UDC 629.7

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FLIGHT PATH CONTROL FOR UNMANNED AERIAL VEHICLE

Проведено исследование оптимизированного по заданному критерию закона управления беспилотного летательного аппарата при наведении его по траектории, заданной опорными точками в инерциальной системе координат. Приводится пример, иллюстрирующий работоспособность предложенных теоретических положений.

Studying the optimized control law specified criteria on UAV while hovering over a path defined by the reference points in the inertial frame. An illustrative example is the theoretical efficiency of the proposed provisions.

Flight path of UAV

Planning UAV missions is a difficult problem due a host of real-world complexities, including the presence of vehicle constraints, poorly known obstacle locations, and dynamic information updates that require re-planning. In addition, these sophisticated optimization problems must be solved efficiently for them to be of any use in real-time planning. Fig. 1 – illustrates a simple abstract UAV mission. In this mission, we have 3 points (A, B, C) which represent the flight path for UAV.

The flight path of the above mission is divided into 3 sections: $R_A(t_0, t_1)$, $R_B(t_1, t_2)$, $R_C(t_2, t_c)$.

R_A – The flight path of UAV into the zone of the main tasks.

R_B – The flight path of UAV to implement main purpose (monitoring the target).

R_C – The end of the mission and trajectory of the UAV to the place of landing.

As a rule, it is the return to the starting point, (t_0) and (t_c), respectively, the start time and the time of ending mission.

Optimization control of flight path

There are a variety of analytical methods of synthesis and optimization control of aircraft. Although theoretically most of them allow an exact solution, in practice the final result depends on the use of mathematical models which are generally not adequate to the actual conditions.

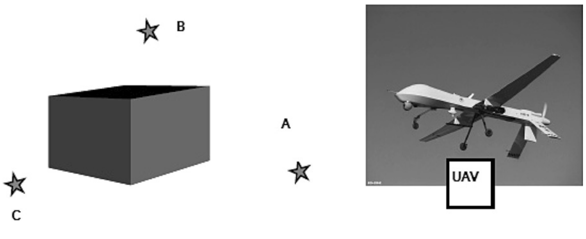


Fig. 1. Simple UAV mission

For an analytical synthesis of control system of UAV, we must have a mathematical model of the flight path of the UAV. The desired flight path can be approximated by different mathematical relations. Convenient use of polynomials, in particular, sections of the path approximated by polynomials of the form [1]

$$R(t) = \sum_{k=0}^{n} C_k t^k \quad (1)$$

$R(t)$ – the time variation of one of the coordinates of the UAV, t – current time of flight of UAVs, Ck ($k = \overline{1,n}$) – given coefficients. In the projections for each of the axes of the coordinate system starting trajectory of the UAV is a polynomial [2]

$$A_3(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3 + C_4 t^4 + C_5 t^5. \quad (2)$$

Differentiating (2) twice time we get the expression for the given projections of the speed and acceleration of the UAV.

$$\dot{A}_3(t) = C_1 + 2C_2 t + 3C_3 t^2 + 4C_4 t^3 + 5C_5 t^4, \quad (3)$$

$$\ddot{A}_3(t) = 2C_2 + 6C_3 t + 12C_4 t^2 + 20C_5 t^3. \quad (4)$$

The coefficients C_3, C_4, C_5 are determined from the system of equations for the, at the time of the end pointing at $t = T$, where T – time of the guidance.

The advantage of this approach is the simplicity of the program path or board UAVs. However, in reality impossible to accurately predict the end time pointing T, since the terminal conditions on the components of the state vector of the UAV may change significantly in the course of its flight.

The main drawback of the implementation of this method is the lack of change in trajectory prediction, which leads to considerable lateral loads UAV. The method is useful for a straight desired trajectory, which is used as a plot great circle passing through the source and destination of the route. In this case, means the part of a great circle arc of a great circle whose center is aligned with the center of the Earth.

Since one of the main tasks is to monitor the drone of individual sections of the earth's surface, it is of interest to UAV guidance on the path specified in the inertial reference points (start) the coordinate system parameters (O^k X^k Y^k Z^k), where k – number of points in space through which the trajectory of the UAV has to go, O^k – the start point of reference. Thus, the trajectory of the UAV consists of separate intervals, which is necessary to ensure optimal guidance of UAVs including the requirements of accuracy and stability of the trajectory guidance.

Let us consider the motion of the UAV relative to a given (inertial) coordinate system for the visualization in the horizontal plane at the k^{th} interval guidance, which is described by a system of linear differential equations. We define the optimal control (acceleration) of the UAV at the k^{th} interval guidance. The index k in this case for the sake of simplicity we omit. The optimization criterion will be considered normal for the problems prompting the quadratic form:

$$J = \frac{1}{2}\left[c_1\left(V_z - V_{init}\right)^2 + c_2\left(Z - Z_{init}\right)^2\right]_{t=t_f} + \frac{1}{2}\int_{t_0}^{t} c_3 a_z^2\,dt. \tag{5}$$

Where: t_f – time of the meeting point of the UAV with the required space.

V_{init} – Set-point speed UAV projection axis O^k Z^k corresponding to the inertial coordinate system at the end pointing to the k$^{\text{th}}$ interval.

Z_{init} – Lateral coordinates a given point of the trajectory. c_1, c_2, c_3 – coefficients optimized functional (6).

The problem of determining the optimal lateral acceleration az(t), minimizing the functional (5) can be solved by the use of different methods of analytical design. In [4], by applying the methods of the calculus of variations obtained a decision that for the formulation of the problem is of the form:

$$\begin{aligned} a_z(V_z, Z, t) = -\Lambda_v(t)[V_z(t) - V_{init}] - \\ -\Lambda_z(t)[Z(t) - Z_{init}]. \end{aligned} \tag{6}$$

Where:

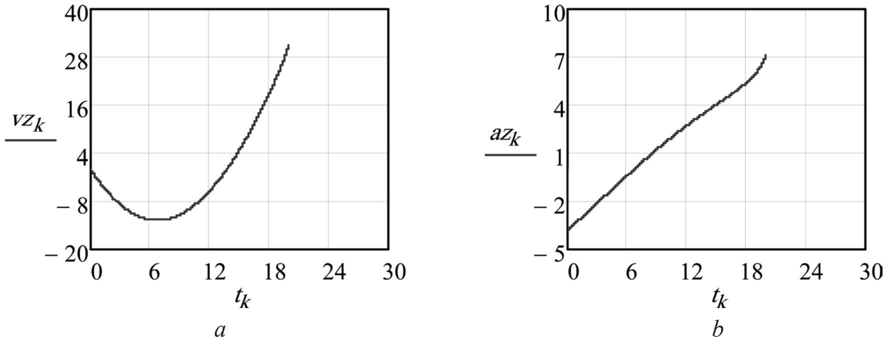
$$\Lambda_v(t) = \frac{(1/c_2) + (1/c_1)(t_f - t)^2 + 1/3(t_f - t)^3}{D(t_f - t)} \tag{7}$$

$$\Lambda_z(t) = \frac{(1/c_1)(t_f - t) + 1/2(t_f - t)^2}{D(t_f - t)}, \tag{8}$$

$$\begin{aligned} D(t_f - t) = \left[\frac{1}{c_2} + \frac{1}{3}(t_f - t)^3\right]\left[\frac{1}{c_1} + t_f - t\right] - \\ -\frac{1}{4}(t_f - t)^4. \end{aligned} \tag{9}$$

In the particular case can take $c_1 \to \infty$ and $c_2 \to \infty$ [2]. This corresponds to the case where the integral term functional (5) can be neglected. This assumption is valid when the UAV has sufficient fuel path at relatively low speed flight unlike missile whose considerable lateral loads lead to significant loss of speed and therefore – to loss of control in the flight path.

Note that the expression (6)–(9) are also valid for the spatial problem of guidance of UAVs. The



a *b*

Fig. 2. The graphical change of (*a*) Velocity (*b*) Acceleration for the UAV

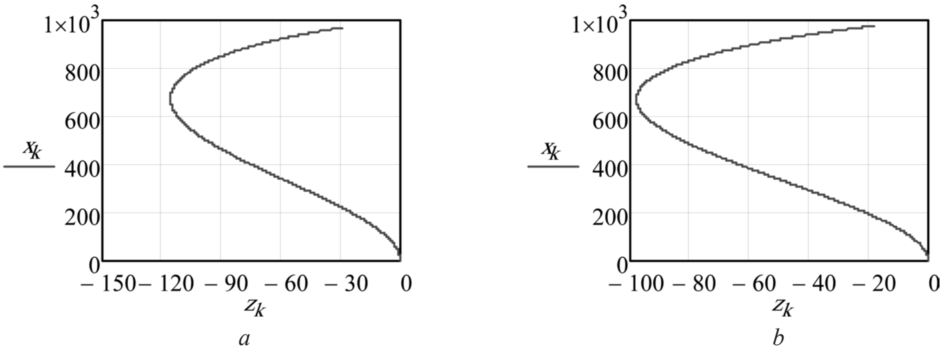


Fig. 3. Trajectories of UAV for (*a*) ν(*k*) = 45º (*b*) ν(*k*) = 30º

corresponding position, velocity and acceleration of the UAV may be replaced by the corresponding three-dimensional vectors.

As an example, we will consider the control of the UAV at the k^{th} interval trajectory for the following initial conditions, as shown in Fig. 2: $X0$ = 1000 m, V = 50 m/s, ν = 45º. Fig. 2 shows graphs of the variation and obtained by modeling in Mathcad.

Fig. 3 shows the trajectories of the UAV $X(Z)$ in meters ν(*k*) = 45º (*a*) and ν(*k*) = 30º (*b*) under the same conditions guidance UAV.

In this case, the error deviation from the UAV endpoint guidance depending on the angle ν(*k*) ranges from 15 to 20 meters, for applications of monitoring the earth's surface using a UAV hundred meters height is quite acceptable.

CONCLUSION

For guidance and navigation control of UAVs, a complete solution which takes into account the equations of motion in conjunction with kinematic constraints is far from implementing in real-time, especially for small UAVs equipped with limited on-board hardware. We propose a path smoothing algorithm using a set of path templates. Instead of smoothing the entire path from an initial position to the goal position, we smooth the path segments over a finite planning horizon with respect to the current position of the UAV.

We present a nonlinear path following control algorithm, to regulate the error distances from a reference path. Based on the kinematic control law. The proposed analytical method for the synthesis of the control law on the stage of the preliminary design of UAV control system allows us to get the best quality for a given criterion of control is when we control the UAV path passing through the given point in space. Follow the practical implementation of the control law in the autopilot unmanned or manned aircraft is a known regulator of the synthesis problem, whose solution is determined by the specified energy ballistic and aerodynamic characteristics of the particular aircraft.

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