the quantity of masses. In this case when $\omega \to 0$ there is one change of sign in the sequence (5), since $\omega = 0$ is a «fictitious» frequency of the system corresponding to the motion of the system as a solid whole. The quantity of real-valued frequencies for such system is equal to $n-1$.

The suggested method is effective not only for calculation of natural frequencies of transmissions but for determination of their quantity in the given frequency ranges. This allows to access the possibility of occurrence of resonance conditions in operation under real loads.

The assessment of reliability of the results obtained through the described method is given in comparison with the accurate analytical solutions. The suggested algorithm is realized in the calculation of tractor-drawn silage combine harvester. The obtained natural frequencies have satisfactorily agreed with those found through other methods.

References. **1.**Przemieniecki J.S. Theory of Matrix Structural Analysis. McGraw-Hill Book Company, New York, Toronto, London, Sidney. 1968. 2. Halfman R. Dynamics. Addison-Wesley Publishing Company, Inc., London. 1971.

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NATURAL OSCILLATIONS OF MODIFIED CONSTRUCTIONS

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Equation of natural oscillations for undamped mechanical system in matrix form is written as:

$$([K]-\omega^2[M])\{q\}=0$$

or

$$([E]-\omega^2[P][M])\{q\}=0 \qquad (1)$$

where

$[E]$ - identity matrix,

$[P]=[K]^{-1}$ - compliance matrix inverse of the stiffness matrix $[K]$,

$[M]$ – mass matrix,

$\{q\}$- displacement vector (at the given natural frequency ω_i the vector $\{q\}_i$ corresponds to the $\boldsymbol{i}$ -th form of oscillations).

If the mechanical system has ***small*** constructional modifications with the compliance and mass matrixes changed to $[\Delta P]$ and $[\Delta M]$, respectively, then equation (1) for the new construction modification is written as:

$$([E]-(\omega^2+\Delta\omega^2)([P]+[\Delta P])([M]+[\Delta M]))(\{q\}+\{\Delta q\})=0 \qquad (2),$$

where $\Delta\omega^2$ and $\{\Delta q\}$ – are changes in the frequency square and vector of form of natural oscillations.

As a first approximation, omitting the small increment products, the matrix equaion (2) is written as:

$$\{A\}+\lambda\{B\}=0 \qquad (3)$$

where

$$\{A\} = [[\Delta P][M] + [P][\Delta M]]\{q\} \tag{4}$$

$$\{B\} = [P][M]\{q\} \tag{5}$$

$$\lambda = \frac{\Delta\omega^2}{\omega^2} \; . \tag{6}$$

Let us assume that the error vector when solving equation (3) is

$$\{\varepsilon\} = \{A\} + \lambda\{B\}. \tag{7}$$

For its minimization the quadratic form extremum conditions are written as

$$\frac{d}{d\lambda}\left(\{\varepsilon\}^T\{\varepsilon\}\right) = 0 \tag{8}$$

($\{\varepsilon\}^T$ - transposed error vector), from whence comes relation

$$\lambda = \frac{-\{A\}^T\{B\}}{\{B\}^T\{B\}} \; . \tag{9}$$

Thus, the change of the frequency square of natural oscillations is

$$\Delta\omega^2 = \lambda\omega^2 \tag{10}$$

To determine the vector of form of natural oscillations of the modified construction the equation (1) is transformed into:

$$(\{q\} + \{\Delta q\}) = ((\omega^2 + \Delta\omega^2)([P] + [\Delta P])([M] + [\Delta M]))(\{q\} + \{\Delta q\}) \quad (11).$$

Neglecting in the right side of equation (11) the increment $\{\Delta q\}$ and increments product of matrixes $[\Delta P]$ and $[\Delta M]$, we obtain expressions for the form vector $\{q\}$ and corresponding value of the frequency square ϖ^2 of natural oscillations of the modified construction:

$$(\{q\} + \{\Delta q\}) = \omega^2(1 + \lambda)(\{A\} + \{B\}),$$

$$\varpi^2 = \omega^2 + \Delta\omega^2 = \omega^2(1 + \lambda).$$

The suggested algorithm allows to accelerate considerably the process of study of frequencies and forms of natural oscillations of various constructions, especially with great quantity of degrees of freedom and for systems with distributed parameters. It is particularly effective in cases when range of changes in stiffness and masses of the studied system is small and pre-determined and the study is connected with fast search of solutions optimal from dynamics point of view.

The report discusses the results of application of the suggested method of assessment of natural frequencies and forms for different mechanical systems and comparative analysis of the computing process acceleration.

References 1. Halfman R. Dynamics.Addison-Wesley Publishing Company, Inc., London. 1971. 2.Przemieniecki JS. Theory of Matrix Structural Analysis. McGraw-Hill Book Company, New York, Toronto, London, Sidney. 1968.