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MATHEMATICAL MODELING OF RANDOM PERIODIC EVENTS

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Any random event can be foreseen and calculated with the help of universal science called “Mathematics”. Let’s take for example “Ergodic Markov chains.”

Markov chain is a sequence of some random events. Its possibilities only depend on previous state of some system but also they don’t depend on events that were before events of previous state.

Ergodic Markov chains only consist of ergodic states and events. That means that system can go to the next state and go back to the previous state. Even if the system in the last state it can come back to the first state after some events or steps.

Ergodic Markov chains are used for modeling and calculating periodic random events such as economic or technical problems: efficiency of economic plan, proving or disproving great performance of machine. But method works if enough observations are made [1].

To show the work of ergodic Markov chain let’s examine abstract gas stations with two service channels and three places in line. Let our system be ergodic so we could use ergodic Markov chains. Let A be the intensity of cars arriving at the stations and let’s B be car refueling intensity. The values of A and B are known from observations and statistical findings, let A be 1 and B be 1/3. Our system has 6 conditions:

1 – two service channels are free;
2 – one service channel is taken;
3 – two service channels are taken and no cars in line;
4 – two service channels are taken and one car in line;
5 – two service channels are taken and two cars in line;
6 – two service channels are taken and two cars in line.

To simplify these conditions, they are collected into one “graph”. Graph is a mathematical picture of a real system (Fig.1).

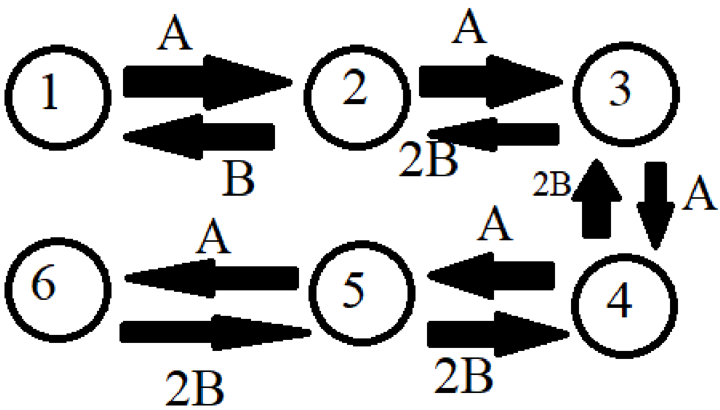

Fig. 1. The Graph of the System

With the help of the graph mathematics makes up differential equations. The solution of the equation system is possibility diagram.

So how do we make up differential equations?

The graph depicts six states and for them we have six possibilities.

For example, let's look at the first possibility – the possibility that our system will be in first state.

In the first differential equation the derivative of the first possibility equals intensity that go to the first state multiplied by the possibility of the other state from which the event goes minus the intensity that go from our state multiplied by our state [2]:

$dp_1/dt = -A*p_1(t) + B*p_2(t)$

If follow such rules for the other states, we will get this system of differential equations:

1. $dp_1/dt = -A*p_1(t) + B*p_2(t)$
2. $dp_2/dt = A*p_1(t) + 2B*p_3(t) - (A + B)*p_2(t)$
3. $dp_3/dt = A*p_2(t) + 2B*p_4(t) - (A + 2B)*p_3(t)$
4. $dp_4/dt = A*p_3(t) + 2B*p_5(t) - (A + 2B)*p_4(t)$
5. $dp_5/dt = A*p_4(t) + 2B*p_6(t) - (A + 2B)*p_5(t)$
6. $dp_6/dt = A*p_5(t) - 2B*p_6(t)$

But also we must consider that the sum of all possibilities equals one:

$p_1 + p_2 + p_3 + p_4 + p_5 + p_6 = 1$

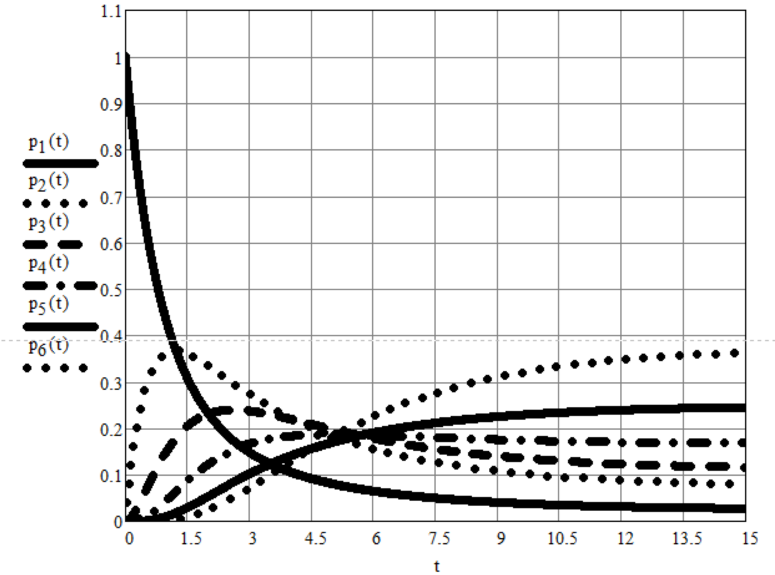

Fig. 2. The Diagram of the System

If we look at the diagram (Fig.2) we can see every possibility strives for exact number and these numbers can be calculated. If we turn our differential equations into simple that equals zero, we can find full possibilities of the system. The solution of the system: 2.5%, 7.4%, 11.1%, 16.6%, 25%, 37.4%. As we can see the possibility of the last condition is significantly big, thus we can say that our gas station is not very effective. So, the owner should provide more service channels or more places in line.

Conclusion: Mathematics is a very powerful system that can predict random events with enough observations of course.

References

1. Ergodic Markov Chain [Electronic resource] – Mode of access: https://www.sciencedirect.com/topics/mathematics/ergodic-markov-chain. – Date of access: 13.03.2024.

2. What is Mathematical Modeling? [Electronic resource] – Mode of access: https://ubcmath.github.io/MATH360/process/overview.html. – Date of access: 14.02.2024.