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ON N-HOMOGENEOUS C*-ALGEBRAS OVER TWO-DIMENSIONAL ORIENTED COMPACT MANIFOLDS

© M. V. Shchukin

Belarusian national technical university
ul. Hmelnizkogo 9, Minsk, 220013, Belarus
e-mail: *mvshchukin@bntu.by*

On n-homogeneous C^*-algebras over a two-dimensional compact oriented connected manifold.

Shchukin M. V.

Abstract. We consider the n-homogeneous C^*-algebras over a two-dimensional compact oriented connected manifold. Suppose A be the n-homogeneous C^*-algebra with space of primitive ideals homeomorphic to a two-dimensional connected oriented compact manifold $P(A)$. It is well known that the manifold $P(A)$ is homeomorphic to the sphere P_k glued together with k handles in the hull-kernel topology. On the other hand, the algebra A is isomorphic to the algebra $\Gamma(E)$ of continuous sections for the appropriate algebraic bundle E. The base space for the algebraic bundle is homeomorphic to the set P_k. By using this geometric realization, we described the class of non-isomorphic n-homogeneous $(n \geq 2)$ C^*-algebras over the set P_k. Also, we calculated the number of non-isomorphic n-homogeneous C^*-algebras over the set P_k.

Keywords: *C^*-algebra, primitive ideals, base space, algebraic bundle, operator algebra, irreducible representation*

Introduction

In [1] I. Gelfand and M. Naimark proved that for any C*-algebra A there exists a Hilbert space H such that A is isomorphic to the algebra $B(H)$ of bounded operators on H. Furthermore, let A be a commutative C*-algebra. Thus there exists a Hausdorff space M such that A is isomorphic to the algebra $C(M)$ of all continuous functions on M. Let π be an irreducible representation of the commutative C*-algebra A. Hence the dimension of $\pi(A)$ equals 1. Moreover, let A be a non-commutative C*-algebra. Consider an irreducible representation π of the algebra. If there exists an integer n such that for any π the dimension of $\pi(A)$ equals n then the algebra A is said to be n-homogeneous. In [2, 3] J. Fell, I. Tomiyama, M. Takesaki proved that a n-homogeneous C*-algebra A is isomorphic to the algebra of all continuous sections for the appropriate algebraic bundle ξ.

In [4] F. Krauss and T. Lawson described the set of algebraic bundles over the spheres S^k.

In [5] A. Antonevich and N. Krupnik described the difference between bundles and algebraic bundles over the sphere S^k. On the other hand, in that paper they introduced some operations on the classes of algebraic bundles over S^k. In [6] S. Disney and I. Raeburn described the set of algebraic bundles over the torus T^2 and T^3. In the present paper we describe the set of algebraic bundles over two-dimensional compact oriented manifolds.

Let us remind that a triple (E, B, p) is called bundle if the following conditions hold:

(I) E and B are topological spaces.

(II) $p : E \to B$ is a continuous surjection.

The space E is called bundle space, the space B is said to be base space. The surjection p is called projection. The set $F = p^{-1}(x)$ is the fiber over a point $x \in B$. For example, consider the product-bundle $E = B \times F$, where B and F are topological spaces. By p denote the projection $B \times F \to B$ to the first multiplier. The bundle ξ is said to be the trivial bundle if it is isomorphic to a product-bundle. On the other hand, consider the Mobius tape M. Note that the Mobius tape M is a non-trivial bundle. The circle S^1 is the bundle space. The interval I is the fiber. However M is not isomorphic to the product-bundle $S^1 \times I$. At the same time M is locally trivial.

A G-bundle $\xi = (E, B, p)$ is called algebraic bundle if the following conditions hold:

(I) The fiber F_x is the algebra $Mat(n) = C^{n\times n}$ of square matrices of order n.

(II) The group G is the group $Aut(n)$ of all automorphisms for the algebra $Mat(n)$.

Two bundles $\xi_1 = (E_1, B_1, p_1)$ and $\xi_2 = (E_2, B_{21}, p_2)$ are said to be isomorphic if there exists a homeomorphism $\gamma : E_1 \to E_2$ such that $\gamma(F_x) = F_{\alpha(x)}$. Here $\alpha : B_1 \to B_2$ is a homeomorphism of the bases, the set $F_{\alpha(x)} = p_2^{-1}(\alpha(x))$ is the fiber over the point $\alpha(x) \in B_2$.

1. Algebraic bundles over the compact connected two-dimensional oriented manifolds

Proposition 1 ([7])**.** If M is a compact connected two-dimensional oriented manifold, then M is homeomorphic to the sphere S^2 with k handles.

We denote it by P_k (k can be equal 0). Let ξ be an algebraic bundle (E, P_k, p) over P_k. Let n be the order of fiber $F \cong Mat(n)$ for ξ. Cut out the part D from P_k, where D is homeomorphic to the disk D^2. Further, we consider the set P_k as the union $(P_k \backslash D) \cup \overline{D}$, where $(P_k \backslash D) \cap \overline{D} = S^1$. The set D is contractible. Therefore, the restriction ξ_D of ξ to D is trivial. Thus, the restriction ξ_D is isomorphic to $Mat(n) \times D$.

Lemma 1. *The restriction $\xi_{P_k \backslash D}$ of the bundle ξ to the set $P_k \backslash D$ is trivial.*

Доказательство. The proof is by induction on number k of handles. Note that the case $B \cong P_0$ was conside red in [5]. First consider the case $B \cong P_1$, where P_1 is homeomorphic to the torus. Now we realize P_1 as torus. Cut out the set D from the set P_1. Now let us prove that the restriction $\xi_{P_1 \backslash D}$ of the bundle ξ to the set $P_1 \backslash D$ is trivial.

We realize the torus P_1 as the square I^2 with conditions of gluing u on its border:

$$u(1, y) = u(0, y); u(x, 0) = u(x, 1)(0 \leq x \leq 1; 0 \leq y \leq 1).$$

Let $I^2_{0.5}$ denotes the square with the side equals 0.5. Suppose that the set $I^2_{0.5}$ has the same center as I^2. Cut out the set $I^2_{0.5}$ from the square I^2. The set $I^2 \backslash I^2_{0.5}$ is homeomorphic to the set $P_1 \backslash D$. The homotopic class of $P_1 \backslash D$ is the same as the homotopic class of the border $\delta(I^2)$ with the functions of gluing u. It is homeomorphic to two circles; these two circles contain a common point. Every algebraic bundle over two circles is trivial [5]. Hence the restriction of ξ to $P_1 \backslash D$ is trivial. The base of the induction is proved.

The induction hypothesis. Let us suppose that for any integer $m \leq k$ any algebraic bundle ξ over $P_m \backslash D$ is trivial.

The step of the induction.

Let us show that the restriction of the algebraic bundle ξ to $P_{k+1} \backslash D$ is trivial. Indeed, cut out a handle $P_1 \backslash D_1$ from the set $P_{k+1} \backslash D$. Here we realize the handle as the set P_1 without the set D_1. Let L be the intersection of $P_{k+1} \backslash D$ and $P_1 \backslash D_1$. Thus the set L is homeomorphic to the unit interval I.

Now we have two sets: $P_k \backslash D_2$ and $P_1 \backslash D_1$ with the gluing function $\nu : L \to Aut(n)$ of the bundle ξ. The restrictions of the bundle ξ to the sets $P_k \backslash D_2$ and $P_1 \backslash D_1$ are trivial by the induction hypothesis. The class of the bundle ξ is determined by the homotopic class of the mapping $\gamma : L \to Aut(n)$[5]. Since the set L is contractible, it follows that the mapping γ is homotopic to the constant mapping. □

Lemma 2. *Let f be a continuous mapping from S^1 to $Aut(n)$, where $S^1 = \delta(P_k \backslash D)$. The identity $[f] = 0$ is a necessary and sufficient condition for the mapping f to have a continuous extension to $f^* : P_k \backslash D \to Aut(n)$. Here $[f]$ denotes the class of f from the group $\pi_1(Aut(n))$.*

Доказательство. The proof is by induction on the number of handles k.

1. The base of induction. The set P_0 is homeomorphic to the sphere S^2. In this case we can extend the mapping $f : S^1 \to Aut(n)$ to the disk D if and only if $[f] = 0([5])$. Moreover, we shall to prove the statement for the set $P_1 \cong T^2$. The set P_1 is homeomorphic to the torus T^2. On the other hand, the set P_1 is homeomorphic to the unit square I^2 with the rule u of gluing on the border: $u(1, y) = u(0, y), u(x, 0) = u(x, 1)(0 \leq x \leq 1, 0 \leq y \leq 1)$. Let us cut out the square

$I^2_{0.5}$ from the set I^2. The square $I^2_{0.5}$ has the same center as I^2. The side of $I^2_{0.5}$ is equal 0.5. The set $I^2 \backslash I^2_{0.5}$ is homeomorphic to the set $P_1 \backslash D$. Now we can consider the function f as the function on the border of $I^2_{0.5}$. Let $U(x,t) = f\left(x(1+t), (y(1+t))\right)$ be a homotopy such that $u(0) = f(x,y), u(1) = f^*(x,y)$, where $(x,y) \in \delta(I^2)$. Here $\delta(I^2)$ denotes the border of I^2. Consider the side $a = (x,0), (0 \leq x \leq 1)$ of the square I^2. The opposite side $c = (x,1), (0 \leq x \leq 1)$ is glued with a. When we move on the border δI^2 we move on the side c in opposite direction. In addition, the same statement is true for two other sides. Therefore, $[f^*] = [f] = 0$.

Otherwise let f be a mapping $f \in C(\delta(I^2_{0.5}), Aut(n))$ and $[f] = 0$. This yields that we can extend the mapping f to all of $I^2_{0.5}$ [5]. Let f^* be the extension of f to the square $I^2_{0.5}$: $f^* \in C(I^2_{0.5}, Aut(n))$. Denote by $f_2 \in C(P_k \backslash D_2, Aut(n)), f_1 \in C(P_1 \backslash D_1, Aut(n))$ the restrictions of f^* to the sets $P_k \backslash D_2$ and $P_1 \backslash D_1$. Consider any point $y \in I^2$. Since $y \in I^2$, we have $y = r \cdot x$, where $x \in \delta\left(I^2_{0.5}\right), r \in [0;2]$. For all r such that $r \in [0;1]$ we have $r \cdot x \in I^2_{0.5}$. By definition, put $f^*(r \cdot x) = f^*((2-r) \cdot x)$ for $r \in [1;2]$. Therefore, the mapping f^* is well defined with respect to the function u of gluing for the square I^2.

The assumption of the induction. Suppose the lemma is true for all $m \leq k$.

The step of the induction. Consider the set $P_{k+1} \backslash D$. Cut out one handle $P_1 \backslash D_1$ from the set $P_{k+1} \backslash D$. Let the set L_1 be the intersection of $P_{k+1} \backslash D$ and $P_1 \backslash D_1$. The set L_1 is homeomorphic to the unit interval I. Now consider the set $P_{k+1} \backslash D$ as a union $P_k \backslash D_2 \cup P_1 \backslash D_1$ $((P_k \backslash D_2) \cap (P_1 \backslash D) = L_1)$. Denote by $f^* \in C(P_{k+1} \backslash D, Aut(n))$ an extension of f to $P_{k+1} \backslash D$, where $f \in C\left(\delta(P_{k+1} \backslash D), Aut(n)\right)$. Let $\alpha_1 : I \to S_1 \cup L_1$ be a parametrization of $\delta(P_1 \backslash D_1)$ such that $\alpha_1\left([0;\frac{1}{2}]\right) = S_1, \alpha_1\left([\frac{1}{2};1]\right) = L_1$, $\alpha_2\left([0;\frac{1}{2}]\right) = L_2, \alpha_2\left([\frac{1}{2};1]\right) = S_2$.

Denote by $g(x)$ the element from the class $[f_1] + [f_2]$ such that

$$g(x) = \begin{cases} f_1\left(\alpha_1(2x)\right), x \in [0;\frac{1}{2}] \\ f_2\left(\alpha_2(2x-1), x \in [\frac{1}{2};1]\right) \end{cases}.$$

Now we define the homotopy by the next rule:

$$F(t,x) = \begin{cases} g\left(\frac{x}{t+1}\right), x \in [0;\frac{1}{2}] \\ g\left(\frac{x+1}{t+1}\right), x \in [\frac{1}{2};1] \end{cases}.$$

Thus $F(0,x) = g(x)$, $F(1;x) = \begin{cases} g\left(\frac{x}{2}\right), x \in \left[0;\frac{1}{2}\right] \\ g\left(\frac{x+1}{2}\right), x \in \left[\frac{1}{2};1\right] \end{cases} = f(x)$, because $g\left(\frac{x}{2}\right) = f(x)$ for all $x \in \left[0;\frac{1}{2}\right]$ and $g\left(\frac{x+1}{2}\right) = f(x)$ for all $x \in \left[\frac{1}{2};1\right]$. Note that $\alpha_1\left(\left[0;\frac{1}{2}\right]\right) = S_1, \alpha_2\left(\left[\frac{1}{2};1\right]\right) = S_2$. This implies that $[f] = [f_1] + [f_2]$.

Otherwise consider a mapping $f \in C(\delta D, Aut(n))$ such that $[f] = 0$. In this case we extend it to the set L_1. Let $S_1(t)\,(t \in [0;1])$ be a parametrization of the set $S_1 = \overline{D}\backslash\overline{D}_2$. Suppose $L_1(t)$ be a parametrization of $L_1 = \overline{D}_1 \cap \overline{D}_2$. Now we define the mapping f^* by the rule: $f^*(L_1(t)) := f(S_1(t))$. It follows in the standard way that $[f^*] = 0$ on the sets $D_1 = S_1 \cup L_1$ and $D_2 = S_2 \cup L_1$.

By the inductive hypothesis extend the mapping f^* to the set $P_k\backslash D_2$, because $[f^*/\delta D_2] = 0$. Finally, extend the mapping f^* to the set $P_1\backslash D_1$. □

Suppose ξ_1 and ξ_2 be two algebraic bundles with fiber $Mat(n)$ over the set $(P_k\backslash D)\cup\overline{D}$. Let $\eta_{12} = \gamma_2^{-1}\gamma_1$ be the gluing function for the bundle ξ_1 and $\mu_{12} = u_2^{-1}u_1$ be the gluing function for the bundle ξ_2. Denote by ν_1 the map from $\xi_1/(P_k\backslash D) \to (P_k\backslash D) \times Mat(n)$ and by ν_2 the map from $\xi_1/\overline{D}$ to $\overline{D} \times Mat(n)$. These maps are well defined by lemma 1. For any point $x \in \delta(D)$ the image of the fiber F_x from the bundle $\xi_1/(P_k\backslash D)$ is the fiber $(F_1)_x$ from the bundle $\xi_1/\overline{D}$. Thus the map η_{12} generates an automorphism $(\gamma_1)_x$ of the algebra $Mat(n)$ over every point $x \in \delta(D)$. The mapping $\gamma_3(x) = (\gamma_1)_x : S^1 \to Aut(n)$ is continuous, because the restriction of the bundle ξ_1 to $\delta(D)$ is trivial. By the same argument, the gluing function μ_{12} generates the mapping $\gamma_4 \in C(S^1, Aut(n))$. Denote by $[\gamma_3]$ the class of mapping γ_3 in the group $\pi_1(Aut(n)) \cong Z/nZ$. In this notation, let $\theta : \pi_1(Aut(n)) \to Z/nZ$ be the corresponded isomorphism. Suppose $-[\gamma_4]$ be the element $\theta^{-1}(-\theta([\gamma_4]))$.

Theorem 1. *A necessary and sufficient condition for the bundles ξ_1 and ξ_2 to be isomorphic is* $[\gamma_3] = \pm[\gamma_4]$.

Доказательство. Denote by $\gamma : \xi_1 \to \xi_2$ the isomorphism of the bundles. Let $\alpha : P_k \to P_k$ be the corresponded homeomorphism of the bases for the bundles. Suppose $\eta_{12} = \gamma_2^{-1}\gamma_1$ be the gluing function for the bundle ξ_1 over $(P_k\backslash D) \cup \overline{D}$. Denote by $\mu_{12} = u_2^{-1}u_1$ the gluing function for the bundle ξ_2 over $(P_k\backslash\alpha(D))\cup\alpha(\overline{D})$. In this notation, $u_1 : \xi_1/(P_k\backslash\alpha(D)) \to (P_k\backslash\alpha(D)) \times Mat(n)$, $u_2 : \xi_2/(\alpha(\overline{D})) \to (\alpha(\overline{D})) \times Mat(n)$.

Let β be a homeomorphism $P_k \to P_k$ such that $\alpha(\overline{D}) = \overline{D}$ and $\alpha(\delta(\overline{D}))$ has the same orientation as $\delta(\overline{D})$.

Denote by β_1 the extension of β to the isomorphism of the bundles: $\beta_1 : \xi_2/\left(P_k\backslash\alpha(\overline{D})\right) \to \xi_2/\left(P_k\backslash\overline{D}\right)$. Let us remark that it is possible, because $\beta(\overline{D}) = \overline{D}$ and $\beta(P_k\backslash D) = P_k\backslash D$. Denote by $\mu_{12}^* : \xi_2/\delta(P_k\backslash D) \to \xi_2/\delta(\overline{D})$ a mapping such that the next diagram is commutative:

$$\begin{array}{ccc} \xi_2/(P_k\backslash\alpha(D)) & \xrightarrow{\mu_{12}} & \xi_2/\left(\alpha(\overline{D})\right) \\ \downarrow\beta_1 & & \downarrow\beta_2 \\ \xi_2/(P_k\backslash D) & \xrightarrow{\mu_{12}^*} & \xi_2/\overline{D} \end{array}$$

In this case, the bundle $\beta_1\left(\xi_2/(P_k\backslash\alpha(D))\right)\underset{\mu_{12}^*}{\cup}\beta_2\left(\xi_2/\alpha(\overline{D})\right)$ is isomorphic to the bundle ξ_2.

Let β_3 be the isomorphism of the bundles.

Suppose α be a homeomorphism $\alpha : P_k \to P_k$ for the bases such that $\beta\circ\alpha\left(\delta(\overline{D})\right)=\delta(\overline{D})$. The restriction of the bundle ξ_1 to the set $P_k\backslash D$ is trivial. Further, let $\beta_5\in C\left(P_k\backslash D, Aut(n)\right)$ be a mapping defined by the isomorphism $\beta_3\circ\gamma$. The image of a fiber F_x under the mapping β_5 is a fiber $\beta_3\circ\gamma(F_x)$ for any point $x\in P_k\backslash D$. In addition, let $\beta_6\in C(\overline{D}, Aut(n))$ be a mapping defined by the isomorphism $\beta_3\circ\gamma$. The image of a fiber F_x under the mapping β_6 is a fiber $\beta_3\circ\gamma\left(F_x\right)$ for any point $x\in\overline{D}$.

We have the next commutative diagram:

$$\begin{array}{ccc} \xi_1/(P_k\backslash D) & \xrightarrow{\eta_{12}} & \xi_1/\overline{D} \\ \downarrow\beta_3\circ\gamma & & \downarrow\beta_3\circ\gamma \\ \xi_2/(P_k\backslash D) & \xrightarrow{\mu_{12}^*} & \xi_2/\overline{D} \end{array}$$

The image of a fiber F_x under the mapping $\beta_3\circ\gamma$ is a fiber $F_{\beta\circ\alpha(x)}$. Therefore we have $\gamma_4\left(\beta\circ\alpha(x)\right)\beta_5(x)=\beta_6\circ\gamma_3(x)$. Further,

$$[\gamma_4(\beta\circ\alpha(x))]+[\beta_5(x)]=[\beta_6(x)]+[\gamma_3(x)] \tag{1}$$

The mappings $\beta_5(x)$ and $\beta_6(x)$ are defined on the sets $P_k\backslash D$ and $\overline{D}$ correspondingly. Therefore we have $[\beta_5(x)]=[\beta_6(x)]=0$ by the lemma 2. Using (1), we get

$$[\gamma_4\left(\beta\circ\alpha(x)\right)]=[\gamma_3(x)] \tag{2}$$

Let the mapping $\beta\circ\alpha$ changes the orientation of the circle $\delta\left(\overline{D}\right)$. Therefore $[\gamma_4(\beta\circ\alpha(x))]=-\left[\gamma_4(x)\right]$.

Otherwise, let the mapping $\beta\circ\alpha$ do not changes the orientation of the circle $\delta(\overline{D})$. In this case, $[\gamma_4(\beta\circ\alpha(x))]=[\gamma_4(x)]$. Actually we obtain $[\gamma_4]=\pm\left[\gamma_3\right]$.

On the other hand, let $[\gamma_4]=\pm\left[\gamma_3\right]$. Suppose we have $[\gamma_4]=-\left[\gamma_3\right]$. Denote by α a homeomorphism $P_k\to P_k$ such that $\alpha\left(\overline{D}\right)=\overline{D}$. Let the homeomorphism α changes the orientation of the circle $S^1=\delta\left(\overline{D}\right)$. In this case, denote by γ_1 the isomorphism $v_1^{-1}\circ u_1$ from $\xi_1/(P_k\backslash D)\to\xi_2/(P_k\backslash D)$. Here, v_1 is the isomorphism of the bundles $\xi_2/(P_k\backslash D)\to(P_k\backslash D)\times Mat(n)$ such that restriction of v_1 to $P_k\backslash D$ equals α. We see that the isomorphism $u_1 : \xi_1/(P_k \backslash D) \to (P_k \backslash D) \times Mat(n)$ produces the identity homeomorphism I for the bundle bases. Note that the isomorphism γ_1 produces a mapping $\gamma_5 \in C(P_k \backslash D, Aut(n))$. Thus the mapping $(\gamma_4(\alpha x))\,\gamma_5(x)\,(\gamma_3(x))^{-1} \in C\left(S^1, Aut(n)\right)$ produces the isomorphism $\eta_{12} \circ \gamma_1 \circ \mu_{12}^{-1} : \xi_1/\delta\overline{D} \to \xi_2/\delta\overline{D}$ for the trivial bundles. In addition, the homeomorphism α changes the orientation for the circle $S^1 = \delta D$. Now we get $[\gamma_4(\alpha x)] = -[\gamma_4(x)]$. We obtain $[\gamma_5(x)] = 0$ by the lemma 2. In addition, the next equality has a place: $[(\gamma_3(x))^{-1}] = -[\gamma_3(x)]$. Denote by γ_7 the extension of $\gamma_4(\alpha x) \circ \gamma_5(x) \circ (\gamma_3(x))^{-1}$ to $\overline{D}$ by lemma 2. This means that $\gamma_7 \in C\left(\overline{D}, Aut(n)\right)$. Define the isomorphism $\gamma_2 : \xi_1/\overline{D} \to \xi_2/\overline{D}$ by the rule $(x, y) \to (\alpha(x), \gamma_7(x) \cdot y)\,, (x \in \overline{D}, y \in F_x)$. The isomorphism is well-defined with respect to the functions of gluing for the bundles ξ_1 and ξ_2. Further, define a map $\gamma : \xi_1 \to \xi_2$ by the next rule: $\begin{cases} \gamma_1 \ on \ \xi_1/(P_k \backslash D) \\ \gamma_2 \ on \ \xi_1/\overline{D} \end{cases}$. This implies that the map γ is an isomorphism of bundles. On the other hand, let $[\gamma_4] = [\gamma_3]$. Define the homeomorphism α as identity $I : P_k \to P_k$. Arguing as before, we get the map $\gamma_4(\alpha x) \circ \gamma_5(x) \circ (\gamma_3(x))^{-1} \in C\left(S^1, Aut(n)\right)$ such that $\left[\gamma_4(\alpha x) \cdot \gamma_5(x) \cdot (\gamma_3(x))^{-1}\right] = [\gamma_4(\alpha x)] + [\gamma_5(x)] + \left[(\gamma_3(x))^{-1}\right] = [\gamma_4] - [\gamma_3] = 0$. Therefore, we can extend the map $\gamma_4(\alpha x) \cdot \gamma_5(x) \cdot (\gamma_3(x))^{-1}$ to a map $\gamma_7 \in C\left(\overline{D}, Aut(n)\right)$. The map γ_7 produces an isomorphism $\gamma_2 : \xi_1/\overline{D} \to \xi_2/\overline{D}$ that is coordinated with gluing functions for the bundles ξ_1 and ξ_2. At the same time the map γ_7 is coordinated with the isomorphism γ_1. These isomorphisms γ_1 and γ_2 produce an isomorphism $\gamma : \xi_1 \to \xi_2$. □

Theorem 2. *Suppose $n = 2l$ or $n = 2l + 1 (l \in N)$. Then there are $l + 1$ non-isomorphic algebraic bundles with fiber $Mat(n)$ over the set P_k.*

Доказательство. Let $n = 2l$. In other notation, we need to find number of classes in $Z/_{nZ}$ with respect to the equality $l = -l$. We have the next classes $\{0\}, \{1, 2l-1\}, \{2, 2l-2\}, \{3, 2l-3\}, ..., \{l-1, l+1\}, \{l\}$. There are exactly $l+1$ such classes.

Further, let $n = 2l + 1$. In this case we have the next classes: $\{0\}, \{1, 2l\}, \{2, 2l-1\}, \{3, 2l-2\}, \{4, 2l-3\}, ..., \{l-1, l+2\}, \{l, l+1\}$. There are $l + 1$ such classes. □

Conclusion

In the work we described the class of non-equivalent algebraic bundles with base space homeomorphic to the two-dimensional compact oriented connected manifold. We calculated the number of non-isomorphic n-homogeneous C^*-algebras with space of primitive ideals homeomorphic to the two-dimensional compact connected oriented manifold. Further, it is interesting to know the structure of n-homogeneous C^*-algebras with its space of primitive ideals homeomorphic to more complicated manifolds, for example, 3-dimensional manifolds and other.

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