

# Prediction of Dynamic Characteristics of Thermocouples with Thin-Wire Sensing Elements

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## Abstract

Thermocouples dynamic characteristics' prediction is one of the relevant directions in the field of dynamic measurements of non-stationary temperatures of liquid and gaseous media. Thermocouples dynamic characteristics' prediction makes it possible to provide effective continuous correction in automatic control systems for non-stationary temperatures. The purpose of this paper was to develop a theoretically justified relation linking the current or expected time constant of fine-wire thermocouples with the known time constant established at known parameters of liquid and gaseous media.

A formula linking the time constant of fine-wire thermocouples with the conditions of heat exchange with the measured medium and the thermophysical characteristics of the thermocouple sensing elements has been deduced. An approximate formula is also given for calculating the internal resistance of wire sensing elements of thermocouples, which must be considered when calculating the time constant of a thermocouple. In consideration of the obtained formulas, a multi-parameter relation linking the current or expected time constant of fine-wire thermocouples with the known time constant set at the known parameters of the measured media has been formed.

It is suggested to simplify the formed multi-parameter relation and make it dependent, for example, on the “expected velocity of the measured medium × expected density of the measured medium” complex ( $V_{m2}\rho_{m2}$ ). Simplified relations in the form of hyperbolic functions with constant parameters and argument in the form of  $V_{m2}\rho_{m2}$  complex were obtained for airflow at different temperatures, pressures, and velocities.

On the example of airflow, it is shown that the complex multi-parametric relation linking the expected and known time constants of thermocouples can be simplified to a hyperbolic dependence, where the argument can be the  $V_{m2}\rho_{m2}$  complex. Moreover, the degree of approximation of hyperbolic dependencies to the exact values of the multi-parametric relation can reach the  $R$ -square = 0.9592 criterion.

A multi-parametric relation has been proposed. That relates the known time constant of a thermocouple to the expected or current time constant of the same thermocouple at other parameters of the measured medium from the point of view of the heat exchange and thermal conduction theory. The proposed relation can be used in automatic control systems of non-stationary temperature of various liquid or gaseous media to provide continuous correction of thermocouples dynamic characteristics. Depending on the number of measured medium parameters, the suggested multi-parameter relation can be replaced by simplified relations with other complexes containing, for example, density, velocity, flow rate and pressure of the measured medium.

**Keywords:** thermocouple, dynamic characteristics, transmission function.

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# Прогнозирование динамических характеристик термопар с тонкопроволочными чувствительными элементами

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Прогнозирование динамических характеристик термопар является одним из актуальных направлений в области динамических измерений нестационарных температур жидких и газообразных сред. Прогнозирование динамических характеристик термопар позволяет обеспечить эффективную непрерывную коррекцию в системах автоматического управления нестационарной температурой. Целью данной работы являлась разработка теоретически обоснованного соотношения, связывающего текущую или ожидаемую постоянную времени тонкопроволочных термопар, с известной постоянной времени, установленной при известных параметрах жидких и газообразных сред.

Выведено выражение, связывающее постоянную времени тонкопроволочных термопар с условиями теплообмена с измеряемой средой и теплофизическими характеристиками чувствительных элементов термопар. Получена также приближённая формула для расчёта внутреннего сопротивления проволочных чувствительных элементов термопар, которое необходимо учитывать при вычислении постоянной времени термопары. С учётом полученных выражений сформировано многопараметрическое соотношение, связывающее текущую или ожидаемую постоянную времени тонкопроволочных термопар, с известной постоянной времени, установленной при известных параметрах измеряемых сред.

Сформированное многопараметрическое соотношение предложено упростить и сделать зависимым, например, от комплекса «ожидаемая скорость измеряемой среды × ожидаемая плотность измеряемой среды» ( $V_{m2}\rho_{m2}$ ). Для воздушного потока при различных температурах, давлении и скорости получены упрощённые соотношения в виде гиперболических функций с постоянными параметрами и аргументом в форме комплекса  $V_{m2}\rho_{m2}$ .

На примере воздушного потока показано, что сложное многопараметрическое соотношение, связывающее ожидаемую и известную постоянные времени термопар, можно упростить до гиперболической зависимости, где аргументом может выступить комплекс  $V_{m2}\rho_{m2}$ . Причём степень приближения гиперболических зависимостей к точным значениям многопараметрического соотношения может достигать по критерию  $R\text{-square} = 0.9592$ .

Предложено точное с точки зрения теории теплообмена и теплопроводности многопараметрическое соотношение, связывающее известную постоянную времени термопары с ожидаемой или текущей постоянной времени этой же термопары при иных параметрах измеряемой среды. Предложенное соотношение может быть использовано в системах автоматического управления нестационарной температурой различных жидких или газообразных сред для обеспечения непрерывной коррекции динамических характеристик термопар. В зависимости от числа измеряемых параметров среды предложенное многопараметрическое соотношение может быть заменено упрощёнными соотношениями с другими комплексами, содержащими, например, плотность, скорость, расход и давление измеряемой среды.

**Ключевые слова:** термопара, динамические характеристики, передаточная функция.

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## Introduction

Thermocouples with sensing thin-wire elements of various diameters are widely used to measure the temperature of air and gas flows in aviation gas turbine engines (GTE). In accordance with GOST 1790-2016 “Wire made of chromel  $T$ , alumel, copel and constantan alloys for thermoelectrodes of thermoelectric converters” thermoelectrode wires for thermocouples are made with a diameter from 0.2 to 5.0 mm.

When thermocouples are used as parts of automatic control systems (ACS) by temperature of gas-air flows there is a need for continuous correction of their dynamic characteristics with changing modes of operation of the GTE. Some design and research issues of various corrective devices and correction methods in the ACS of GTE and in other subjects where measurement of non-stationary temperatures is needed were explained in the works [1–9].

For optimal and continuous correction of dynamic characteristics of thermocouples in the ACS, it is necessary to set the structure and parameters of transfer functions of thermocouples with required accuracy. Meanwhile, changes of functions in changing operating modes of the GTE must also be taken into account. Works [10–17] are devoted to identification of dynamic characteristics of various temperature sensors and other measuring instruments.

In works [1–17] compensation of inertia of thermocouples is produced by using a dynamic first order model. This transfer function is:

$$W(p) = \frac{k}{Tp + 1}, \quad (1)$$

where  $k$  is the static conversion coefficient of a thermocouple, mV/°C;  $T$  is the time constant or an indicator of thermal inertia which depends on gas-air flows influencing parameters, s;  $p$  is the Laplace variable, 1/s.

There are several different ratios for accounting changes in the time constant  $T$  because of influencing parameters of gas-air flows in (1).

In the TGM 1594-79 “Measurement of non-stationary air flow temperature during bench tests of the GTE. Thermometers” the following ratio is given:

$$\varepsilon_1 = \varepsilon_0 \left( \frac{V_0 p_{0, st}}{V_1 p_{1, st}} \right)^n, \quad (2)$$

where  $\varepsilon_0$  is the indicator of thermal inertia of the thermometer at velocity  $V_0$  and static pressure  $p_{0, st}$ ;  $\varepsilon_1$  is the indicator of thermal inertia of the thermometer at velocity  $V_1$  and static pressure  $p_{1, st}$ ;  $n = 0.488$  is the empirical coefficient for an open transversely streamlined sensing element of a thermometer.

There are relations in works [4, 9]:

$$T_{tc} = T_{tc c} \left( \frac{G_{g c}}{G_g} \right)^{0.5}, \quad (3)$$

where  $T_{tc c}$  is the calculated time constant of the thermocouple at the estimated flow rate  $G_{g c}$  of the gas flowing around it;  $T_{tc}$  is current (expected) time constant of the thermocouple at the current (expected) consumption  $G_g$  of the gas flowing around it.

There is also a relation<sup>1</sup>:

$$\tau = \tau_0 \left( \frac{\rho_0 V_0}{\rho_i V_i} \right)^n \frac{A(\rho_i V_i)^n + 1}{A(\rho_0 V_0)^n + 1}, \quad (4)$$

where  $\tau$  is the current (expected) time constant of the thermocouple;  $\rho$  and  $V$  are air density and velocity that is blowing over the thermocouple; indexes 0 and  $i$  are related to the conditions of the experiment and flight;  $n = 0.6–0.85$ ;  $A = 0–0.05$  are constants determined by the design features of the thermocouple.

The theoretical justification of the relations (2)–(4) and the accuracy of calculating current or expected time constants of thermocouples based on them (indicators of thermal inertia), as well as their application areas had not been found. Analysis of (2)–(4) relationships also showed that time constants or indicators of thermal inertia established experimentally on certified installations at known speeds, air flow pressures and temperatures [18] are used as the initial data for the calculation.

Theoretical justification of the dependence of the time constant of the thermocouple on the expected operating conditions are given in the works [19–21]. This dependence has the following form:

$$T(\alpha) = \frac{1}{\alpha \Psi_T} + T_\infty, \quad (5)$$

where  $\alpha$  is the heat transfer coefficient of the sensing element of the thermocouple with the measured medium, W/(m<sup>2</sup>·K);  $\Psi_T$  is the criterion that

<sup>1</sup> Sinyakov, A.N. Automatic control systems for aircraft and their power plants / A.N. Sinyakov, F.A. Shaymardanov. – M.: Mechanical engineering, 1991.

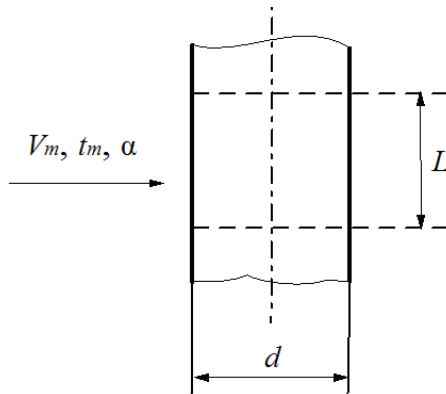
characterizes the uneven distribution of temperatures in a thermocouple and depends on the geometrical and physical features of the thermocouple and the conditions of its heat exchange;  $T_\infty$  is the value of the time constant with  $\alpha \rightarrow \infty$ .

The purpose of this work was to develop theoretically justified ratio, linking the current or expected time constant of thermocouples, described by a dynamic model (1), with a time constant established experimentally on air installations certified for these purposes.

## The main part

Time constant of thermocouples with wire sensing elements depends on the heat exchange with the measured medium (by air or gas flow), dimensions of sensing elements and their thermophysical characteristics.

Figure 1 shows the scheme of a transversely streamlined continuous cylinder of infinite length. The scheme is used for theoretical justification heat exchange of wire sensing elements of thermocouples with the measured medium.



**Figure 1** – Diagram of a transversely streamlined infinite cylinder:  $V_m$  – the velocity of the medium;  $t_m$  – the temperature of the medium;  $\alpha$  – the average heat transfer coefficient on the cylinder surface;  $d$  – the diameter of the cylinder;  $L$  – the length of the elementary section of the cylinder

The following expression defines time constant during heating or cooling of the cylinder illustrated on Figure 1:

$$T = (R_\alpha + R_T)C_T = R_\alpha C_T + R_T C_T = R_\alpha C_T + T_\infty, \quad (6)$$

where  $R_\alpha$  is the thermal resistance on the cylinder surface at length  $L$ ;  $R_T$  is the internal thermal resistance of the solid homogeneous cylinder;  $C_T$  is the

thermal capacity of the elementary section of the cylinder;  $T_\infty = R_T C_T$  is the time constant with  $\alpha \rightarrow \infty$ .

Thermal resistance  $R_\alpha$  on the surface of the cylinder at length  $L$  is determined by the following known hyperbolic dependence on the heat transfer coefficient  $\alpha$ :

$$R_\alpha = \frac{1}{\alpha S} = \frac{1}{\alpha \pi d L} \text{ [K/W]}.$$

The thermal capacity of the elementary section of the cylinder is equal to:

$$C_T = c \rho U = c \rho \frac{\pi d^2}{4} L \text{ [J/K]}, \quad (7)$$

where  $c$  is the specific heat capacity coefficient of the cylinder material, J/(kg·K);  $\rho$  is the cylinder material density, kg/m<sup>3</sup>;  $U$  is the volume of the elementary section of the cylinder, m<sup>3</sup>.

There is no exact formula for the calculation of the internal thermal resistance  $R_T$  of solid homogeneous cylinder. An approximate formula with a high degree of approximation can be obtained as follows.

There are precise calculation formulas for temperatures coaxial section and the average temperature of the cross-section of a cylinder of infinite length in the theory of thermal conductivity of solid bodies in non-stationary modes. The thermal EMF developed by thermocouples is determined by the average cross-section temperature of the  $\bar{\Theta}(\tau)$  of the sensing elements. Therefore the following known dependence is used to derive the calculation formula of the internal thermal resistance:

$$\bar{\Theta}(\tau) = (\Theta_m - \Theta_0) \cdot \left[ 1 - \sum_{n=1}^{\infty} \frac{4}{\mu_n^2} \exp(-\mu_n^2 \text{Fo}) \right], \quad (8)$$

where  $\Theta_0$  is the initial temperature of the solid cylinder;  $\mu_n$  are the roots of the Bessel function of the first kind of zero order; Fo is Fourier number.

For a solid cylinder Fourier number defined by the following expression:

$$\text{Fo} = \frac{\lambda}{c \rho R^2} \tau,$$

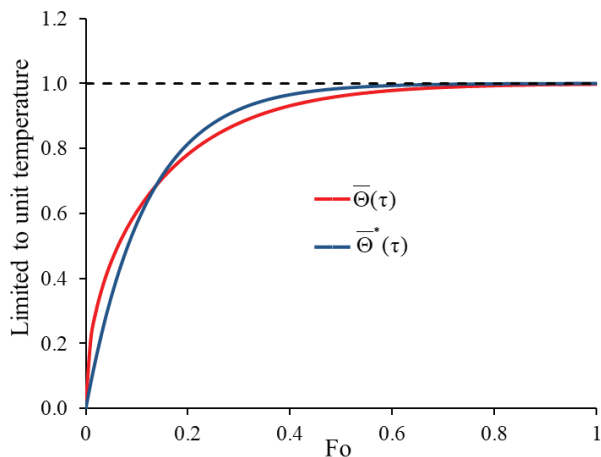
where  $\lambda$  is the coefficient of thermal conductivity of the solid cylinder material, W/(m·K);  $R = d/2$  is the radius of a solid cylinder, m;  $\tau$  is the time of the transition process of heating or cooling, s.

The following expression for the reduced average temperature over the cross section of the solid

cylinder will be obtained if in (8) ratio was accepted that  $\Theta_0 = 0$  and  $\Theta_m = 1$ :

$$\bar{\Theta}(\tau) = 1 - \sum_{n=1}^{\infty} \frac{4}{\mu_n^2} \exp(-\mu_n^2 \text{Fo}). \quad (9)$$

Figure 2 illustrates a graph of the dependence of the average cross-section temperature  $\bar{\Theta}(\tau)$  from the Fo number.



**Figure 2** – Dependences of temperatures reduced to unity on the Fourier number

The average cross-section temperature  $\bar{\Theta}(\tau)$  is formed from the sum of an infinite number of exponents with different bases and weight coefficients, as can be seen from (9). We propose to replace this dependency with a single exponent, the basis of which provides the best approximation to its dependence (9). The following regression function was obtained  $\bar{\Theta}^*(\tau)$  with the use of regression analysis:

$$\bar{\Theta}^*(\tau) = 1 - \exp(-8.422 \cdot \text{Fo}). \quad (10)$$

Figure 2 illustrates a graph of the installed function  $\bar{\Theta}^*(\tau)$ , which provides an approximation to the function  $\bar{\Theta}(\tau)$  at least 95.8 % on the interval Fo from 0 to 1.0.

The base of the exponent in (10) is transformed as follows:

$$-8.422 \cdot \text{Fo} = -8.422 \frac{\lambda}{c\rho R^2} \tau = -\frac{\tau}{\left(\frac{c\rho R^2}{8.422 \cdot \lambda}\right)} = -\tau / (R_T C_T) = -\tau / T_{\infty}. \quad (11)$$

The following approximate expression for the internal thermal resistance of a solid cylinder was obtained from (11) with (7) taken into account:

$$R_T = \frac{0.1187}{\pi \lambda L} \text{ [K / W]}.$$

At the same time, the time constant  $T_{\infty}$  with  $\alpha \rightarrow \infty$  will be equal to:

$$T_{\infty} = \frac{0.029675 c\rho d^2}{\lambda} \text{ [s]}.$$

There is a main formula for calculating the average coefficient of convective heat transfer on any cylindrical surface of the examined body in the case of transverse air (gas) flow for practical calculations:

$$\alpha = \lambda_m \frac{C \text{Re}^n \text{Pr}^m}{d} = \lambda_c \frac{C \left(\frac{V_m d \rho_m}{\eta_m}\right)^n \left(\frac{\eta_m c_p}{\lambda_m}\right)^m}{d}, \quad (12)$$

where  $C = 0.5$ ,  $n = 0.5$ ,  $m = 0.38$  with Re is from 5 to  $10^3$ ;  $C = 0.25$ ,  $n = 0.6$ ,  $m = 0.48$  with Re is from  $10^3$  to  $2 \cdot 10^5$ ;  $C = 0.023$ ,  $n = 0.8$ ,  $m = 0.37$  with Re is from  $2 \cdot 10^5$  to  $2 \cdot 10^9$ ;  $\lambda_m$  is the coefficient of thermal conductivity of the medium, W/(m·K);  $\eta_m$  is the coefficient of dynamic viscosity of the medium, Pa·s;  $c_p$  is the heat capacity of the medium at constant pressure, J/(kg·K);  $\text{Re} = V_m d / \nu_m$  is Reynolds number;  $\text{Pr}$  is Prandtl number.

Formula (12) is valid for thermocouples with open transversely streamlined wire sensing elements.

If the wire sensing elements of thermocouples are placed in the braking chambers, then it is necessary to use the speed in the braking chamber  $V_{kt}$  instead of the speed of  $V_m$  in formula (12). This is proposed to be calculated by the following expression in the work [20]:

$$V_{kt} = V_m \sqrt{\frac{1 - \xi_{tc}}{1 - \xi_{se}}},$$

where  $\xi_{tc}$  is the recovery coefficient of a thermocouple with a braking chamber;  $\xi_{se}$  is the recovery coefficient of the thermocouple sensor;  $V_m$  is the velocity of the measured medium flowing around the thermocouple.

The values of  $\xi_{tc}$  and  $\xi_{se}$  of specific types of thermocouples are established experimentally on certified installations [18].

#### Approach to solving the problem

Let's consider the expression (6) for two different heat exchange conditions corresponding to the heat exchange coefficients  $\alpha_1$  and  $\alpha_2$ :

$$T_1(\alpha_1) = \frac{0.25 c\rho d}{\alpha_1} + \frac{0.029675 c\rho d^2}{\lambda} \text{ [s]};$$

$$T_2(\alpha_2) = \frac{0.25c\rho d}{\alpha_2} + \frac{0.029675c\rho d^2}{\lambda} \text{ [s]},$$

where  $T_1(\alpha_1)$  is the time constant of a thermocouple with a wire sensing element with a heat transfer co-

$$\frac{T_2(\alpha_2)}{T_1(\alpha_1)} = \frac{\frac{0.25c\rho d}{\alpha_2} + T_\infty}{\frac{0.25c\rho d}{\alpha_1} + T_\infty} = \frac{\alpha_1(0.25c\rho d + \alpha_2 T_\infty)}{\alpha_2(0.25c\rho d + \alpha_1 T_\infty)} =$$

$$= \left( \frac{V_{m1}}{V_{m2}} \right)^n \left( \frac{\rho_{m1} \eta_{m2}}{\rho_{m2} \eta_{m1}} \right)^n \left( \frac{c_{m1} \eta_{m1} \lambda_{m2}}{c_{m2} \eta_{m2} \lambda_{m1}} \right)^m \left( \frac{\lambda_{m1}}{\lambda_{m2}} \right) \left[ \frac{c\rho d^2 + 4C\lambda_{m2}T_\infty \left( \frac{c_{m2} \eta_{m2}}{\lambda_{m2}} \right)^m \left( \frac{V_{m2} d \rho_{m2}}{\eta_{m2}} \right)^n}{c\rho d^2 + 4C\lambda_{m1}T_\infty \left( \frac{c_{m1} \eta_{m1}}{\lambda_{m1}} \right)^m \left( \frac{V_{m1} d \rho_{m1}}{\eta_{m1}} \right)^n} \right].$$

If we assume that the time constant  $T_1(\alpha_1)$  of the thermocouple is determined by the experimental transient characteristic obtained on a certified installation with known parameters of the air flow, then the expected time constant  $T_2(\alpha_2)$  with the

efficient  $\alpha_1$ ;  $T_2(\alpha_2)$  is the time constant of the same thermocouple with the heat transfer coefficient  $\alpha_2$ .

It can be seen that the time constants  $T_1(\alpha_1)$  and  $T_2(\alpha_2)$  agree well with the dependency (5).

Take the relation  $T_2(\alpha_2)$  to  $T_1(\alpha_1)$  taking into account the heat transfer coefficients according to (12):

expected parameters of the air flow can be calculated by the following ratio, similar in appearance to the ratios (2)–(4):

$$T_2(\alpha_2) = T_1(\alpha_1) \cdot \Psi, \quad (13)$$

where

$$\Psi = \left( \frac{V_{m1}}{V_{m2}} \right)^n \left( \frac{\rho_{m1} \eta_{m2}}{\rho_{m2} \eta_{m1}} \right)^n \left( \frac{c_{m1} \eta_{m1} \lambda_{m2}}{c_{m2} \eta_{m2} \lambda_{m1}} \right)^m \left( \frac{\lambda_{m1}}{\lambda_{m2}} \right) \left[ \frac{c\rho d^2 + 4C\lambda_{m2}T_\infty \left( \frac{c_{m2} \eta_{m2}}{\lambda_{m2}} \right)^m \left( \frac{V_{m2} d \rho_{m2}}{\eta_{m2}} \right)^n}{c\rho d^2 + 4C\lambda_{m1}T_\infty \left( \frac{c_{m1} \eta_{m1}}{\lambda_{m1}} \right)^m \left( \frac{V_{m1} d \rho_{m1}}{\eta_{m1}} \right)^n} \right].$$

The ratio (13) is an accurate from the point of view of the theory of heat transfer and thermal conductivity within the accuracy of calculating the internal thermal resistance of wire sensing elements of thermocouples. It is also can be used for gaseous media in which the thermophysical characteristics are known.

### Research methodology

Relation (13) is a complex multiparametric expression. It is wise to simplify it without significant loss of accuracy for easier practical use.

If we use the ratio (13) to track the change in the time constant of the thermocouple with continuous correction of its dynamic characteristic in the ACS GTD, then it is desirable to simplify it to two measured parameters. For example, to the current velocity  $V_{m2}$  and the current density  $\rho_{m2}$ . One of the options of this simplification might involve establishing the parameters of some regression function  $F$  where the argument is a  $V_{m2} \rho_{m2}$  complex.

Let's consider a particular case when the measured medium is an air flow with a speed ranged from 100 to 250 m/s, with a pressure ranged from 0.1 to 2 MPa and a temperature ranged from 20 to 727 °C.

As the object of research thermocouples with wire thermoelectrodes of chromel-alumel calibration with diameters of 0.2 and 1.2 mm were selected.

It was assumed that the time constants  $T_1(\alpha_1)$  of thermocouples were determined by experimental transient characteristics obtained on a certified installation with known parameters of the air flow, specifically when  $V_{m1} = 100$  m/s,  $p_{m1} = 0.1$  MPa,  $t_{m1} = 20$  °C and  $\rho_{m1} = 1.205$  kg/m<sup>3</sup>.

In our studies the number of Re was in the range from  $10^3$  to  $15 \cdot 10^3$ , which in (13) corresponded to  $C = 0.25$ ,  $n = 0.6$  and  $m = 0.48$ .

The results of calculating the function  $\Psi$  of the complex  $V_{m2} \rho_{m2}$  at  $d = 0.2$  mm and  $T_1(\alpha_1) = 0.092$  s are given in Table 1.

Table 1

The results of calculating the function  $\Psi$  of the complex  $V_{m2} \rho_{m2}$  at  $d = 0.2$  mm and  $T_1(\alpha_1) = 0.092$  s

| $V_{m2}$ ,<br>m/s | $p_{m2} = 0.1$ MPa<br>$t_{m2} = 20$ °C |  | $p_{m2} = 0.1$ MPa<br>$t_{m2} = 227$ °C |  | $p_{m2} = 1$ MPa<br>$t_{m2} = 227$ °C |  | $p_{m2} = 0.1$ MPa<br>$t_{m2} = 727$ °C |  | $p_{m2} = 2$ MPa<br>$t_{m2} = 627$ °C |  |
|-------------------|--|--|---|--|---------------------------------------|--|---|--|---------------------------------------|--|
|                   | $\Psi$                                 | $V_{m2}\rho_{m2}$ ,<br>kg/(m <sup>2</sup> s) | $\Psi$                                  | $V_{m2}\rho_{m2}$ ,<br>kg/(m <sup>2</sup> s) | $\Psi$                                | $V_{m2}\rho_{m2}$ ,<br>kg/(m <sup>2</sup> s) | $\Psi$                                  | $V_{m2}\rho_{m2}$ ,<br>kg/(m <sup>2</sup> s) | $\Psi$                                | $V_{m2}\rho_{m2}$ ,<br>kg/(m <sup>2</sup> s) |
| 100               | 1.0000                                 | 120.5  | 1.1634                                  | 70.0   | 0.2946                                | 694.0  | 0.3357                                  | 347.0  | 0.2186                                | 769.0  |
| 150               | 0.7846                                 | 180.8  | 0.9128                                  | 105.0  | 0.2316                                | 1041.0                                       | 0.2638                                  | 520.5  | 0.1720                                | 1153.5                                       |
| 200               | 0.6607                                 | 241.0  | 0.7685                                  | 140.0  | 0.1953                                | 1388.0                                       | 0.2224                                  | 694.0  | 0.1452                                | 1538.0                                       |
| 250               | 0.5782                                 | 301.3  | 0.6725                                  | 175.0  | 0.1712                                | 1735.0                                       | 0.1949                                  | 867.5  | 0.1273                                | 1922.5                                       |

The results of calculating the function  $\Psi$  of the complex  $V_{m2}\rho_{m2}$  at  $d = 1.2$  mm and  $T_1(\alpha_1) = 1.130$  s are given in Table 2.

Table 2

The results of calculating the function  $\Psi$  of the complex  $V_{m2} \rho_{m2}$  at  $d = 1.2$  mm and  $T_1(\alpha_1) = 1.130$  s

| $V_{m2}$ ,<br>m/s | $p_{m2} = 0.1$ MPa<br>$t_{m2} = 20$ °C |  | $p_{m2} = 0.1$ MPa<br>$t_{m2} = 227$ °C |  | $p_{m2} = 1$ MPa<br>$t_{m2} = 227$ °C |  | $p_{m2} = 0.1$ MPa<br>$t_{m2} = 727$ °C |  | $p_{m2} = 2$ MPa<br>$t_{m2} = 627$ °C |  |
|-------------------|--|--|---|--|---------------------------------------|--|---|--|---------------------------------------|--|
|                   | $\Psi$                                 | $V_{m2}\rho_{m2}$ ,<br>kg/(m <sup>2</sup> s) | $\Psi$                                  | $V_{m2}\rho_{m2}$ ,<br>kg/(m <sup>2</sup> s) | $\Psi$                                | $V_{m2}\rho_{m2}$ ,<br>kg/(m <sup>2</sup> s) | $\Psi$                                  | $V_{m2}\rho_{m2}$ ,<br>kg/(m <sup>2</sup> s) | $\Psi$                                | $V_{m2}\rho_{m2}$ ,<br>kg/(m <sup>2</sup> s) |
| 100               | 1.0000                                 | 120.5  | 1.1625                                  | 70.0   | 0.2983                                | 694.0  | 0.3392                                  | 347.0  | 0.2228                                | 769.0  |
| 150               | 0.7858                                 | 180.8  | 0.9132                                  | 105.0  | 0.2356                                | 1041.0                                       | 0.2677                                  | 520.5  | 0.1764                                | 1153.5                                       |
| 200               | 0.6625                                 | 241.0  | 0.7697                                  | 140.0  | 0.1995                                | 1388.0                                       | 0.2265                                  | 694.0  | 0.1497                                | 1538.0                                       |
| 250               | 0.5805                                 | 301.3  | 0.6743                                  | 175.0  | 0.1755                                | 1735.0                                       | 0.1991                                  | 867.5  | 0.1319                                | 1922.5                                       |

Figure 3 shows the results of calculating the function  $\Psi$  as a function of the complex  $V_{m2}\rho_{m2}$  in the form of point values according to the data from Tables 1 and 2.

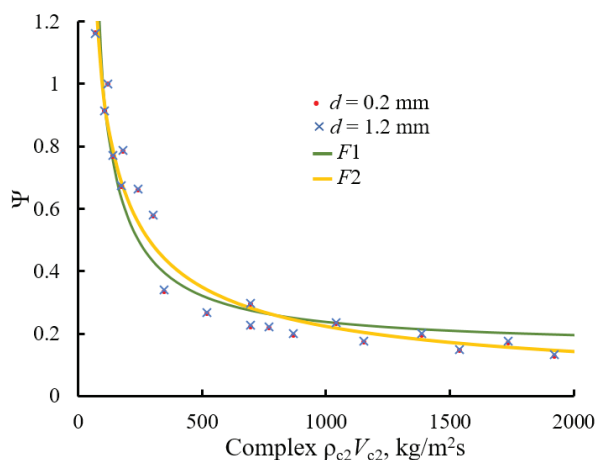


Figure 3 – Dependence of the function  $\Psi$  on the complex  $V_{m2}\rho_{m2}$  at  $d = 0.2$  mm and  $d = 1.2$  mm

As it can be seen from Figure 3 and Tables 1 and 2, the results of calculating the function  $\Psi$  for the same values of the complexes  $V_{m2}\rho_{m2}$  almost do not depend on the selected diameters of the wire thermoelectrodes of the thermocouples.

In addition, there is a hyperbolic law of dependence of the function  $\Psi$  on complexes  $V_{m2}\rho_{m2}$  that may be the basis for choosing a regression function.

The following two functions are selected as a regression functions:

$$F1 = \frac{A}{\rho_{m2} V_{m2}} + B \quad \text{and} \quad F2 = \frac{A}{(\rho_{m2} V_{m2})^t} + B.$$

The following parameters of the selected regression functions were determined with an accuracy estimate based on the  $R$ -square criterion with the use of the regression analysis:

$$F1 = \frac{84.05}{\rho_{m2} V_{m2}} + 0.1529 \quad \text{with } R\text{-square} = 0.9184;$$

$$F2 = \frac{19.22}{(\rho_{m2} V_{m2})^{0.6445}} \text{ with } R\text{-square} = 0.9592.$$

Graphs of functions  $F1$  and  $F2$  are also shown in Figure 3.

If the regression function  $F2$  is chosen as the most accurately describing function  $\Psi$ , then the relation (13) will take the following simplified form:

$$T_2(\alpha_2) = T_1(\alpha_1) \cdot \frac{19.22}{(\rho_{m2} V_{m2})^{0.6445}}.$$

According to (13) the accuracy of determining the time constant  $T_2(\alpha_2)$  also depends on the accuracy of establishing  $T_1(\alpha_1)$  according to experimental transient characteristics obtained on installations certified for these purposes.

As is known, experimental transient characteristics always contain interference of various levels and spectra, which reduce the accuracy of determining time constants.

A spectral method for determining the time constants of various dynamic models of thermocouples based on experimental transient characteristics containing high-level interference lying outside the information part of the amplitude spectrum of the signal  $s(\tau)$  and formed according to the established rules from the transient characteristic is proposed in work [22].

For transfer functions of thermocouples of the (1) type in [22], the following relation is proposed to determine the time constant  $T$  under known test conditions:

$$S(j\omega) = \frac{U_m \cdot T}{\sqrt{1 + \omega^2 T^2}}, \quad (14)$$

where  $S(j\omega)$  is the signal amplitude spectrum  $s(\tau)$ ;  $\omega$  is the frequency, rad/s;  $U_m$  is the amplitude of the  $s(\tau)$  signal with  $\tau = 0$ ;  $T$  is the time constant, s.

In cases where the experimental transient characteristics contain harmonic interference of any level and frequency the work [23] proposes a spectral method insensitive to harmonic interference for determining time constants of different dynamic models of thermocouples.

The following relation is proposed for transfer functions of thermocouples of the (1) type in [23] to determine the time constant  $T$  under known test conditions:

$$\text{Im}(\omega) = \frac{U_m T}{1 + \omega^2 T^2}, \quad (15)$$

where  $\text{Im}(\omega)$  is the imaginary part of the amplitude spectrum  $S(j\omega)$ ;  $\omega$  is the frequency, rad/s;  $T$  is the time constant, s.

The determination of time constants  $T$  in (14) and (15) in [22, 23] is proposed to calculate with the use of regression analysis on the calculated amplitude spectrum and its imaginary part.

According to the methods proposed in [22, 23] the established time constants of the experimental transient characteristics can be used as time constants  $T_1(\alpha_1)$  in (13) to increase the accuracy of forecasting the expected time constants.

## Conclusion

Ratio linking the known time constant of a thermocouple obtained during tests on certified installations with known thermophysical characteristics, velocity and pressure of the gas flow with the expected or current time constant of the same thermocouple with other thermophysical characteristics, velocities and pressures of the gas flow has been developed.

This relation was developed according to the theory of the heat transfer theory and the thermal conductivity in the external flow of cylindrical bodies.

As opposed to the known equivalent ratios this developed ratio takes into account the entire complex of gas flow parameters, thermophysical characteristics and the diameter of the sensor element of the thermocouple that determines values of the time constants of the thermocouple.

Simplified relations linking the known time constant of a thermocouple obtained during tests on certified installations at a known speed and pressure of the gas flow with the expected or current time constant of the same thermocouple at different speeds and pressures of the gas flow are proposed. Simplified ratios describe the initial ratio with an accuracy no worse than  $R\text{-square} = 0.9184$ . It was shown with the use of changing the temperature of the air flow by a thermocouple with wire thermoelectrodes of chromel-alumel calibration with diameters 0.2 and 1.2 mm as example.

Further research in this field can be aimed at establishing simplified dependencies of the function  $\Psi$  for gas flows that are products of combustion of various fuels in atmospheric air and at replacing simplified ratios that contain pressure, flow rate, velocity and density of gas flows.



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