1. Introduction

Nd- and Yb-doped potassium gadolinium tungstate, Nd: KGD(WO4)2 (KGW), is an attractive material for cw [1–4] and pulsed [5–8] low-threshold diode-pumped lasers operating in the vicinity of 1 and 1.3 μm. KGW is a biaxial crystal, with three crystal orientations having been employed so far for laser engineering: along the b (Np) axis [1–3, 5–8], along the Ng axis [9–12] and along the optical axis (O) [13, 14], i.e. Ng + 43° in the Ng–Nf plane [15]. Strong astigmatism of the thermo-optical properties of the material results in significant variation in the sign and value of the thermal lens in crystals cut for light propagation along the Np and Ng axes under diode- or flash-lamp pumping [9, 12, 15, 16]. Thermal lens measurements were reported previously for the Np- and Ng-cut Nd: KGW crystals at the wavelength of 1.06 μm under the flash-lamp pumping [9, 15, 16] and at the wavelength of 1.35 μm under the diode pumping [12].

O-cut KGW laser crystals have recently attracted significant attention due to the so-called ‘conical refraction’ effect [17]. This effect is thought to be responsible for polarization tuning of the laser emission [13], and the 'excellent' laser beam quality [14] in Nd: KGW lasers using this crystal cut. However, to the best of our knowledge, thermo-optical properties of the O-cut KGW crystal have never been taken into account when analyzing conical refraction effects in these lasers.

In this letter we present for the first time the results of experimental and theoretical analysis of thermal lens in the
Figure 1. Orientation of studied Nd:KGd(WO₄)₂ laser crystals with respect to axes of optical indicatrix (N_p, N_m, N_g) and crystallographic axes (a₁, b₁, c₁–f₂/c).

diode-pumped Nd:KGW laser with the crystal cut for light propagation along the optical axis (the so-called ‘conical refraction laser’). A direct comparison of thermal lens properties under identical experimental conditions is made for Nd: KGW lasers with crystals cut along the N_p and N_g axes of optical indicatrix. It was found that, as has been reported in Nd: YLF [18], the photo-elastic effect and its anisotropy in KGW affects the sign and value of the thermal lens in the crystal. The contribution of the photo-elastic effect to the thermal lens at 1 μm was calculated for three orientations of the Nd: KGW crystal.

2. Experimental details

KGW crystals with Nd ion concentration of 3 at% were grown by the technique described in [19]. Three different crystal orientations were studied, namely N_p-, N_g- and O-cut (figure 1). The crystal thickness was 1 mm and the aperture was 3 x 3 mm².

The thermal lens in diode-pumped Nd: KGW laser crystals was characterized by analyzing changes in the spatial profile of the output laser beam (figure 2). For cw pumping, a fiber-coupled AlGaAs laser diode producing unpolarized emission was used (λ_p = 810 nm, FWHM ~ 3 nm). Pump radiation was focused into the laser crystal by a pair of spherical lenses (collimating lens, CL, and focusing one, FL). The radius of the pump beam 0_p inside the crystals was measured to be 180μm. Pump absorption in all the Nd: KGW crystals under study was measured directly using the pump laser diode emission to be more than 97%. The input and output faces of the crystals were anti-reflection (AR) coated for the pump and laser (1067 nm) wavelengths. The crystals were kept in thermal contact with a copper heat-sink by means of heat grease; its temperature was maintained at ~17°C.

The laser cavity was formed by a concave HR@1067 nm mirror (R = 50 mm, HT@810 nm) and a flat output coupler, OC (T = 0.5% @1067 nm). The air gap between the crystal and the HR mirror was 13 mm; the total cavity length was 47 mm. The radius of the TEM₀₀ cavity mode in the laser crystals was calculated by the ABCD method to be ~180μm. The polarization of the output beam was naturally selected by the anisotropy of the laser gain (it was E∥N_m for N_p- and N_g-cut crystals). For the O-cut crystal near the threshold, the orientation of the vector of polarization was very sensitive to the pump power (the similar behavior was observed previously in [13] for O-cut Yb-doped KGW). However, at higher pump levels the laser polarization stabilized to E⊥N_m. The output beam was near-Gaussian with the M² < 1.1. This M² value was found to be unchanged within the pump powers used in the experiments. The spatial profile of the output beam at a distance of 11 cm from the OC was measured using a CCD camera. The second moment diameter of the beam was measured.

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Significant astigmatism of the thermal lens in KGW crystals was reported in [9, 12]. This means that the laser output beam is, in general, elliptical. That is why the measurements of the laser beam sizes were performed along the principal meridional planes A and B, i.e. planes which correspond to major and minor semi-axes of these ellipses. The dependence of the radii of the laser beam ω_A(B) along directions A (B) on the absorbed pump power P_abs was measured. The optical power of the thermal lens D_A(B), and the sensitivity factor, M_A(B) = dD_A(B)/dP_abs, can then be calculated from the laser resonator parameters by the ABCD method [12, 20]. The sensitivity factor shows the increase of the thermal lens optical power on absorption of 1 W of pump power. The difference S = 1/M_p - M_b is called the astigmatism degree (for a spherical thermal lens S = 0). The orientation of A and B planes, M_A(B) and S values make the complete set of thermal lens parameters.

3. Results

Typical spatial profiles of the output beam of the Nd: KGW lasers captured with the CCD camera are presented in figure 3. Here the upper and lower images correspond to the low and high values of absorbed pump power of P_abs = 0.1 and 0.5 W. The profiles are near-circular for low pump level (near the laser threshold). With increase of the pump power, the profiles become elliptical.

The measured dependencies of the output beam radii, ω_A(B), on the absorbed pump power, P_abs, are shown in figure 4. Here the points are the experimental data; curves are the results of modeling with the ABCD method. For the N_p- and O-cut crystals, the thermal lens results in expansion of the laser beam along the A direction and in its compression in orthogonal B direction. In contrast, for the O-cut crystal the thermal lens leads to expansion of the laser beam along both A and B directions.
In general, the orientation of the A and B planes is not directly related with polarization of the laser emission. Indeed, for the N_p-cut crystal, the A (B) directions make the angle of ~30° with the N_m (N_g) axes; for the O-cut crystal, they make the angle ~25° with the N_m (⊥N_m) axes; and for the N_g-cut crystal, AllN_m and BiLN_p. The scheme illustrating this difference is presented in figure 5 (circles and ellipses correspond to spatial profiles of the output beam at low and high pump levels).

It was demonstrated recently that the orientation of principal meridional planes for the flash-lamp-pumped Nd:KGdW crystal is determined by the anisotropy of the thermal expansion [15]. According to our data, this is also true for the diode pumping. Indeed, maximum and minimum values of the thermal expansion coefficient in the N_m–N_g plane correspond to the X_1 and X_3 principal axes, with N_m^X_1= 32.1° [21]. This is in agreement with our data, A^N_m ~ 33°. As a result, for the N_p-cut crystal, A||X_1 and B||X_3. For the O-cut crystal, directions with max and min thermal expansion are

![Figure 3](image3.png)

**Figure 3.** Spatial profiles of the output beam of a diode-pumped Nd:KGd(WO_4)_2 laser based on N_p-, N_g- and O-cut crystals; upper and lower images correspond to low and high pump levels.

![Figure 4](image4.png)

**Figure 4.** Radii of the TEM_00 laser mode, ω_A/B, as a function of the absorbed pump power for N_p-cut (a), O-cut (b) and N_g-cut (c) Nd:KGd(WO_4)_2 crystals; (d) beam ellipticity ω_A/ω_B.

![Figure 5](image5.png)

**Figure 5.** Distortion of the output beam of the Nd:KGd(WO_4)_2 laser due to the influence of the thermal lens: black circles and red ellipses correspond to beam profiles at low and high pump levels, dotted lines correspond to principal meridional planes (A and B directions).
rotated by ~30º-anticlockwise from the \( N_m \) (\( \perp N_m \)) axes which is again in agreement with our data. Maximum and minimum values of the thermal expansion coefficient in the \( N_m-N_p \) plane \((N_g\)-cut crystal\) correspond directly to \( N_m \) and \( N_p \) axes. This also agrees with the observed coincidence of \( A \) and \( B \) directions with \( N_m \) and \( N_p \) axes (figure 3).

The ratio of the radii of an elliptical Gaussian beam, \( \frac{\omega_A}{\omega_B} \), is denoted as the beam ellipticity, see figure 4(d). It increases monotonically with the increase of the pump power. For the \( N_p \)- and \( O \)-cut crystals, the beam ellipticity reaches ~1.2 at \( P_{\text{abs}} = 1 \) W. The \( N_g \)-cut crystal allows one to obtain lower distortion of the output laser beam (as the corresponding value of the beam ellipticity is less than 1.1).

The calculated dependencies of the optical powers of the thermal lens in the two meridional planes, \( D_A \) and \( D_B \), on the absorbed pump power are presented in figures 6(a)–(c). ABCD modeling of the laser resonator used (figure 6(d)) indicates that expansion (compression) of the output laser beam corresponds to the positive (negative) thermal lens in Nd: KGW crystal. Moreover, the value of \( \omega \) is more sensitive to a positive lens. Thus, for the \( N_p \) and \( O \)-cut crystals, the thermal lens is positive (negative) for rays lying in the \( A \) (\( B \)) planes; it is positive for rays lying in all meridional planes for the \( N_g \)-cut crystal. These findings are supported by the previous results reported for the \( N_p \) and \( N_g \)-cut Nd: KGW crystals under the flash-lamp and diode pumping [9, 12, 15, 16].

The slopes of the dependencies in figures 6(a)–(c) are called thermal lens sensitivity factors, \( M_{AB} = \frac{\partial D_A}{\partial D_B} \). These values are summarized in table 1. The difference of principal sensitivity factors is denoted as astigmatism degree, \( S = |M_A-M_B| \). For the \( N_p \)-cut and \( O \)-cut crystals, the values of \( S \) are close and equal to ~1.6 m\(^{-1}\) W\(^{-1}\). In contrast, the \( N_g \)-cut crystal offers near eight times reduced astigmatism, \( S \approx 0.2 \) m\(^{-1}\) W\(^{-1}\).

### Table 1. Parameters of thermal lens for Nd: KGD(WO\(_4\))\(_2\) crystals at 1\( \mu \)m.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( N_p )-cut</th>
<th>( O )-cut</th>
<th>( N_g )-cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_{AB} = \frac{\partial D_A}{\partial D_B} ) (at ( \omega_p=180 \mu m ))</td>
<td>+0.57</td>
<td>+0.72</td>
<td>+2.35</td>
</tr>
<tr>
<td>( S =</td>
<td>M_A-M_B</td>
<td>) (m(^{-1}) W(^{-1}))</td>
<td>+1.60</td>
</tr>
</tbody>
</table>

### 4. Discussion

Formation of the thermal lens in solid-state lasers is determined by three main effects, namely (i) the temperature variation of refractive index \( \frac{dn}{dT} \), (ii) the photo-elastic effect (dependence of refractive index on thermally induced stress, \( P_{\text{PE}} \)) and (iii) the bulging of crystal end faces due to non-uniform thermal expansion \( Q_{\text{dist}} \) [22]. These contributions are summarized in the so-called ‘generalized’ thermo-optic coefficient [23], \( \Delta = \frac{dn}{dT} + P_{\text{PE}} + Q_{\text{dist}} \). For diode pumping of bulk crystals (plane stress approximation), optical power of the thermal lens can be determined as [23]

\[
D = \frac{P_{\text{abs}} \eta_h}{2 \pi \omega_p^2} \left( \frac{dn}{dT} + P_{\text{PE}} + Q_{\text{dist}} \right)
\]

(1),

where \( \eta_h \) is the fractional head load (for Nd\(^{3+} \) ions, it is usually estimated as the quantum defect, \( 1-\lambda_p/\lambda_i \), where \( \lambda_p \) and \( \lambda_i \)
are the pump and laser wavelengths, correspondingly; \( o_\| \) is the pump spot radius (top-hat profile); \( \kappa \) is the thermal conductivity. It should be noticed that in previous publications on the thermal lens in cubic Y\(_3\)Al\(_2\)O\(_12\), the \( Q_{\text{dist}} \) term was evaluated as \((n-1)/n\), where \( \nu \) is the Poisson ratio, \( n \) is the refractive index and \( \alpha \) is the thermal expansion coefficient in the direction of light propagation [22]. However, for KGW crystal the tensor of elastic constants \( C_{ij} \) is highly anisotropic and does not satisfy basic relations for high-symmetry materials like Y\(_3\)Al\(_2\)O\(_12\) (even in rough approximation) [24], that is why the bulging of crystal end faces is described in KGW as \( Q_{\text{dist}} = (n-1)/n \). Thermal conductivity \( \kappa \) is a scalar quantity for isotropic materials. However, for crystals with low anisotropy of thermal conductivity like double tungstates, a directionally averaged value of \( \kappa \) can be used [23]. Under longitudinal diode pumping, heat flow in the crystal is predominantly transverse. Thus, \( \kappa \) should be averaged over the plane perpendicular to the light propagation direction.

To date, the thermal conductivity for KGd(WO\(_4\))\(_2\) has been measured only for a few directions in the crystal [25, 26]. Complete characterization of the \( \kappa \) tensor has been performed for another representative of double tungstates, isostructural KLu(WO\(_4\))\(_2\) [27]. Since the thermal properties within this crystal family are very similar, we can use this data in the present paper. Directionally averaged values of \( \kappa \) for \( N_g \) - cut are summarized in table 2. The so-called thermal coefficient of the optical path \( W = du/dT + (n-1)\alpha \) was measured in this study at 1064 nm in the \( O \)-cut Nd: KGW crystal for \( E\parallel N_m \) by the beam deviation method [28] under the homogeneous heating to be \((-7 \pm 1)\times 10^{-6} \text{K}^{-1} \). The \( W \)-coefficient for the \( O \)-cut Nd: KGW can also be calculated from known values of the thermo-optic coefficients \( d\alpha/d\omega \) and \( d\kappa/d\omega \), index of refraction \( n_\omega \) and thermal expansion \( \alpha_\omega \). In general, the index of refraction \( n(E\parallel N_m) \) of a KGW crystal cut for light propagation in the \( N_g\parallel N_p \) plane can be calculated as [29]

\[
n(E\parallel N_m) = \left( n_p^2 \cos^2 \phi + n_\varphi^2 \sin^2 \phi \right)^{1/2}
\]

with the \( dn/d\omega \) being, after differentiation,

\[
\frac{dn}{d\omega} (E\parallel N_m) = \frac{dn_p}{d\omega} n_p^2 \sin^2 \phi + \frac{dn_\varphi}{d\omega} n_\varphi^2 \cos^2 \phi
\]

where \( n_p \) and \( n_\varphi \) are the indices of refraction and thermo-optic coefficients, respectively, for \( E\parallel N_p \) and \( \varphi \) is the angle between the \( N_g \) axis and the direction of light propagation in the \( N_g\parallel N_p \) plane (\( \varphi = 43^\circ \) in KGW crystal cut along the optical axis). The set of thermo-optical parameters of Nd: KGW at 1 \( \mu \text{m} \) is summarized in table 2. The index of refraction in the \( O \)-cut crystal does not depend on polarization, i.e. \( n_\parallel(E\parallel N_m) = n_m = 2.01 \) at 1067 nm [30] at \( \varphi = 43^\circ \). The thermo-optic coefficient \( d\alpha/d\omega \) does depend on polarization due to the different physical nature of these dependencies [31–33]. The value of the thermal expansion \( \alpha_\omega \) along the optical axis can be calculated from the following expression [21]:

\[
\alpha_\omega = \alpha_\| \cos^2 \phi + \alpha_\parallel \sin^2 \phi \]
not expect a direct relation between the latter and the value of the photo-elastic effect.

The values of the $P_{\text{PE}}$ term, to the best of our knowledge, have never been reported previously for any crystal orientation of Nd: KGW. For the $N_d$-cut Nd: KGDW crystal, $P_{\text{PE}}$ is positive and comparable in absolute value with $dn/dT$. As a result, the optical power of the thermal lens is determined by the opposite action of temperature- and stress-dependent variation of the refractive index. This leads to different signs of the opposite action of temperature- and stress-dependent variations. It increases when the crystal cut changes counterclockwise from being along the $N_d$ axis to along the $N_A$ axis.

In the case of an optical axis-oriented Nd: KGW crystal and for light propagation along the $N_m$, the minimum and maximum values of the photo-elastic contribution were evaluated for the positive and negative thermal lens, respectively. The contribution of the photo-elastic effect to the thermal lens (10$^{-6}$ K$^{-1}$) in diode-pumped Nd: KGDW lasers at 1.06 µm was calculated based on measurements of the output laser beam distortions for light propagation along the $N_m$, $N_g$, and optical axes of the crystal for the first time, to the best of our knowledge. The photo-elastic contribution was found to be positive for all studied crystal orientations. It increases when the crystal cut changes counterclockwise from being along the $N_d$ axis to along the $N_A$ axis.

The contribution of the photo-elastic effect to the thermal lens in diode-pumped Nd: KGDW crystal, $P_{\text{PE}}$, is positive and comparable in absolute value with $dn/dT$. As a result, the optical power of the thermal lens is determined by the opposite action of temperature- and stress-dependent variations. It increases when the crystal cut changes counterclockwise from being along the $N_d$ axis to along the $N_A$ axis.

For the $O$-cut crystal the impact of positive $Q_{\text{dist}}$ becomes more significant, but still is not enough for the crystal to demonstrate positive thermal lensing along both $A$ and $B$ directions for $E \parallel N_m$. In the case of $E \parallel N_m$ the net effect of $dn/dT$ and $Q_{\text{dist}}$ results in a relatively small value of the thermal coefficient of $1.7 \times 10^{-6}$ K$^{-1}$. However, the photo-elastic effect in the general case should be different for different polarizations of light [16, 18]; therefore the values previously calculated for $E \parallel N_m$ will not be valid for $E \parallel N_m$ and, strictly speaking, not only the value, but even the sign of the lens cannot be predicted from the data available in this study. However, our measurements of thermal lens in the $O$-cut Nd: KGW crystal in $E \parallel N_m$ configuration under the flash-lamp pumping shows that it is positive along direction $A$ and negative along direction $B$ [15]. This allows us to estimate the maximum and minimum values of the photo-elastic contribution to the thermal lens (see table 3) under the current pump conditions.

For the $N_g$-cut crystal, the photo-elastic effect also results in an increase of refractive index ($P_{\text{PE}} > 0$). However, the absolute value of $P_{\text{PE}}$ is small (compared with the $dn/dT$ and especially the $Q_{\text{dist}}$ terms). Thus, the optical power of the thermal lens is mainly determined by the dominance of the positive $Q_{\text{dist}}$ over the negative $dn/dT$ term, and the thermal lens is positive. The small difference between the $P_{\text{PE}}$ values for the $A$ and $B$ planes leads to small astigmatism. Reduction of $\Delta$ to $dn/dT + (n-1)\alpha$ is acceptable only for $N_g$-cut Nd: KGDW, but this approach will result in some underestimation of optical power of the thermal lens. In contrast, for $N_p$-cut and $O$-cut crystal, the $P_{\text{PE}}$ term cannot be omitted if the thermo-optic effects are to be described correctly.

### Table 3. Contribution of the photo-elastic effect to thermal lens optical power in diode-pumped Nd: KGD(WO$_4$)$_2$.

<table>
<thead>
<tr>
<th>Crystal</th>
<th>$dn/dT$</th>
<th>$P_{\text{PE}}$</th>
<th>$Q_{\text{dist}}$</th>
<th>$\Delta$ (10$^{-6}$ K$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_p$-cut, $E \parallel N_m$</td>
<td>$-11.7$</td>
<td>$+11.2$ (A)</td>
<td>$+1.8$</td>
<td>$+1.3$ (A)</td>
</tr>
<tr>
<td>$O$-cut, $E \parallel N_m$</td>
<td>$-16.6$</td>
<td>$+7.3$ (B)</td>
<td>$+10.0$</td>
<td>$-2.6$ (B)</td>
</tr>
<tr>
<td>$E \parallel N_m$</td>
<td>$-11.7$</td>
<td>$&lt;+1.7$ (A)</td>
<td>$+10.0$</td>
<td>Positive (A) [15]</td>
</tr>
<tr>
<td>$N_g$-cut, $E \parallel N_m$</td>
<td>$-11.7$</td>
<td>$+0.6$ (A)</td>
<td>$+17.0$</td>
<td>$+5.9$ (A)</td>
</tr>
</tbody>
</table>

### 5. Conclusions

The contribution of the photo-elastic effect to the thermal lens in diode-pumped Nd: KGW lasers at 1.06 µm was calculated based on measurements of the output laser beam distortions for light propagation along the $N_g$, $N_m$, and optical axes of the crystal for the first time, to the best of our knowledge. The photo-elastic contribution was found to be positive for all studied crystal orientations. It increases when the crystal cut changes counterclockwise from being along the $N_d$ axis to along the $N_A$ axis.

**References**


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