Refractive index determination for a plane dielectric layer using the measurements of transmitted beam intensity

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Abstract

The aim of the present work is the theoretical justification of new refractive index determination technique for a homogeneous transparent plane dielectric layer. It uses intensity measurements for two polarizations of transmitting electromagnetic beam and does not take into consideration phase relationships and phase parameters of testing field at unknown thickness of a layer.

For this purpose, the layer transmission energy coefficients for two linear polarizations of an electromagnetic beam, orthogonal and parallel to the plane of incidence, are studied to be dependent on the layer thickness and its refractive index. We have found the function of energy transmission coefficients for these polarizations, which does not depend on the layer thickness and is characterized by monotonic dependence on its refractive index.

It is shown that this function provides the opportunity to determine the layer refractive index uniquely. It can be made analytically using the inverse function, and also with the help of experimental calibration technique for the initial function. The influence of losses on the method efficiency is investigated, and it is established, that the presence of absorption causes appearance of separated zones of refractive index variation, where the method becomes inoperative. However, at absorption index values of the order of 10^{-5} the method can be applied, but in the bounded domain of refractive index variation.

So, it is established that the proposed methods provides the opportunity to determine the refractive index of a plane dielectric layer under conditions of low and null value of absorption using the intensity measurements for two orthogonal polarizations of transmitting electromagnetic radiation.

Keywords: refractive index measurement, energy transmission coefficients, transparent dielectric, low absorption.

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Определение показателя преломления плоского диэлектрического слоя методом измерения интенсивностей проходящих пучков

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Целью работы является теоретическое обоснование нового метода определения показателя преломления однородного прозрачного плоского диэлектрического слоя по измерению интенсивности двух поляризаций проходящего пучка электромагнитного излучения без учета фазовых соотношений и фазовых параметров тестирующего поля при неизвестной толщине слоя.

Для этого рассматривались энергетические коэффициенты прохождения слоя для двух линейных поляризаций пучка электромагнитного излучения, перпендикулярной и параллельной плоскости падения, и изучались их зависимости от показателя преломления слоя и его толщины. Найдена функция энергетических коэффициентов прохождения двух разных поляризаций, которая не зависит от толщины слоя и характеризуется монотонной зависимостью от его показателя преломления.

Показано, что применение данной функции позволяет однозначно определить искомый показатель преломления слоя по измерениям интенсивностей двух ортогональных поляризаций проходящего излучения. Это можно сделать аналитически, используя обратную функцию, а также с помощью экспериментальной калибровки исходной функции. Изучено влияние поглощения на эффективность применения метода и установлено, что его присутствие вызывает появление отдельных зон изменения показателя преломления, где метод перестает работать. Однако если его величина не превышает значений порядка 10^{-5}, то метод может применяться, но уже в ограниченном интервале изменения показателя преломления.

Таким образом, доказано, что предлагаемый метод дает возможность определять показатель преломления диэлектрического слоя в условиях малого и нулевого поглощения по измерениям интенсивностей проходящего электромагнитного излучения двух ортогональных поляризаций.

Ключевые слова: измерение показателя преломления, энергетический коэффициент прохождения, прозрачный диэлектрик, малое поглощение.

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Introduction

The problem of refractive index measurement for various dielectric materials and substances is of great importance for all fields of science and industry, which use and study the phenomenon of electromagnetic waves propagation through matter [1–3]. For its solving, one applies various techniques. There are refractometric ones (using also the phenomenon of total internal reflection) [1, 2–4], interferometric methods (utilizing the phase relationships between various coherent fields after transmission and reflection from the testing material) [1, 2, 5, 6] and others [7–9]. This topic acquires new importance last years due to development of study of heterogeneous disperse systems, i.e. materials produced by macro- and microparticles of different phases having developed interfaces (soils, clouds, food-stuffs, cosmetic products, building, wood, paper materials and so on) [10, 11]. Using measurements of averaged dielectric permittivity for such materials under various conditions and over various regions of electromagnetic radiation, one can investigate their physical properties, for instance, concentration and relative position of compounding phases, physical state and etc (see, for example, [11]).

Among the collection of refractive index determination techniques one can highlights the ellipsometric ones [8], which allows for solving the problem of refractive index determination for various homogeneous and layered media in the wide region of electromagnetic spectrum at unknown thickness of different layers. The name of the technique reveals its essence, when sought parameters of tested medium are determined by measurements of polarization ellipse parameters for reflected or transmitted beam. However, the ellipsometric technique produces low accuracy at very small absorption [12], besides, it uses complicated algorithm of refractive index computation. In the present work, we present an additional method of this index determination for transparent and low absorbing plane materials, based on field intensity measurements without taken into account phase relationships, but distinguished by simplicity of realization. It uses results of intensity measurements for two transmitting orthogonal polarizations, parallel and orthogonal to the plane of beam incidence, and employs simple analytical equations at determining sought refraction index and does not require additional information about thickness of a medium and another its physical properties.

Description of the method

Let a beam of electromagnetic radiation of the frequency ω be incident on a plane dielectric layer of the thickness h. As it is known, for two orthogonal polarizations of a plane wave, the amplitude transmission coefficient for a dielectric layer is determined by the expression [13–15]:

$$T_{H,E} = D^{-1} T_{vd} T_{dv} \exp(ik\gamma h), \qquad (1)$$

where $i = (-1)^{1/2}$ is the imaginary unite; $k = \omega/c$ is the wavenumber;

$$D = 1 - R_{vd}^2 \exp(2ik\gamma h),$$

R_{vd}, T_{vd} and T_{dv} are the amplitude coefficients of plane wave reflection and refraction on the plane boundaries «air (vacuum) – dielectric» and «dielectric – air»:

$$R_{vd} = \frac{\varepsilon^{\theta}\beta - \gamma}{\varepsilon^{\theta}\beta + \gamma}; \quad T_{vd} = \frac{2\beta}{\varepsilon^{\theta}\beta + \gamma}; \quad T_{dv} = \frac{2\varepsilon^{\theta}\gamma}{\varepsilon^{\theta}\beta + \gamma};$$

(the Fresnel formulae), written in terms of the normal propagation parameters [12–15]. Here, ε is the dielectric permittivity of layer material at the frequency of incident radiation, $\beta = \cos\varphi$ and $\gamma = (\varepsilon - 1 + \beta^2)^{1/2}$ are the parameters of normal propagation for a plane wave in air and in a dielectric, respectively, φ is the angle of wave incidence on the surface of a dielectric layer, $\theta = 0$ for the H (or TE) polarization of incident wave, when its electric vector is orthogonal to the plane of incidence, and $\theta = 1$ for the E (or TM) polarization, whose electric vector lies in this plane.

Usually, the Fresnel formulae are derived theoretically for incident field presented by a plane electromagnetic wave, i.e. for monochromatic radiation with determined direction of propagation in space [12–15]. However, scientific experience show, that as these formulae, as the formula (1) are valid for more general case, when radiation is presented by a spatially bounded beam or by superposition of waves with various frequencies from narrow bounded frequency region. Then, for the temporal ω and spatial β frequencies of propagation, one takes averaged values of these parameters over the temporal and spatial (angular) spectrum of incident field (see, for example, [16]).

The expression (1) can be transformed to the form:

$$T_{H,E} = \left[\cos k\gamma h - i(2\varepsilon^{\theta}\beta\gamma)^{-1}(\varepsilon^{2\theta}\beta^2 + \gamma^2)\sin k\gamma h\right]^{-1}. \quad (2)$$

Assume that a layer is transparent, i.e. the permittivity ε is real number. Then, the parameter γ of wave propagation inside a layer is also real number, and the coefficient of energy transmission through the layer, which equals to the squared absolute value of the amplitude coefficient (2), is determined by the equation:

$$|T_{H,E}|^2 = \left[1 + (2\varepsilon^{\theta}\beta\gamma)^{-2}(\varepsilon^{2\theta}\beta^2 - \gamma^2)^2\sin^2 k\gamma h\right]^{-1}. \quad (3)$$

The plots on Figure 1 demonstrate the dependence of the energy transmission coefficients (3) on the refractive index of a layer. From here one can see that this dependence is almost periodic.

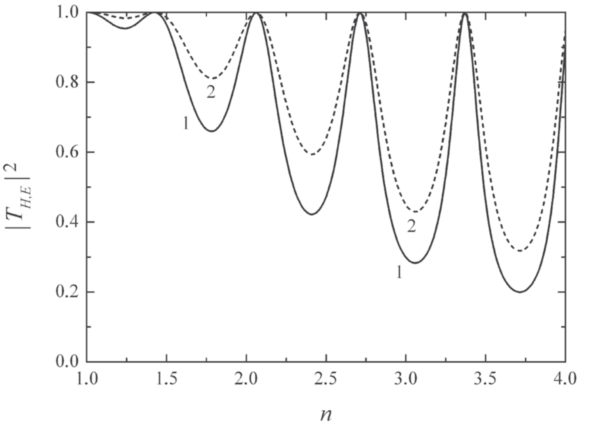

Figure 1 – Energy coefficients of transparent dielectric layer transmission for two various polarizations of incident radiation $|T_H|^2$ (curve 1) and $|T_E|^2$ (curve 2) as functions of the refractive index n of a layer at its thickness h = 0.75λ, when the angle of incidence φ = 30º

The analogous dependences on the layer thickness are wholly periodic with the small period. This circumstance is inhibitory to solve an inverse problem of refractive index determination for a testing dielectric layer over the wide region, if one can use the energy transmission coefficient only of one polarization. However, using of data on measuring of such coefficients for two different polarizations, provides the opportunity to solve this problem independently on the layer thickness, like it made in [17, 18] for determining of the absorption coefficient using the amplitude coefficients of reflection and refraction. Note that in the values $|T_H|^2$ and $|T_E|^2$ (3), one can construct the function, which is not dependent on the thickness:

$$f(T_H, T_E) = \frac{g(T_H)}{g(T_E)} = \frac{\varepsilon}{\beta^2(\varepsilon + 1) - 1}, \quad (4)$$

where:

$$g(T_{H,E}) = \left(|T_{H,E}|^{-2} - 1\right)^{1/2}. \quad (5)$$

The obtained simple equation allows for calculation of the refractive index for a dielectric layer using numerical value of the function (4):

$$n = \sqrt{\varepsilon} = \left(\frac{(1-\beta^2)f(T_H, T_E)}{\beta^2 f(T_H, T_E) - 1}\right)^{1/2}. \quad (6)$$

It is obvious that the case of normal incidence on a layer (φ = 0) is excluded here, because in this case β = 1, and the energy transmission coefficients (1), (2) for two orthogonal polarizations are equal one to the other.

Figure 2 demonstrates the dependences of functions $f(T_H, T_E)$ (4) and $g(T_{H,E})$ (5) on the refractive index of a layer. The first of them is monotonic, what provides the opportunity to use that for refractive index determination as by the method of experimental calibration directly on a curve, as using analytical technique according equation (6) (the corresponding curve is also shown on Figure 2).

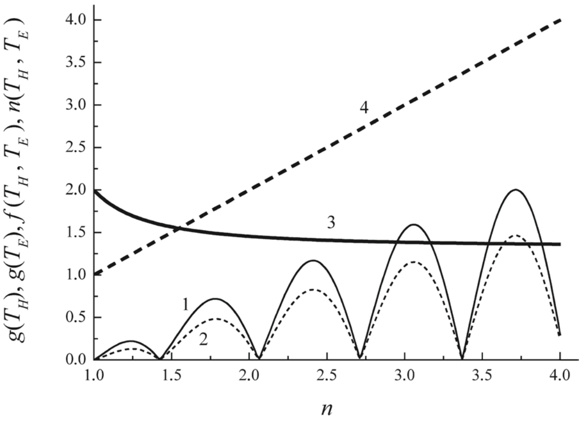

Figure 2 – Dependence of functions $g(T_H)$ (curve 1), $g(T_E)$ (curve 2), $f(T_H, T_E)$ (curve 3) and $n(f(T_H, T_E))$ (curve 4), computed by (5), (4) and (6), on the refractive index n of a transparent dielectric layer, when its thickness h = 0.75λ and the angle of incidence φ = 30º

The inclination angle for the curve, depicting the function $f(T_H, T_E)$ (4), decreases under increase of the refractive index, what corresponds to decrease of sensitivity of the discussed technique. To improve its sensitivity and decrease calculation error at great values of the refractive index, one can increase the angle of incidence for radiation on the surface of the testing layer. Under increase of this angle, the difference between transmission coefficients for various polarizations (3) increases, what causes to increase of derivative value for the curve, representing the function (4).

Up to here, we have considered the case of a perfectly transparent testing layer at the absence of absorption. Now consider the influence of low-absorption on the opportunity to solve the inverse problem of refractive index determination using energy transmission coefficient measurements for various polarizations. On Figure 3, the plots of the functions $g(T_{H,E})$, $f(T_H, T_E)$ and $n(f(T_H, T_E))$, calculated by equations (4), (5), (6) and (2), are shown for the case, when the absorption coefficient is not vanish: $\kappa = 2.0 \times 10^{-5}$ ($\varepsilon = (n + i\kappa)^2$). In this case, the function $n(f(T_H, T_E))$, as before, regenerates linearly growth of the refractive index, with the exception of several narrow regions of anomalous behavior of the pointed functions, where the magnitudes of transmission coefficients (2) for various polarizations reach their maximum values.

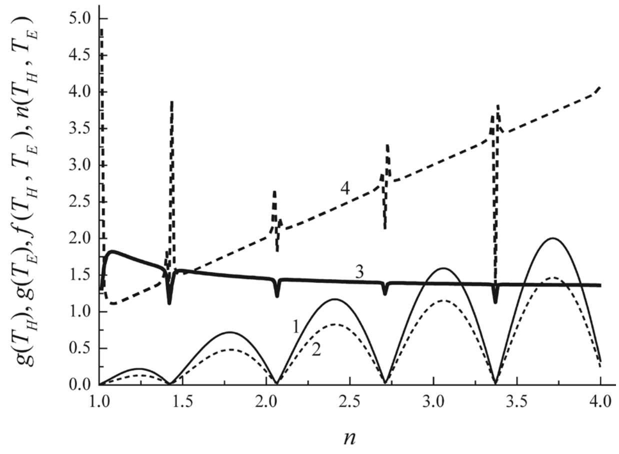

Figure 3 – Dependence of functions $g(T_H)$ (curve 1), $g(T_E)$ (curve 2), $f(T_H, T_E)$ (curve 3) and $n(f(T_H, T_E))$ (curve 4), computed by (5), (4) and (6), on the refractive index n of a low-absorbing dielectric layer, when its thickness $h = 0.75\lambda$, the absorption index in a layer $\kappa = 2.0 \times 10^{-5}$ and the angle of incidence $\varphi = 30^{\circ}$

Here, the real part of the sine of $k\gamma h$ equals or almost equals to zero, and then the contribution of components caused by the imaginary part of ε and γ to (2), becomes noticeable. If it is necessity to carry out the measurements directly in one of these anomalous regions, we can shift them, changing the angle of incidence, whose value depends on their position, so that the energy transmission coefficient of the E polarization to be essentially differs from the unite. Hence, the proposed technique can be applied also in the case of low-absorption.

Under increase of absorption in a layer, anomalous regions become wider, and even at $\kappa \approx 1.0 \times 10^{-3}$ they annex in the aggregate all region of refractive index change, so that the pointed algorithm stops to work under real conditions. Besides, usually in practice the coefficient of absorption in material is also unknown parameter, which should be determined together with the refractive coefficient, so that two measured parameters (the energy transmission coefficients) can occur not sufficient for correct solving of corresponding equations, which include also unknown thickness of a layer.

Conclusion

Studying the dependence of energy transmission coefficients of a plane transparent dielectric layer on its thickness and refractive index, we have established that, using two transmission energy coefficients for two orthogonal polarizations of inclined incidence electromagnetic beam, one can form a function, which monotonically depend on the refractive index of a layer over wide domain of its variation and does not depend on the layer thickness. Application of this function provides the opportunity to solve the inverse problem of refractive index determination on measurements of field intensities and using the layer energy transmission coefficients under conditions of incoherent beam and of unknown physical parameters of a dielectric layer. For solving this problem, one can applied the analytical method of inverse function computation, or can utilize the direct experimental calibration method for initial function. It emerges that accuracy of the method using calibration function depends on the value of the angle of beam incidence on a layer. If the refractive index value varies over the range from 1 to 2, then one can use the small incident angle of 30º, but for more great values of refractive index (from 2 up to 4 and more), it is appropriate to increase this angle up to the value of 60º. Using the analytical method of inverse function computation, the refractive index can be computed with any degree of accuracy, hence, accuracy of the presented method on the whole is determined by the measurement accuracy for electromagnetic radiation intensity.

So, in the present work, an unconventional technique of refractive index determination for a transparent dielectric layer on measurement of transmitting electromagnetic beam intensity is proposed. It differs from another techniques by the object of measurements, when one measures intensities of two linear polarizations of transmitting beam, which are orthogonal and parallel to the plane of its incidence, and distinguishes by simplicity of the analytic formulae (4) and (6), using for refractive index determination with the help of results for these intensity measurements without taken into account and introducing any phase field parameters.

The presence of absorption in a layer has very negative influence on application efficiency of the proposed method of refractive index determination. Even weak absorption produces appearance of narrow anomalous zones, where the dependence of the function being considered on refractive index is not monotonic. Distance between them is about 0.4–0.7 of refractive index value, and their width is equal to 0.1–0.3 for the absorption index κ having the order of 10^{-5} and 10^{-4}. It allows us to use the proposed method of refractive index determination also under presence of absorption, but over narrow domain of its variation. If necessary, the position of anomalous zones can be shifted away from the assuming domain of refractive index variation by changing the angle of beam incidence on a layer. However, at the growth of absorption up to $\kappa \approx 10^{-3}$ and more, the width of anomalous zones of monotonicity violation appreciably increases, and the method becomes to be practically inapplicable. In this case, substantial modification of the proposed method is required to take into consideration complexity of electromagnetic field propagation parameters in a dielectric.

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