UDC 539.3

STUDY OF BENDING DEFORMATIONS OF A TWO-SUPPORTED BEAM USING ANSYS 22.2

[1]Khvasko V. M., [2]Bandara H. M. S. H.
[1]Belarusian National Technical University,
Minsk, Belarus, hvasko.victoriya@gmail.com,
[2]Belarusian National Technical University,
Minsk, Belarus, bsanduni8@gmail.com

Abstract. The article considers the bending of a two-support beam using the example of the simplest load option - a concentrated force applied in the middle of the span. The values of deformations in the middle of the span are calculated for various variants of the beam cross section. The calculation results were modeled using the Ansys program, version 22.2

In the course of strength of materials, one often encounters bars that are subjected to various types of deformations. A rod is a body, one dimension of which is greater than the other two. Bending is a type of deformation when bending moments occur in cross sections [1]. In the simplest cases, rods of a solid cross section, made of one material, are considered. A beam is a bar working in bending [2].

In this paper we study the bending of a beam on two supports using the simplest example of a load - one concentrated force is applied in the middle of the beam span. Three variants of the beam cross section are considered: 1) a solid rectangular section; 2) a rectangular section composed of two rods of the same material; 3) a rectangular section composed of two rods of dissimilar materials.

The beam loading scheme and options for its cross section are shown in Fig. 1. We assume that the following characteristics and dimensions are given: Young's modulus for steel $E_1 = 2 \cdot 10^5$ MPa, Young's modulus for copper $E_2 = 1.2 \cdot 10^5$ MPa [3], the width b = 100 mm, the height h = 80 mm, the length l = 2 m.

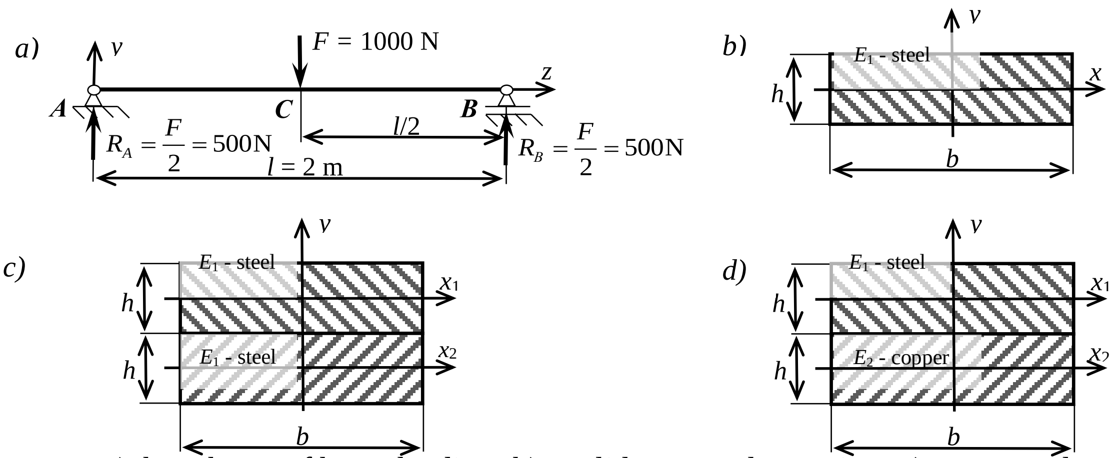

Figure 1 – *a)* the scheme of beam loading; *b)* a solid rectangular section; *c)* a rectangular section composed of two rods of the same material; *d)* a rectangular section composed of two rods of dissimilar materials

To determine the deformations that occur in the cross section of the beam, we can use the method of initial parameters [1]. Then the equation for deflections will look like:

$$E_1 I_x y = E_1 I_x y_0 + E_1 I_x \theta_0 \cdot z + R_A \cdot \frac{z^3}{6} - F \cdot \frac{(z-1)^3}{6}, \quad (1)$$

where $I_x = \frac{bh^3}{12} = \frac{100 \cdot 80^3}{12} = 4.267 \cdot 10^6 \text{mm}^4$ – moment of inertia [2] of one rectangular section shown in fig. 1*b*,

y_0, θ_0 – the initial parameters assuming the origin on support *A* (so $E_1 I_x y_0 = 0$).

Let's express the deformation at point *C* using the equation (1) when *z* = 1 m:

$$E_1 I_x y_C = E_1 I_x y_0 + E_1 I_x \theta_0 \cdot z + R_A \cdot \frac{z^3}{6} = 0 + E_1 I_x \theta_0 \cdot 1 + 500 \cdot \frac{1^3}{6}. \quad (2)$$

For defining the initial parameter $E_1 I_x \theta_0$ let's calculate the deformation on support *B* using the equation (1) when *z* = 2 m:

$$E_1 I_x y_B = E_1 I_x y_0 + E_1 I_x \theta_0 \cdot z + R_A \cdot \frac{z^3}{6} - F \cdot \frac{(z-1)^3}{6} = 0 + E_1 I_x \theta_0 \cdot 2 + 500 \cdot \frac{2^3}{6} - 1000 \cdot \frac{(2-1)^3}{6} = 0, \quad (3)$$

whence

$$E_1 I_x \theta_0 = \frac{\frac{1000}{6} - \frac{4000}{6}}{2} = -250 \text{N} \cdot \text{m}^2. \quad (4)$$

Now we can define the deflection at point *C* using the formula (2):

$$E_1 I_x y_C = E_1 I_x \theta_0 \cdot z + R_A \cdot \frac{z^3}{6} = -250 \cdot 1 + 500 \cdot \frac{1^3}{6} = -166.67 \text{N} \cdot \text{m}^3, \quad (5)$$

so

$$y_C = -\frac{166,67 \cdot 10^9}{E_1 I_x} = -\frac{166.67 \cdot 10^9}{2 \cdot 10^5 \cdot 4.267 \cdot 10^6} = -0.195 \text{mm} = -1.95 \cdot 10^{-4} \text{m} \quad (6)$$

The results of theoretical calculations were also used to model a two-support beam using the Ansys Mechanical program, version 22.2. The resulting diagram is shown in fig. 2.

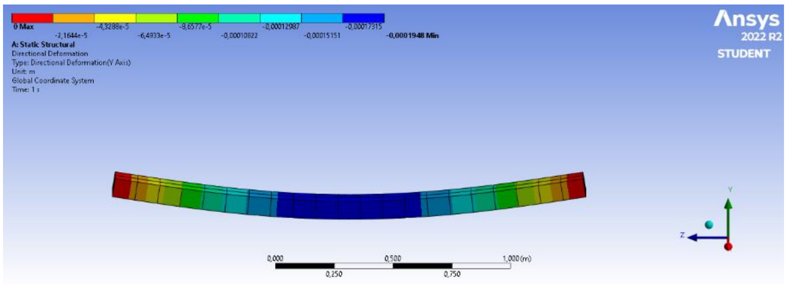

Figure 2 – The diagram of deformations of two-supported beam of solid rectangular section, using the Ansys 22.2

Then we will consider the case when the cross section of the beam is composed of two rods made of the same material – steel (shown in Fig. 1*c*). Let's express the deformation at point *C* using the equation (1):

$$(E_1 I_x)_t \cdot y_C = (E_1 I_x)_t \cdot y_0 + (E_1 I_x)_t \cdot \theta_0 \cdot z + R_A \cdot \frac{z^3}{6}, \quad (7)$$

where $(E_1 I_x)_t = E_1 \cdot (I_x + I_x) = E_1 \cdot 2I_x = 2 \cdot 10^5 \cdot 2 \cdot 4.267 \cdot 10^6 = 17.068 \cdot 10^{11}\,\mathrm{N \cdot mm^2}$ - total stiffness of two bars [4].

Now we can calculate the deflection at point *C* using the formulas (5) and (7) when z = 1 m:

$$y_C = -\frac{166.67 \cdot 10^9}{(E_1 I_x)_t} = -\frac{166.67 \cdot 10^9}{17.068 \cdot 10^{11}} = -0.098\mathrm{mm} = -0.98 \cdot 10^{-4}\mathrm{m} \quad (8)$$

The results of theoretical calculations were also used to model a two-support beam using the Ansys Mechanical program, version 22.2. The resulting diagram is shown in fig. 3.

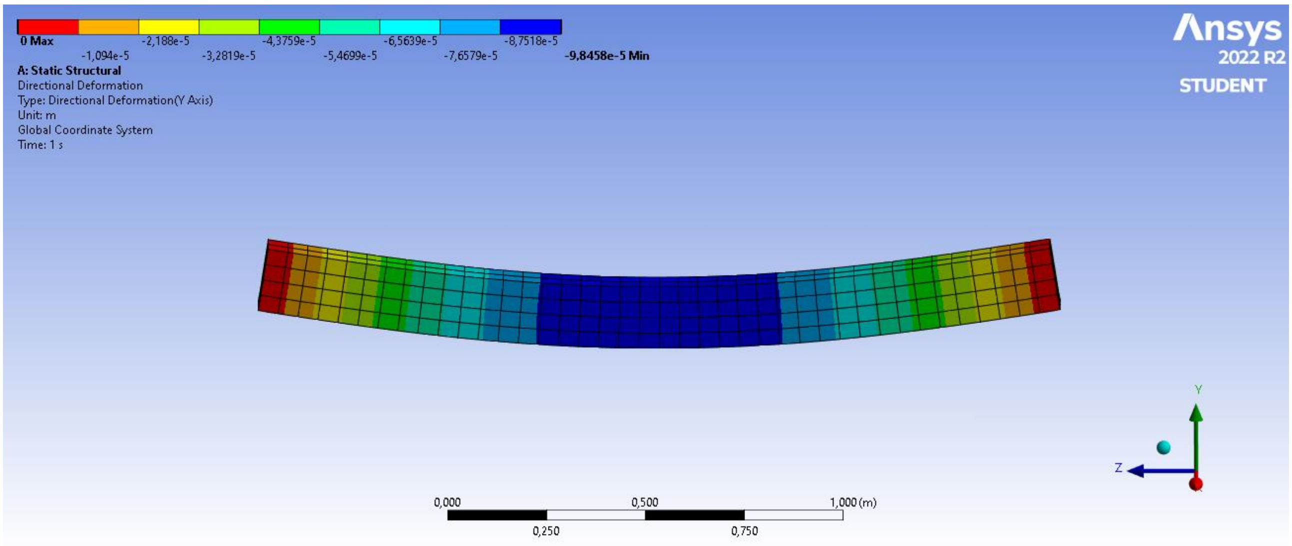

Figure 3 – The diagram of deformations of two-supported beam of rectangular section composed of two rods of the same material, using the Ansys 22.2

Now let's consider the case when the cross section of the beam is composed of two rods made of the dissimilar materials – steel and copper (shown in Fig. 1*d*). We will define the deformation at point *C* using the equation (1):

$$(EI_x)_t \cdot y_C = (EI_x)_t \cdot y_0 + (EI_x)_t \cdot \theta_0 \cdot z + R_A \cdot \frac{z^3}{6}, \tag{9}$$

where $(EI_x)_t = E_1 \cdot I_{x1} + E_2 \cdot I_{x1} = 2 \cdot 10^5 \cdot 4.267 \cdot 10^6 + 1.2 \cdot 10^5 \cdot 4.267 \cdot 10^6 = 13.654 \cdot 10^{11}\,\mathrm{N \cdot mm^2}$ – total stiffness of two bars [4].

So, let's calculate the deflection at point *C* using the formulas (5) and (9) when $z = 1$ m:

$$y_C = -\frac{166.67 \cdot 10^9}{(EI_{x1})_t} = -\frac{166.67 \cdot 10^9}{13.654 \cdot 10^{11}} = -0.122\,\mathrm{mm} = -1.22 \cdot 10^{-4}\,\mathrm{m}. \tag{10}$$

The results of theoretical calculations were also used to model a two-support beam using the Ansys Mechanical program, version 22.2. The resulting diagram is shown in fig. 4.

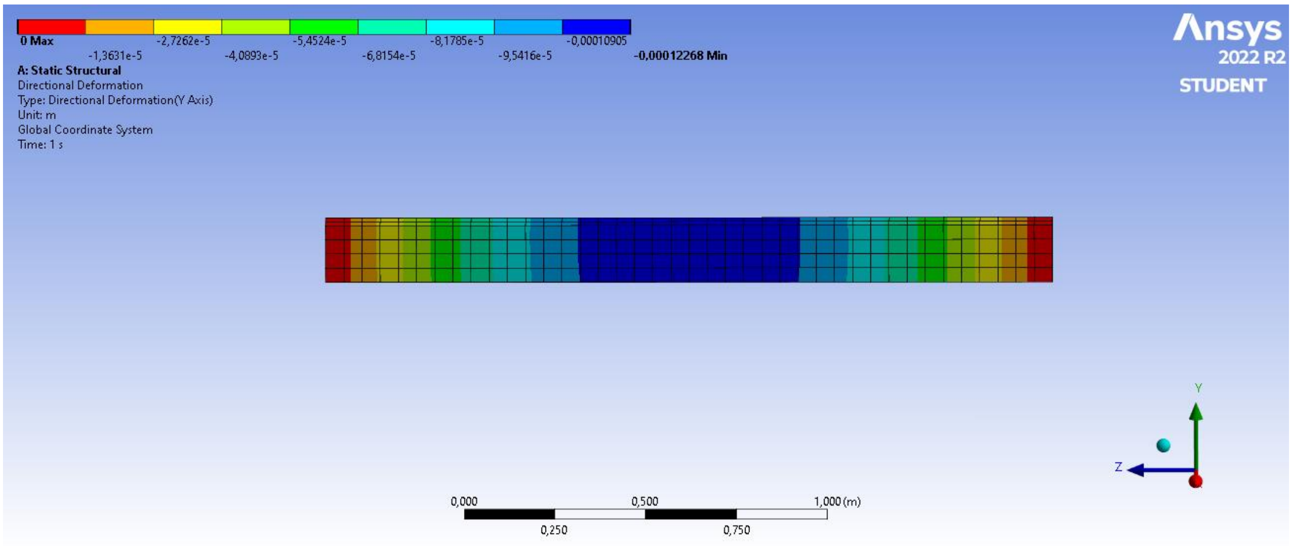

Figure 4– The diagram of deformations of two-supported beam of rectangular section composed of two rods of the dissimilar materials, using the Ansys 22.2

Thus, let's compare the results obtained for directional deformations that occur when a two-supported beam is bent under the action of a concentrated force applied in the middle of the span. The smallest values occur when the beam section is made up of two rectangular bars of the same size made of the same material (steel). When combining materials for the sections of composite bars (steel and copper), smaller deformations also occur compared to a solid rectangular section (steel). This is important to consider when designing parts of engineering structures.

Using the Ansys Mechanical program, version 22.2 during the studying of the course of strength of materials can help students for visualizing the beam bending for better understanding how directional deformations occur and how they change the shape of the beam.

References

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