APERIODIC PALINDROMES AND CONNECTED CIRCULANT GRAPHS

Dongseok Kim dongseok@kgu.ac.kr

Kyonggi University, Korea

A compositionσ = σ1σ2 . . . σm of n is an ordered word of one or more positive integers whose sum is n. The number of summands m is called the number of parts of the composition σ. A composition of n without order gives a partition of n. A composition of n with k parts is aperiodic if its period is k. In other words, a composition is aperiodic if it is not the concatenation of a proper part of given composition. A composition σ = σ1σ2 . . . σm is called prime if gcd(σ) = gcd{σ1, . . . , σm} = 1. A numeral palindrome (or simply, palindrome) is a composition in which the summands in given order are the same with those in the reverse order (i.e., σ = σ−1 where σ−1 = σm . . . σ2σ1 for σ = σ1σ2 . . . σm). It is known that the number of palindromes of n ≥ 2 is 2⌊ n/2 ⌋. The first 30 palindromes in decimal can be found as the sequence A002113 in OEIS. Several types of palindromes are studied. The number of aperiodic palindromes of n with k parts (1 ≤ k ≤ n) is studied but the numbers are known only for n ≤ 55 (see OEIS). However it is still unknown whether there exist infinitely many palindromic primes or not, where a palindromic prime is a positive integer which is prime and also a palindrome. Although palindromes are often considered in the decimal system, the concept of palindromicity can be generalized to the natural numbers in any numeral system. An integer m>0 is called palindromic in base b≥2 if it is written in standard notation.

A circulant graph is a graph whose automorphism groupincludes a cyclic subgroup which acts transitively on the vertex set of the graph. For a subset S ⊆ Zn satisfying S = -S mod n, a circulant graph of order n denoted by G(n,S) is a graph with vertex set {0, 1, …, n-1} and edge set E, where{i, j}is in E if and only if i is not equals to j and j-i is in S mod n. A circulant digraph is also defined without the condition S=-S. That is, for a subset S ⊆ Zn a circulant digraph G(n,S) is a digraph with vertex set {0, 1, …, n-1} and arc set A, where (i,j) is in A if and only if i is not equals to j and j-i is in S mod n. Isomorphism problem of circulant graphs had been studied by several authorsand it is completely solved by Muzychuk. Recently, Kim, Kwon and Lee found degree distribution polynomials for the equivalence classes of circulant graphs of several types of order. They also found an enumeration formula for the number of equivalence classes of circulant graphs and they listed the degree distribution polynomials and the number of equivalence classes of circulant graphs for 1 ≤n ≤20. We observe that the number of equivalence classes of circulant graphs is equal to the number of aperiodic palindromes of n for 1 ≤n ≤ 20. This leads us to study a coincidence between the set of circulant graphs of order n and the set of aperiodic palindromes of n. In this paper, we study the coincidence and we extend this to circulant digraphs of order n and compositions of n.

We first shows that there is a one-to-one correspondence between the set of compositions of n and the set of circulant digraphs of order n. In particular, we also show that this bijection guarantees a one-to-one correspondence between the set of prime compositions of n and the set of connected circulant digraphs of order n. As an application, we enumerate the number of connected circulant digraphs (i.e., the number of prime compositions), disconnected circulant digraphs and circulant digraphs of outdegree k. Next we first show that there is a one-to-one correspondence between the set of palindromes of n and the set of circulant graphs of order n. In particular, we also show that this bijection in Theorem 3.1 guarantees a one-to-one correspondence between the set of aperiodic palindromes of n and the set of connected circulant graphs of order n. As a corollary, we give an enumeration formula of the number of aperiodic palindromes of n.