HIGH-FREQUENCY PARAMETRIC OSCILLATIONS OF THE ELEMENTS TRACKED VEHICLE RUNNING GEAR

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Most of the modern high-speed track-type machine is equipped with tracks fitted out with rubberrized tread of variable thickness of the elastic layer. It is considered, that this salvation reduces mass per unit length, reduces dependence of the strength from wear, reduces level of tires thermostension, noise and electromagnetic radiations. The choice of the settings of supporting rollers and rubberized running tracks depends on the condition of supplying of the supporting ability and limitation of the condition of temperature within the resource. When using these machines with rubberized running tracks on roads with poorly deformable base (rocky, crashed rock road, snowless winter road with a tire tread, frozen soil, damaged asphalt, etc…)[1], we can see fatigue distraction of the balance-bobs, made of high-alloy steel 40hn2ma-sh (40ХН2МА-Ш), in the zone of concentration, which is an area where balance-bob’s axes transform into a fillet’s stem.

The dependence of the chassis parts resource on the stiffness of soils (roads) is well known, confirmed by many experimental data and experience in operating high-speed tracked vehicles. In the situation of moving on a deformable substrate collision of the first wheel on the truck makes it plunged into the ground as into a discrete elastic medium and accompanied by vertical and angular displacement. The following road wheels (2 ... 6) are moving along relatively smooth canvas treadmill. Therefore, the variable elasticity «tyre - rubberized running track» less effect on the dynamics of contact interaction and the nature of loading elements chassis.

On the solid roads the nature of the interaction of tires with track is extremely unstable[2]. Such conditions makes rubber pads aggravated in case of of the interaction «tyre - rubberized running track» and increases dynamic loading of balance-bob.

Considered design of the balance-bobs has a high probability of failure-free operation within a given resource. However, when driving on the hard road destruction of the balance-bobs comes in 2 ... 5 thousand kilometers. Thus, this work is devoted to analyze this phenomenon and identify ways to improve resource. Performed experimental researches and corresponding rating data, obtained in accordance with the methodology [4], shows that the magnitude and rigidity of the elastic convergence in contact «tyre - rubberized running track» on the length of truck are variable due to different thickness of rubber layer (fig.1). The function of the elastic convergence is defined by varying the thickness of the elastic rubberized running track layer \( H_D \in (0; 21; 27) \) mm and geometrical sizes of tires 95×45; 110×41; 110×33.75; 95×30 with an estimated diameter of 560 mm when changing the vertical load from 0 to static value - 10 kN. The greatest value of the elastic convergence is observed at zero \( H_D \). This value along the truck is constantly and is 5 mm (graph.1, fig1) and the smallest - at \( H_D = 27 \) mm (tire dimensions are 95 × 30 mm) and varies from 1.0 to 2.5 mm (graph.5, fig1). Most elastic convergence is observed in the area of hinge track, where the thickness of rubber layer and the contact patch is minimal. For other values of the tire size elastic convergence has an intermediate value, preserving the above mentioned character. The Obtained results and the nature of the destruction can hypothesize about the destruction of the balance-bobs when driving on solid ground by dynamic load, which is not covered previously, formed by parametric variations of road wheels. The condition which is necessary for excitation of such oscillations is variable stiffness in touch «tyre - rubberized running track» [3].

To study a new dynamic phenomena and solving the inverse task of providing the required resource necessary to develop a mathematical model and determine the stability region of parametric resonance. Most mathematical models of cushioning body allows to study the motion of a high-speed tracked vehicle as a linear or nonlinear mechanical system with deterministic or random perturbations, to iden-
tify patterns that reflect the connection of parameters ride with the design parameters of the suspension system in specific driving conditions, loading items of chassis.

Figure 1: Definition of the functions of the elastic convergence «tyre - rubberized running track»:

1. tyre 560×95×45 (metal running track);
2. tyre 560×110×33.75;
3. tyre 560×110×41;
4. tyre 560×95×45 (for plots 2, 3, 4 $H_D=21$ mm.);
5. tyre 560×95×30 ($H_D=27$ mm.)

This approach takes the assumption that the caterpillar is deformable homogeneous tape, although the writings of V. P. Abramov has been a significant influence on the dynamic of track vehicle. Issues of perturbations caused by relief of running track, dedicated many works of Omsk Tank Engineering Institute scientists. However, the study built on the basis of kinetostatic models, i.e. without the dynamic properties of the system. A model of contact interaction «tyre - rubberized running track», including in the area hinges tracks composed approximately, based on static data without rolling and load fluctuations. In fact, the interaction of «tyre - rubberized running track» corresponds to the operation of narrow-band filter. However, an analysis conducted on qualitative relationships, taking the vertical axis acceleration road wheels in the form of harmonic or constant functions do not take into account the rate of deformation and relaxation of the materials, i.e. absorbing and smoothing ability. This leads to significant discrepancies (50%) amplitudes of acceleration, and the resonant mode is impossible to imitate. The above determines the need to develop a mathematical model that better reflects the formation of dynamic loading.

In the work adopted a single-mass design scheme and mathematical model of the dynamic process of vibration impact formation on suspension block, generated by the convergence of elastic track roller with a relief of the running track. This system includes roller mass with a mass of the hub axle, part of the mass of the rocker lever and the moving parts, shock absorbers. Interaction of rubber solid tire track roller and the deformable layer of rubberized running track, forming a variable stiffness, and the coefficient of viscous resistance, taking into account the hysteresis losses in rubber.

The function of the elastic convergence along the tracks $y_{D1}$ determined by solving the contact problem [4] and experimental data(fig.1, graph.4). In the simulation data taken for the reference roller with the
parameters of $560 \times 95 \times 45$ (the value of the elastic convergence $y = y_0 \pm \Delta y$, $y_0 = 4\text{mm}$, $\Delta y = 0.5 \sin (pt)$, stiffness $c(t) = \frac{F}{y} = c_0 \pm \Delta c(t)$, $c(t) = (2.5 + 0.5 \sin (pt)) \cdot 10^6$ N/m, the frequency of excitation $p = 2\pi \frac{V}{t_r}$, where $V$ - velocity, and $t_r$ - step of track. Excluding the viscous differential equation of motion of the linear system is given by the Mathieu-Hill [3]:

$$\ddot{Z} + \omega_0^2 [1 + 2\mu \Phi(t)]Z = 0,$$

where $\ddot{Z}$, $Z$ – vertical acceleration and displacement of the reference roller;

$\omega_0^2$ – natural frequency of the system,

$$\omega_0 = \frac{c}{m};$$

$\mu$ – parameter modulation stiffness, $\mu = \frac{\Delta c}{c_0};$

$\Phi(t)$ – some periodic function.

Changing the stiffness $\Phi(t)$ can be described by an arbitrary periodic function, for example, Fourier series However, to obtain analytical solutions modified stiffness was adopted by a harmonic law. In this equation, viscous forces, which are known [3], has not yet taken into account. As we know they reduce the instability region, i.e. have a stabilizing effect. However, linear forces are not able to limit the amplitude of the oscillations at parametric resonance. When the harmonic change $c(t)$ Mathieu equation reduces to:

$$\ddot{Z} + \omega_0^2 [1 + 2\mu \cos (pt)]Z = 0.$$

To analyze the stability of Mathieu equation is usually given to the shape parameters of the chart Ince-Stretto (диаграммы Айнса-Стретта):

$$\frac{d^2Z}{dt^2} + \left[a + 2h \cos (2\tau)\right]Z = 0,$$

where $a = \left(\frac{2\omega_0}{p}\right)^2$, $a = \frac{c_0^2}{\pi^2 m V^2}$, $h = a \mu$, $h = \frac{c_0^2}{\pi^2 m V^2}$, $2\tau = pt$. In this form of Mathieu's equation is possible to analyze the stability of parametric oscillations without solving it. The value of the above parameters for the problem with the speed variation is given in the table and figure Ince-Stretto.

<table>
<thead>
<tr>
<th>Table 1: Stability analysis of solutions of the Mathieu equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed, $V$, м/с</td>
</tr>
<tr>
<td>-----------------</td>
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<tr>
<td>Value diagram options</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Location parameters and the conclusion of the stability</td>
</tr>
</tbody>
</table>
Figure 2: The results of the calculation:
1 - Evaluation of the stability of parametric oscillations of the diagram Ince-Stretto (the numbers 1, 2, 3, 4 correspond to T1, T2, T3, T4, Table 1);
2 - Dependence of the critical value of the parameter modulation and velocity on the ratio of frequencies.

From the data table and figure 2.1 shows that the majority of speed limits is not a solution is stable and is accompanied by a parametric resonance. Points 2 and 4, the corresponding rate of 7 and 20 m/s are in the field of sustainability. But this region is quite narrow and with little variation of system parameters (heating tire wear elastic layers) increases the probability of excitation of parametric resonances.

The inverse problem - determination of the conditions that exclude the excitation of parametric resonances is based on the analysis of the parameter modulation stiffness $\mu$ and the permissible speed $V$ as a function of frequency ratio

$$\frac{p}{2\omega_0} = k$$

($k=0.5, 1, 2, \ldots$).

To perform such an analysis, the differential equation (1) Mathieu complemented component of viscous resistance $2\varepsilon Z$. The movement track roller in this case is described by the equation

$$\ddot{Z} + 2\varepsilon \dot{Z} + \omega_0^2 [1 + 2\mu \cos(pt)]Z = 0.$$  

(2)

Boundary core, the most important area of instability in

$$\frac{p}{2\omega_0} = 1,$$

is defined as follows. The solution of differential equation on the boundary in the first approximation is taken in the form,

$$Z = C \cdot \cos \left( \frac{pt}{2} \right) + D \cdot \sin \left( \frac{pt}{2} \right)$$

where $C, D$ - arbitrary constants. Substituting the solution in (2), after conversion, and application of the method of harmonic balance differential equation is reduced to the system:
\[
\begin{aligned}
&\left\{\omega_0^2 - \frac{p^2}{4} + \omega_0^2 \mu\right\} C + \varepsilon p D = 0 \\
&\varepsilon p C + \left\{\omega_0^2 - \frac{p^2}{4} - \omega_0^2 \mu\right\} D = 0
\end{aligned}
\]

(3)

The conditions under which (2) can be represented in approximate form corresponds to zero value of the determinant of system (3):

\[
\begin{aligned}
&\left\{\omega_0^2 - \frac{p^2}{4}\right\} - \omega_0^4 \mu^2 + \varepsilon^2 p^2 = 0.
\end{aligned}
\]

Dividing this equation on \(\omega^4\) and expressing \(2\varepsilon / \omega\) in terms of the logarithmic decrement \(\Delta = b / m\omega_0\), with discrete values neglecting small quantities of higher order for \(k = 1\):

\[
\frac{p}{2\omega_0} = \left\{1 \pm \left(\mu^2 - \left(\frac{\delta}{\pi}\right)^2\right)^{0.5}\right\}^{0.5}
\]

When \(k \approx 1\), critical value of stiffness modulation is \(\mu = \frac{\delta}{\pi}\), and the corresponding speed limits is

The results of the calculation of the required functions for the drive wheels with the above parameters are given in Table 2 and Figure 2.2.

<table>
<thead>
<tr>
<th>Parameter stability area (k)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>The ratio of frequencies (p = \frac{2\omega_0}{k})</td>
<td>(p = 2\omega_0)</td>
<td>(p = \omega_0)</td>
<td>(p = \frac{2}{3}\omega_0)</td>
<td>(p = 0.5\omega_0)</td>
</tr>
<tr>
<td>The critical value of stiffness modulation (\mu_{kp})</td>
<td>0,1</td>
<td>0,32</td>
<td>0,46</td>
<td>0,56</td>
</tr>
<tr>
<td>Accordingly speed, (V), m/s</td>
<td>32</td>
<td>16</td>
<td>11</td>
<td>8</td>
</tr>
</tbody>
</table>

The graph shows that to avoid parametric oscillations drive wheels when driving cars at speeds above 15 m/s value for the depth of modulation \(\mu\) not exceed 0,10 ... 0,12. Given the tendency to further enhance the qualities of high-speed vehicles, the problem of exclusion of parametric resonance becomes even more relevant. To assess the correctness of the assumptions and the adequacy of the model to the real process performed experimental studies. In the process of the vehicle moving with rubberized running tracks on a flat road with low deformable basis for a smooth change of speed from 5 to 50 mph and back check the balance of vertical acceleration of road wheels. Fragments of oscillograms characterizing the oscillatory processes of extreme (1) and medium (3) balances are shown in Figure 3.
A) speed 22...25 km/h;

B) speed 32...35 km/h;
3.2

Figure 3: Results of experimental researches:

3.1 – Fragments of oscillograms changes in vibration acceleration in the balance-bobs of extreme and average road wheels;

3.2 – The amplitude spectrum of acceleration for the three ranges of velocities.

Fragments of oscillograms shows that the process of movement is accompanied by significant fluctuations of road wheels with increasing amplitude up to 25 ... 30 g (250 ... 300 m/c²) and track frequency. The parameters correspond to the motion described by Mathieu’s equation, significantly different from the oscillations, defined by differential equations with constant coefficients. The difference lies in the fact that the parametric oscillations occur over a wide area, not only at the coincidence of natural frequencies of the perturbing. The value of additional previously ignored by the high dynamic load formed by parametric variations in the mass of the wheels in 50 kg up to 15 kN (the amplitude commensurate with the calculated static load). This explains the limitation of the balance-bobs resource when driving on the road with low deformable base. Consequently, to ensure the required resources needed to eliminate the condition of excitation of parametric oscillations. It should be noted that the variable stiffness not only limits the durability of the elements of the running gear, but also leads to vibration load of other vehicle systems. At certain speeds the movement can generate vibrations in the assembly units of high-precision laser optical-equipment control systems installed in the average office machine, disrupting the efficiency of its functioning.

For the numerical estimates of the parameters of oscillatory processes carried out related pilot studies in which the recorded vibroacceleration on the mirror of the optical device. Amplitude-frequency characteristics of vibration accelerations for the three ranges of velocities are shown in Figure 7 and contains several distinct peaks. The first two with a frequency of up to 2 and 21 Hz, respectively, low-frequency vibrations caused by the body and motor disturbance. Others correspond to track frequency and multiples it (Table 3).

<table>
<thead>
<tr>
<th>Speed, km/h</th>
<th>Track frequency, Hz</th>
<th>Multiple frequencies, Гц</th>
</tr>
</thead>
<tbody>
<tr>
<td>22...25</td>
<td>42...44 (majeure)</td>
<td>84...88; 126...132</td>
</tr>
<tr>
<td>32...35</td>
<td>60</td>
<td>120 (majeure)</td>
</tr>
<tr>
<td>50</td>
<td>70...90</td>
<td>140...180</td>
</tr>
</tbody>
</table>

Eliminating the above vibrations can be supplied with appropriate filtering in an electromagnetic actuator mirror.
It should be noted that parametric resonance leads to a significant heating of the tires. In determining the heat-stressed tire wheels on the resonant modes of operation blocks suspension arising from the movement of vehicles at a speed of 25 ... 30 km / h, it was found that the mileage on the road 6 km long heating of the tire occurs as well as at the maximum speed at which resonance is absent, a distance of 18 km (temperature of the internal busses 2,3,4-loaded drive wheels was 128 ... 135 °C from the ambient temperature - 23 °C).

**Conclusions**

1. Variable stiffness in contact «tire - rubberized running track» lead to the excitation of parametric resonances.

2. When vehicles are moving on roads with a low deformable ground parametric oscillation drive wheels limit the longevity of the elements of chassis, disrupting the efficient operation of the laser optical-equipment control systems. In addition, these fluctuations increase heat-stressed tires significantly.

3. The procedure of design calculation of the elements chassis cars regarding the choice of parameters of «tire - rubberized running track», providing the required bearing capacity, durability and allowable temperature schedule, it is necessary to add exceptions to the condition of parametric resonance.

**Reference**


