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Several Problems of the Graph Theory

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In mathematics graph theory is the study of graphs, which are mathematical structures used to model pairwise relations between objects. A graph in this context is made up of vertices, nodes, or points which are connected by edges, arcs, or lines, so a graph is a representation of a set of points and of how they are joined up, and any metrical properties are irrelevant [1].

In recent years, graph theory has established itself as an important mathematical tool in a wide variety of subjects, ranging from operational research and chemistry to genetics and linguistics, and from electrical engineering and geography to sociology and architecture. At the same time it has also emerged as a worthwhile mathematical discipline in its own right [2].

There are several problems in the Graph Theory, which demonstrate the specifics of the problems in this field of mathematics.

The first problem is about seven bridges of Konigsberg: the city of Konigsberg, Prussia (now Kaliningrad, Russia) was set on both sides of the Pregola river. There were two islands on the river and there were seven bridges connecting them and the mainland.

Citizens observed that they could not cross all the bridges only once. They had to skip one bridge or cross some bridges twice. Some of them conjectured that it was impossible to cross the seven bridges once and only once, but they could not explain why.

The problem was submitted to Leonard Euler, one of the most famous mathematicians that time. Euler proved that there was no solution to the problem; that is, there was no way to cross the seven bridges exactly once.

The second problem called “the Travelling Salesman problem” (TSP) is about finding a route: given a list of cities and the distances between each pair of cities. What is the shortest possible route that visits each city exactly once and returns to the origin city? It is an NP-hard problem in combinatorial optimization, important in operations research and theoretical computer science [3].

The TSP has several applications even in its purest formulation, such as planning, logistics, and the manufacture of microchips. Slightly modified, it appears as a sub-problem in many areas, such as DNA sequencing. In these applications, the concept city represents, for example, customers, soldering points, or DNA fragments, and the concept distance represents travelling times or cost, or a similarity measure between DNA fragments. The TSP also appears in astronomy, as astronomers observing many sources will want to minimize the time spent moving the telescope between the sources. In many applications, additional constraints such as limited resources or time windows may be imposed [4].

The last problem is about a graph coloring, which is an assignment of labels or colors to each vertex of a graph such that no edge connects two identically colored vertices. The most common type of vertex coloring seeks to minimize the number of colors for a given graph. Such a coloring is known as a minimum vertex coloring, and the minimum number of colors which with the vertices of a graph may be colored is called the chromatic number [5].

There is a theorem known as “The Four Color Theorem” that associated with the graph coloring problem. The search for a proof of the four color theorem — stating that every planar map can be colored with four colors such that adjacent countries receive different colors has certainly been one of the driving sources of graph theory for a long time. Presently, graph coloring plays an important role in several real-world applications and still engages exciting research.

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