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# Algorithm for Measuring the Geometric Parameters of Optical Freeform Surfaces Using a Coordinate Measuring Machine

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**Abstract**

Nowadays use of freeform surfaces in optical systems for various purposes – from mobile displays to space optics – becomes increasingly popular. This design solution makes it possible to create a system with a minimum number of components, low weight, and high image quality. However, as the surfaces become more complex, problems arise with their control. Aim of the study was to develop an algorithm for inspecting such surfaces using a coordinate measuring machine and to formulate technical requirements for the measurement process. An analysis of the advantages of freeform optical surfaces is presented. It is noted that due to lack of symmetry and variable profile height, classical measurement methods are not applicable for such parts. The inspection algorithm using a coordinate measuring machine involves two main approaches: high-performance "fast scanning" to assess overall deviations from the nominal value, and more accurate but time-consuming sectional scanning which allows obtaining detailed quantitative data on the surface profile. The paper is mainly focused to the measurement algorithm and process requirements: preliminary temperature adaptation of equipment, mandatory calibration of probes using a ceramic reference ball, and software-controlled limitation of tool movements to protect polished surfaces. The proposed algorithm, based on the use of CAD-models, makes it possible to achieve the required accuracy of monitoring complex surfaces on standard shop-floor equipment, despite significant time costs compared to aspherical surfaces' measuring.

**Keywords:** freeform surfaces, surface quality control, control and measuring machine, measurement algorithm

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# Алгоритм измерения геометрических параметров оптических поверхностей freeform с использованием координатно-измерительной машины

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В настоящее время становится всё более популярным применение в оптических системах различного назначения поверхностей freeform (свободной формы) – от мобильных дисплеев до космической оптики. Это конструкторское решение позволяет приблизиться к мечте: создать систему с минимальным числом компонентов, малым весом и высоким качеством изображения. Однако с усложнением поверхностей возникают проблемы с их контролем. Целью исследования являлась разработка алгоритма измерения таких поверхностей с использованием координатно-измерительных машин и формулирование технических требований к процессу измерений. Приведён анализ преимуществ оптических поверхностей freeform. Отмечается, что из-за отсутствия симметрии и переменной высоты профиля классические методы измерений для таких деталей неприменимы. Алгоритм контроля на координатно-измерительной машине предполагает два основных подхода: высокопроизводительное «быстрое сканирование» для оценки общих отклонений от номинала и более точное, но трудоёмкое сканирование по сечениям, позволяющее получить детальные количественные данные о профиле поверхности. Основное внимание в работе уделено алгоритму проведения измерений и требованиям к процессу: предварительной температурной адаптации оборудования, обязательной калибровке щупов по керамическому эталонному шару и программному ограничению перемещений инструмента для защиты полированных поверхностей. Предложенный алгоритм, основанный на использовании CAD-моделей, позволяет достичь требуемой точности контроля сложных поверхностей на стандартном заводском оборудовании, несмотря на значительные временные затраты по сравнению с измерением асферических поверхностей.

**Ключевые слова:** поверхности freeform, контроль качества поверхностей, контрольно-измерительная машина, алгоритм измерений

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## Introduction

Currently, the use of freeform surfaces in optical systems is becoming increasingly popular. This design solution allows creating systems with a minimum number of components, low weight, and high image quality.

Systems containing parts with freeform surfaces are in demand in mobile displays (including smart glasses), space remote sensing of the Earth, medical endoscopy, automotive lidars, etc. [1–4].

Unlike traditional spherical (one radius of curvature), aspherical (axial symmetry, variable curvature), anamorphic (different image scale in two perpendicular directions) and diffractive (contain micro-relief for wavefront control) surfaces, freeform surfaces have neither axes of rotation nor translational symmetry (Figure 1) [5, 6]. Their shape can be arbitrary, described by high-order polynomials or even by a set of points.

This feature of freeform surfaces gives the designer complete freedom, but creates three main problems:

1. Calculation of the surface shape is labor-intensive and takes a lot of time;
2. Manufacturing requires special methods, e. g., diamond turning on CNC machines or injection molding, since the standard grinding-polishing method is insufficient to obtain the required shape;
3. Inspection becomes more difficult due to the complexity of implementing reference surfaces.

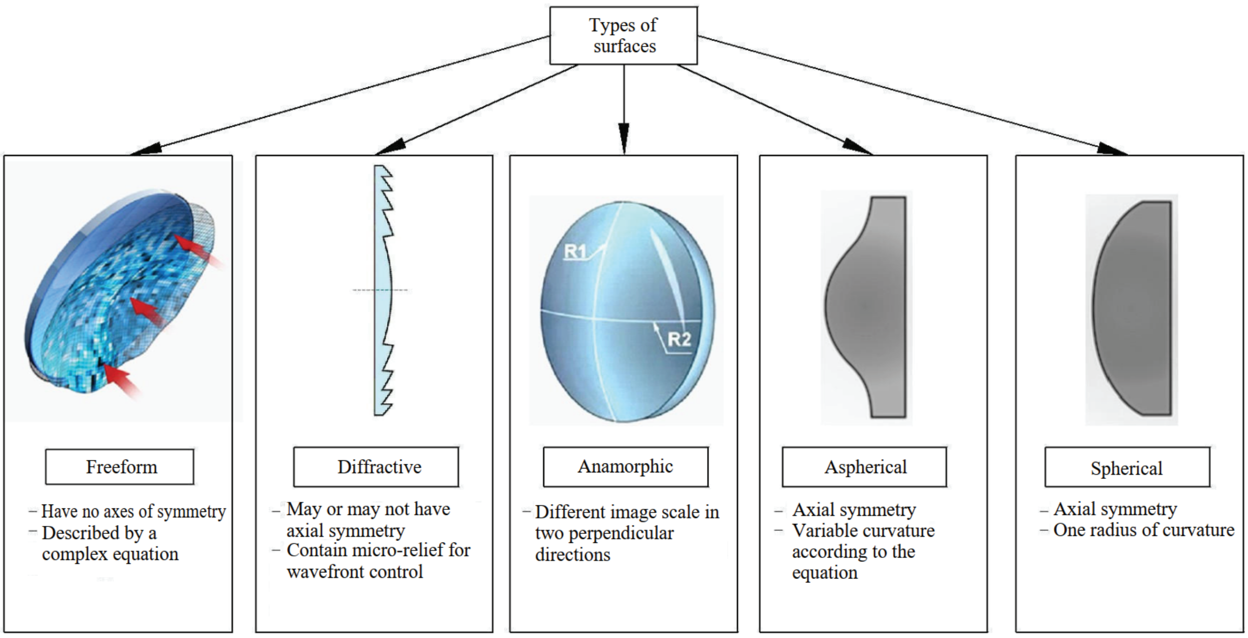


**Figure 1** – Comparison of optical surface types

Therefore, it can be confidently stated that the manufacturing technology of systems with this type of surfaces requires special knowledge in design, manufacturing and inspection [7].

As already noted, as freeform structures become more complex, it becomes increasingly difficult to control the final shape of a part with the same accuracy as for traditional optical parts (flat and spherical). A specific feature of controlling freeform surfaces is that they have variable height over the entire surface (from a peak to a valley). This makes it impossible to use classical measurement methods, creates problems in manufacturing a reference comparison surface, and greatly complicates the inspection procedure.

The aim of the work was to develop an algorithm for inspecting such surfaces using coordinate measuring machines and to formulate technical requirements for the measurement process.

In this work an algorithm is understood as a set of interrelated elements: equipment preparation, measurement algorithm, scanning modes, data processing, technical requirements and accuracy analysis. It is the integrated approach that considers the specificity of freeform surfaces and allows obtaining reproducible and reliable results.

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## Methods for controlling freeform surfaces

Based on the works [8, 9], inspection of freeform surfaces is carried out by mechanical methods using profilographs-profilometers and CMMs, as well as by optical methods, namely the interferometric method using computer-generated holograms.

The creation of synthesized holograms is a complex and time-consuming task. A synthesized hologram for controlling freeform surfaces is more complex in design, represents a composite hologram and requires special calculation methods, as well as manufacturing and certification. Nevertheless, inspection with their help provides high accuracy and allows controlling the entire surface of the part.

Surface topography is controlled by mechanical methods using a sensor that moves along the surface along a given trajectory. 3D and 2D models of the controlled surface are created based on the obtained surface profile values after performing the corresponding calculations. The mechanical method is labor-intensive, does not allow controlling the entire surface and is inferior to the optical one in accuracy. However, for its implementation it is sufficient to use the equipment available at the enterprise – profilographs-profilometers or a CMM, into the CAD software of which the calculated profile of the freeform surface can be loaded.

Among all the proposed methods for controlling freeform surfaces, an algorithm implemented on a coordinate measuring machine has been developed and substantiated in this work.

## Information about the coordinate measuring machine

Coordinate measuring machines are designed to measure the geometric dimensions of parts of complex shape, deviations of shape and position of part surfaces.

The CMM measuring probe consists of a head with a measuring stylus and a sensitive element.

The functionality of the measuring head largely determines the functionality of the CMM, the classes of surfaces and the number of product parameters available for monitoring.

The measuring stylus provides primary measurement information on the basis of which the dimensions of the part are determined. This information can be obtained either as the actual coordinates of points on the surface under test or as deviations of these coordinates from those specified in a certain direction.

The specific accuracy characteristics achieved within the framework of the proposed algorithm depend on the CMM model and the type of measuring probe – they are discussed below.

## Developed algorithm using a coordinate measuring machine

The developed algorithm is based on known approaches to controlling aspherical surfaces using CMMs, which involve comparing a cloud of measured points with a CAD model and analyzing deviations along sections [7, 10, 11]. However, unlike axisymmetric aspherical parts, the control of freeform surfaces is associated with a number of additional difficulties due to the lack of any symmetry. This required the following key modifications:

1. Change in scanning strategy: introduction of a two-stage procedure ("fast scanning" for general assessment and "sectional scanning" for precision control) instead of a single mode, which allows optimizing measurement time and accuracy.
2. Tightening of requirements: increase in the density of measuring points and the number of control sections for correct description of complex topology, as well as development of special requirements considering the fragility and high cleanliness of polished optical surfaces (e.g., a software "safety cube" and strict temperature stabilization).

### *Preparation for measurements and algorithm*

Before any actions, the CMM operator receives a task to perform the necessary work. The task usually contains the name of the part, its description, previous data on its control (if it was carried out), controlled parameters of the part, accuracy and characteristics of the room in which the control is to be carried out. In addition to the task, the part drawings must be attached, which indicate all controlled parameters and their tolerances, as well as its 3D model. After receiving the necessary documentation, measurements are carried out in accordance with the algorithm shown in Figure 2.

### *Scanning modes and data processing*

The first measurement method (fast scanning) consists of contact scanning of the surface using a grid containing a small number of points. The method is applicable only if a CAD model is available, since it defines the ideal (calculated, nominal) surface shape. After scanning, the values of the maximum and minimum deviations from the nominal value are displayed on the monitor. As the measurements are performed, a drawing of the measured surface is superimposed on the ideal model, where deviations from the required profile are highlighted in a certain color.

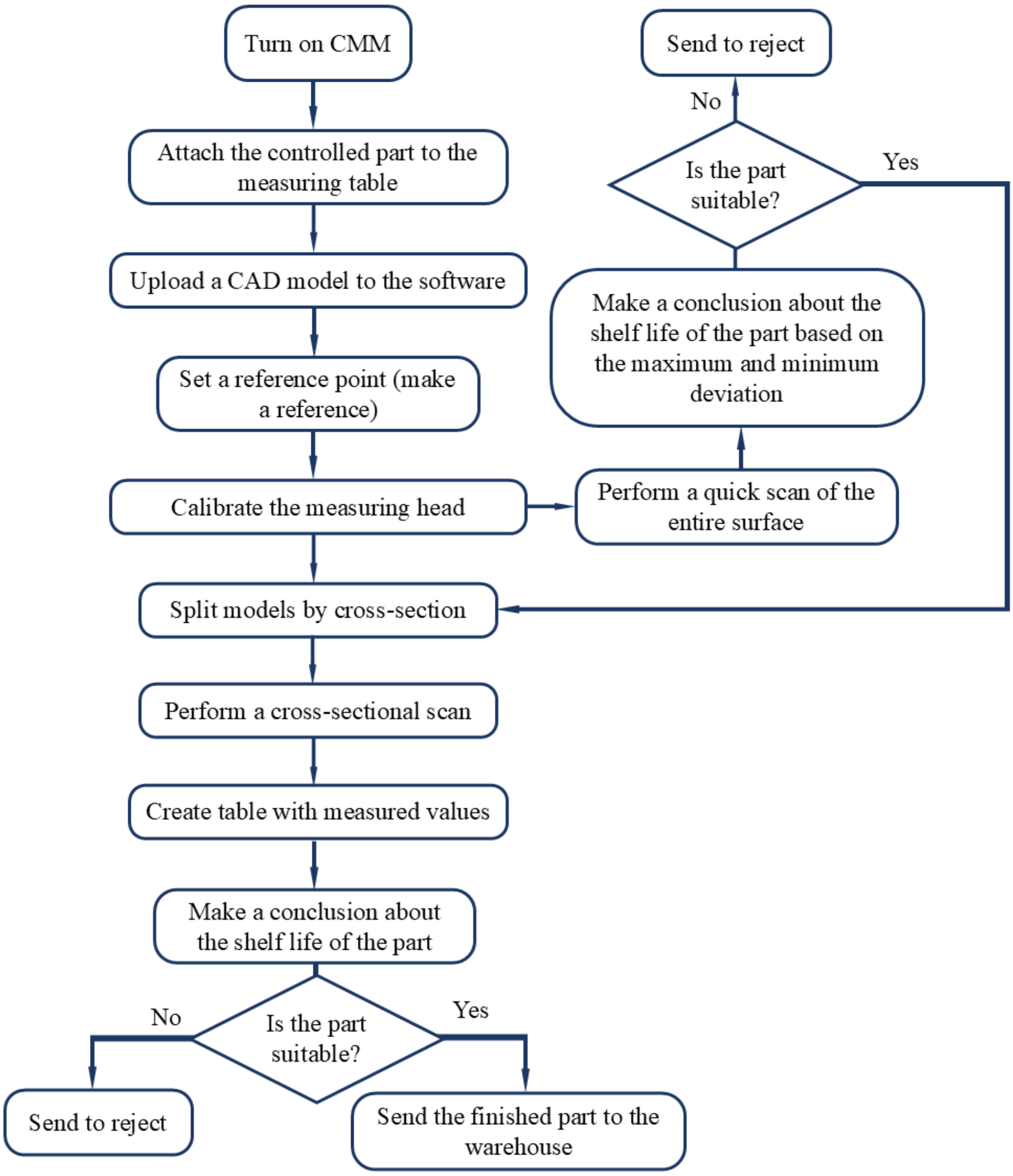


**Figure 2** – Algorithm for the shape of freeform surfaces' measuring using coordinate measuring machine

This method is visual and productive, but it does not provide high accuracy and allows only obtaining the maximum and minimum deviations from the specified surface profile.

The second measurement method is more labor-intensive and requires dividing the measured surface into sections and scanning them with a given number of points. As a result of the measurements, the coordinates of the measured points relative to the reference point are displayed on the monitor.

After obtaining the array of coordinates of the measured points, they are compared with the CAD model. The deviation at each point is calculated as the shortest distance to the nominal surface. An example of such a model with control sections and visualization of deviations is shown in Figure 3 (blue color shows local deviations from the nominal shape). The permissible deviation limits are specified in the design documentation for the part. The control result is considered positive if all measured deviations do not exceed the established tolerances.

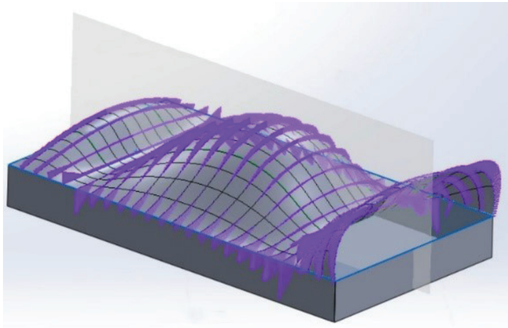

**Figure 3** – CAD model of a freeform surface with reference sections

It is also possible to visualize the obtained values. The image of the measured points is superimposed on the model. However, unlike the first method, the second method does not create a "portrait" of the entire surface shape, but shows the shape in sections.

This method is accurate and allows presenting measurement results in both quantitative and qualitative form, but it takes a lot of time. Especially for freeform surfaces, the measurement time increases many times over, since a larger number of points are required to be measured than for aspherical surfaces.

*Accuracy factors and recommendations for mode selection*

The accuracy of these methods is influenced by a number of factors [11].

1. Measuring probe errors. For example, for the VAST XT measuring probe of the ACCURA II coordinate measuring machine, the linear measurement error is 1.6 µm, the probing deviation is 1.7 µm, the scanning probing deviation is 2.5 µm, and the form measurement error is 1.7 µm.

It should be noted that when measuring freeform surfaces, one of the influencing factors on the measurement error is the tip radius when measuring with large local slopes. To minimize the influence of this factor, it is recommended to use styli with a ball diameter of no more than 0.5–1 mm. In the case of particularly steep areas (>45°), software correction of the tip radius should be applied.

2. Volumetric length measurement error ($MPE_E$). Unlike the local probe error, this parameter considers the geometric deviations of the CMM guides and the errors of the reading scales along the *X*, *Y*, *Z* axes. For the modification used, the permissible absolute error limit is $MPE_E = 1.6 + L/333$ µm, where $L$ is the measured length in mm.

3. Effect of scanning discreteness. The approximation error of the shape between sections is estimated at 0.2–0.5 µm depending on the surface curvature.

4. Influence of the external environment and thermal compensation. Despite the presence of a built-in automatic temperature expansion correction system, the accuracy of the method is guaranteed if the temperature gradient does not exceed 1 °C/h.

5. Algorithmic component. The use of certified Calypso software (version 5.x) eliminates subjective errors in point cloud processing and ensures reproducibility of results when compared with the CAD model.

6. Dynamic characteristics. The granite base of the machine on anti-vibration supports minimizes the influence of workshop vibrations on the stability of the scanning process.

Thus, the accuracy capabilities of the algorithm are determined by the combination of characteristics of the measuring system "frame-drive-scales-probe-software", which makes it possible to measure complex-shaped parts with micron accuracy.

The choice of measurement mode is determined by the requirements for the part. Fast scanning is recommended for preliminary sorting or control of large batches, when it is sufficient to estimate the maximum deviations. Sectional scanning is mandatory for the final control of critical optical parts, as well as when it is necessary to construct a map of shape deviations.

*Advantages of the developed algorithm*

The proposed algorithm has the following advantages.

– High accuracy. Sectional scanning allows recording shape deviations at the micron level, which meets the requirements for most optical surfaces.

– Economic affordability. Standard factory coordinate measuring machines are used, which eliminates the need to manufacture expensive synthesized holograms for each new part.

– Measurement safety. The software "safety cube" prevents accidental damage to polished surfaces during automatic movement of the probe.

*Limitations of the algorithm*

The limitations of the proposed algorithm related to the accuracy of controlling freeform optical surfaces should be noted.

– Discrete nature of measurements. The algorithm is based on scanning along individual sections, not over the entire area. Local deviations between sections may remain undetected.

– Influence of the coordinate measuring machine error and surface geometry. When controlling the shape in areas with local inclination angles greater than 30°, the deviation measurement error may increase by 0.5–1 μm due to projection effects (non-collinearity of the measurement direction and the normal to the surface).

– Sensitivity to external conditions and CAD model. The requirement of a temperature gradient of no more than 1 °C/h is critical for parts made of materials with a high coefficient of thermal expansion (polymers, some IR crystals). In addition, the algorithm requires a high-precision CAD description of the surface: the model error (rounding of polynomial coefficients, discretization) can reach 0.1–0.3 μm.

– Surface cleanliness. The contact method is sensitive to contamination – scratches, residues of polishing paste or dust cause false coordinate spikes. The surface must be cleaned before measurements.

– Time consumption. 20–90 minutes per part depending on the size and scanning density limit the application of the algorithm in large-scale production, but are acceptable for one-off and small-batch production.

– Equipment requirements. The declared accuracy is achievable only on coordinate measuring machines of a class not lower than 2 μm (e. g., ACCURA II with VAST XT probe); the use of less accurate machines does not guarantee correct control of optical surfaces.

## Technical requirements for the algorithm

When controlling freeform optical surfaces using a CMM, the following requirements are imposed.

1. The coordinate measuring machine must be switched on 30 minutes before the start of measurements to ensure that the working area of the machine adapts to the ambient temperature.

2. The probe holder and the probe itself must be installed in the machine at least 15 minutes before the start of measurements, since heating from the hands can affect the properties of the components.

3. It is necessary to perform coordinate referencing (find the reference point). It corresponds to the machine zero point and is located in the upper left corner of the measuring range (point 1 in Figure 4). Before starting measurements, the reference point must be determined.

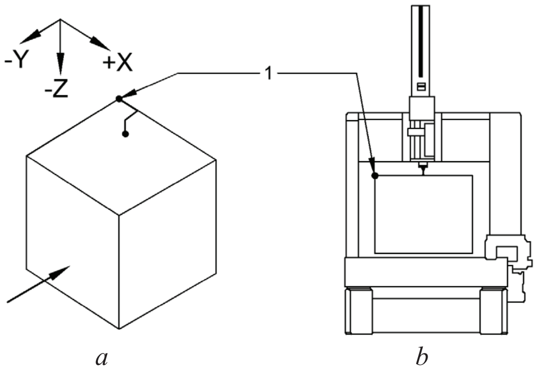


**Figure 4** – Reference point: *a* – location of the reference point in the measurement area; *b* – location of the reference point on thr coordinate measuring machine

4. Before starting measurements, each measuring stylus used for measurements must be calibrated in accordance with the calibration standard, setting it to the reference point.

A high-precision ball is used as a calibration standard (Figure 5). The ball is made of ceramic material and has a diameter of 30 mm. The ball is attached to the calibration ball holder.

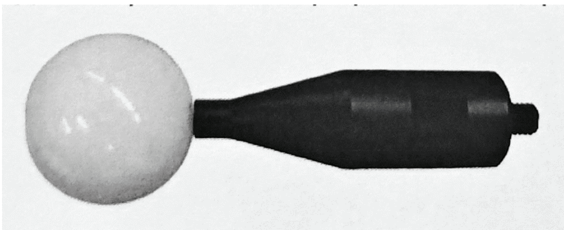

**Figure 5** – Calibration ball with pin

The pin of the calibration ball can be screwed to the holder in different positions. The holder allows the installation of several calibration balls.

5. It is necessary to limit the movement of the measuring head stylus, since uncontrolled movement can damage the polished working surfaces of the part. This is an important part when controlling the working surfaces of optical parts.

For this purpose, a so-called "safety cube" is created. It defines the limits of probe movement be-

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yond which it cannot go. This ensures that the probe is safely retracted.

The size of the "safety cube" is determined by the size of the part being measured (it passes along its edges). The safe probe withdrawal zones are defined by the corner points (10 mm from the edge of the part) (points (1) and (2) in Figure 6).

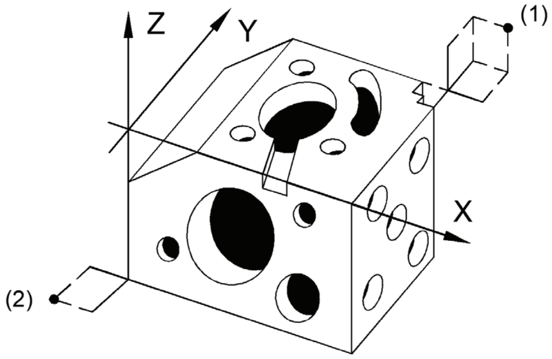


**Figure 6** – Security cube

6. Measurements of freeform surfaces (as well as aspherical surfaces) are performed only using a CAD model (measurements of such surfaces are performed along sections). It is also used for coordinate referencing, creating a "safety cube" and dividing the controlled surface into sections.

7. Tolerances for deviation of the surface shape from the nominal value are specified in the design documentation for the specific part. The algorithm does not impose restrictions on their numerical values within the accuracy capabilities of the CMM used.

After performing the specified preparatory operations, the measurement algorithm according to Figure 2 is implemented.

The given numerical requirements (warm-up time, probe diameter, temperature gradient) are substantiated by the results of preliminary experiments at the enterprise and guarantee repeatability of measurements.

## Conclusion

An algorithm for controlling freeform surfaces using a coordinate measuring machine has been developed. A generalized step-by-step algorithm for controlling freeform surfaces on a coordinate measuring machine is proposed, including stages from CAD model alignment to quantitative analysis of deviations along sections. The algorithm includes a comparison of two measurement modes – "fast scanning" for visual assessment and "sectional scanning"for obtaining high-precision data. A set of technical requirements (mandatory 30-minute temperature stabilization, calibration of probes using a ceramic reference ball, etc.) is formulated.

Scientific novelty:

- a two-stage algorithm for controlling freeform surfaces on a coordinate measuring machine, combining fast scanning for rejection and precision sectional scanning for quantitative evaluation, is proposed for the first time;
- a "safety cube" – a software limitation of the working area that prevents damage to polished surfaces (no analogues found in the literature) – has been developed and formalised;
- quantitative requirements for temperature stabilisation (30 min) and probe adaptation time (15 min), as well as for the probe diameter (0.5–1 mm) to minimise errors on steep areas, have been established.

The algorithm is intended for use in high-tech industries producing compact optical systems: from medical endoscopes and wearable electronics (smart glasses) to space remote sensing systems and lidars.

Further development of the algorithm may be associated with automatic adaptive assignment of the section density depending on the local curvature of the surface, as well as with the integration of non-contact optical sensors for preliminary topography assessment.

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