – *Kx*, the angle of yaw or slip (the angle of the path) – *Ry*, the pitch or attack angle (inclination of the trajectory) – *Tz*, then the dynamic equation of motion in general form will look as follows [10]:

$$\begin{cases} m\dot{V} = \sum Fx, \\ mV\dot{T}z = \sum Fy, \\ -mV\dot{R}y\cos(Tz) = \sum Fz, \end{cases} \qquad (4)$$

Figure 4 shows a comparison of the trajectory and terrestrial coordinate systems, as well as the

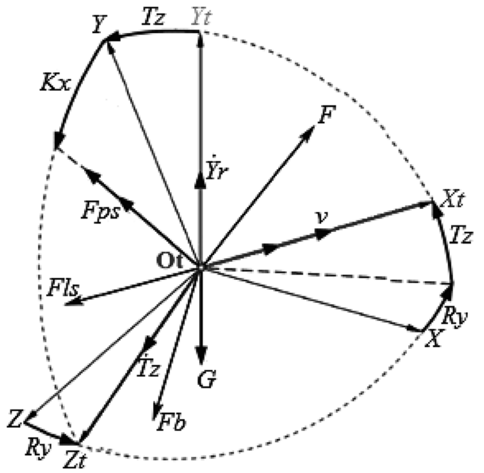


Fig. 4. Comparison of the trajectory and terrestrial coordinate systems with the projections of the forces acting on the UAV

basic forces acting on the UAV in flight (with a projection onto the trajectory coordinate system) and the angles of deviation from the axes of the normal coordinate axis:

In Figure 4, the following notations are introduced:

– *Fls* – force of frontal resistance of air;

– *Fps* – lifting force;

– *Fb* – lateral force;

– *F* – power point force of the UAV;

– *G* – gravity force is multiplication of the mass of the UAV and acceleration of gravity *g* (9.81 m/s^2);

– *v* – course speed.

Denote the pitch angle of the UAV – *Tza*, the yaw angle of the UAV – *Ryb* and, taking into account the forces acting on the UAV (Figure 4), we rewrite equation 4 in the following form:

$$\begin{cases} m\dot{V} = F\cos(Tza)\cos(Ryb) - FlsG\sin(Tz), \\ \mathrm{mV\dot{T}z} = \left(Fps + F\sin(Tza)\cos(Kx) - \right. \\ \left. -\left(Fb - F\cos(Tza)\sin(Ryb)\right)\sin(Kx) - G\cos(Tz)\right), \\ -mV\dot{R}y\cos(Tz) = \left(F\sin(Tza) + Fps\right)\sin(Kx) + \\ +\left(Fb - F\cos(Tza)\sin(Ryb)\right)\cos(Kx). \end{cases} \tag{5}$$

Software development

The resulting equations of flight dynamics were used as the basis for the creation of a mathematical model of the UAV flight of. The mathematical model was implemented on an object-oriented high-level language – C # 5.0 [11, 12].

To build a graphical user interface, the program used WPF technology [13].

The program is designed to measure the distance of flight of an unmanned aerial vehicle with a limited fuel, on ideal conditions flight with rectilinear motion in steady state (after all the adjustments associated with flying to a given course) and to compare the distance of flight of the UAV with wind influence.

The algorithm of the program includes the following basic steps:

1) input of initial data, including:

– current speed (km / h);

– direction of flight (in degrees);

– minimum and maximum values of wind power (m/s);

– flight time (*Tmax*) (s), etc.

2) calculation of a number of parameters (the main ones are listed):

– power point speed (according to the condition of the simulation, they correspond to the set speed);

– wind power at each iteration (a randomly generated sequence of values between the limits (*WPmin* and *WPmax*), specified by user entering data).

The wind model obtained at this stage is shown in Figure 5;

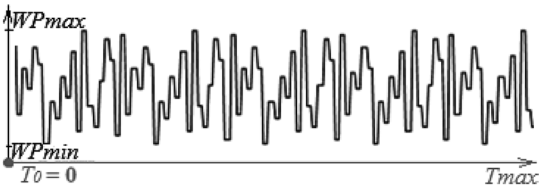
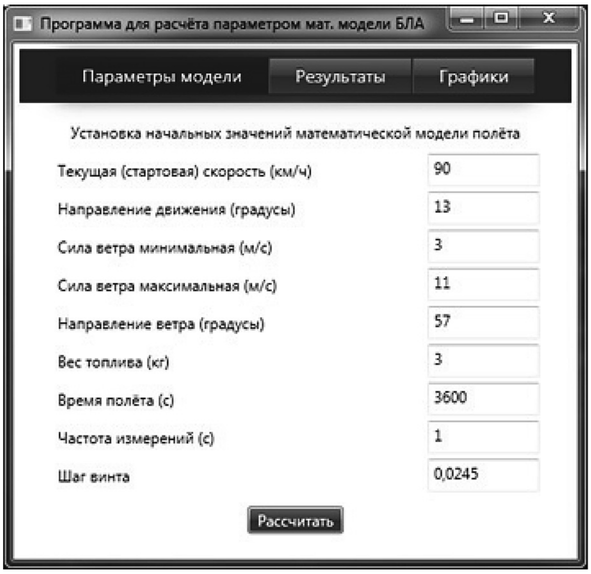


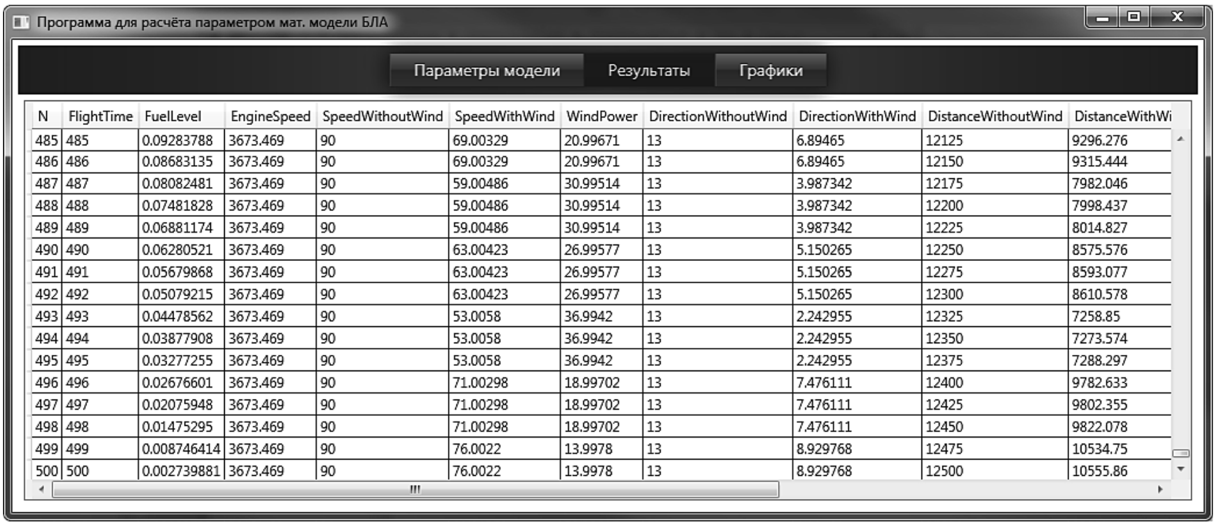
Fig. 5. Wind model obtained by applying the algorithm of random numbers generation in C#



Установка начальных значений математической модели полёта

Текущая (стартовая) скорость (км/ч) 90
Направление движения (градусы) 13
Сила ветра минимальная (м/с) 3
Сила ветра максимальная (м/с) 11
Направление ветра (градусы) 57
Вес топлива (кг) 3
Время полёта (с) 3600
Частота измерений (с) 1
Шаг винта 0,0245

Fig. 6. Main program window



N	FlightTime	FuelLevel	EngineSpeed	SpeedWithoutWind	SpeedWithWind	WindPower	DirectionWithoutWind	DirectionWithWind	DistanceWithoutWind	DistanceWithWi
485	485	0.09283788	3673.469	90	69.00329	20.99671	13	6.89465	12125	9296.276
486	486	0.08683135	3673.469	90	69.00329	20.99671	13	6.89465	12150	9315.444
487	487	0.08082481	3673.469	90	59.00486	30.99514	13	3.987342	12175	7982.046
488	488	0.07481828	3673.469	90	59.00486	30.99514	13	3.987342	12200	7998.437
489	489	0.06881174	3673.469	90	59.00486	30.99514	13	3.987342	12225	8014.827
490	490	0.06280521	3673.469	90	63.00423	26.99577	13	5.150265	12250	8575.576
491	491	0.05679868	3673.469	90	63.00423	26.99577	13	5.150265	12275	8593.077
492	492	0.05079215	3673.469	90	63.00423	26.99577	13	5.150265	12300	8610.578
493	493	0.04478562	3673.469	90	53.0058	36.9942	13	2.242955	12325	7258.85
494	494	0.03877908	3673.469	90	53.0058	36.9942	13	2.242955	12350	7273.574
495	495	0.03277255	3673.469	90	53.0058	36.9942	13	2.242955	12375	7288.297
496	496	0.02676601	3673.469	90	71.00298	18.99702	13	7.476111	12400	9782.633
497	497	0.02075948	3673.469	90	71.00298	18.99702	13	7.476111	12425	9802.355
498	498	0.01475295	3673.469	90	71.00298	18.99702	13	7.476111	12450	9822.078
499	499	0.008746414	3673.469	90	76.0022	13.9978	13	8.929768	12475	10534.75
500	500	0.002739881	3673.469	90	76.0022	13.9978	13	8.929768	12500	10555.86

Fig. 7. Output results after calculation of UAV flight at 90 km/h speed in ideal conditions and with the wind power between 3 and 11 m/s influence

– shift angle of the UAV (calculated in the program as multiplication of a 60 constant value to the quotient of the wind speed divided by the speed of the flight, multiplied by the sine of the wind angle);

– speed of flight with an additive wind application;

– directions of UAV flight with continuous wind influence;

– the distance in meters (until the end of the UAV fuel or the completion of the preset simulation time);

3) the output of the results in a tabular (in the form of a series of columns of parameters grouped by pairs (for comparison) describing the UAV flight in ideal and with influence of wind conditions) and, in part, graphically.

The main program window is shown in Figure 6.

An example of the output results from the program is shown in Figure 7.

Conclusion

In this paper, the UAV was considered as description of its dynamics during the flight in ideal conditions and with wind influence to it.

Further research will continue in the field of modeling the automatic control system in relation to the mathematical model of an unmanned aerial vehicle described in this paper, and the described model is improved to construct a more realistic wind model with consideration of longitudinal overload (occurs when the power point thrusting and flight shifting) and lateral overload (occurs when flying with slip) and other influences, and power point, navigation equipment, etc.

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Степанов В. Ю.

МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ДИНАМИКИ БЕСПИЛОТНОГО ЛЕТАТЕЛЬНОГО АППАРАТА

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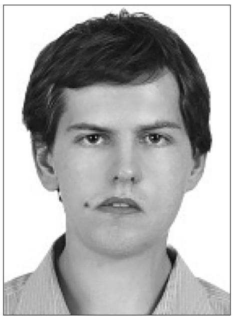
В статье даётся классификация основных компонентов систем беспилотного летательного аппарата (БЛА), приводятся области человеческой деятельности, в которых актуально применение БЛА на практике уже сегодня. Далее БЛА рассматривается более подробно с точки зрения анализа его динамики полёта, даётся уравнение, необходимое для создания математической модели, а также модели обыкновенной динамической системы как нестационарного нелинейного управляемого объекта. Далее даётся описание разработанного программного обеспечения для моделирования и приводится описание алгоритма работы программы. В конце делается вывод о необходимых направлениях дальнейших научных исследований.

***Ключевые** слова: беспилотный летательный аппарат, математическое моделирование, стохастическую модель состояния системы.*



Степанов В. Ю. Аспирант кафедры «Информационные системы и технологии» Белорусского национального технического университета. Получил высшее образование и окончил магистратуру по направлению «Информационные системы и технологии» в 2013 и 2014 годах соответственно. На данный момент аспирант ведёт исследования в направлениях цифровой обработки сигналов и систем управлений, включая параметрическую оптимизацию систем и программирование. Интересы в целом касаются ИТ, программирования и робастного управления.

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