MINISTRY OF EDUCATION OF THE REPUBLIC OF BELARUS
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APPLIED MATHEMATICS
PLASTIC DEFORMATION OF METALS

Approved by the Educational and Methodical Association
of education institutions in the field of instrumentation
as educational and methodical manual for students
of specialty 1-38 80 01 “Instrumentation”

Minsk
BNTU
2022

УДК 51-7 (075.8)
ББК 22.19я7
Г94

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Hundzina, M. A.
Г94 Applied mathematics plastic deformation of metals : educational and methodical manual for students of specialty 1-38 80 01 “Instrumentation” / M. A. Hundzina, O. V. Yuhnovskaya. – Minsk : BNTU, 2022. – 67 p.
ISBN 978-985-583-745-0.

This educational and methodical manual is intended for students of engineering specialties of the instrument-making faculty of BNTU, who studies the discipline “Applied mathematics” for students of the second degree of study of engineering specialties. The tasks are developed taking into account the recommendation of the department of Engineering Mathematics of the instrument-making faculty of Belarusian National Technical University, they are consistent with the requirements for standard in mathematics.
The educational and methodical manual is intended for researchers and students of technical specialties.

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INTRODUCTION

The lecture is devoted to the description of plasticity characteristics in different kinds of materials.

It is known that if the stress applied to a certain structural element exceeds the elastic limit, the relationship between loads and displacements ceases to be unambiguous. The displacements corresponding to a given system of loads depend on the order of their application. After removing the load, the deformations caused by it do not disappear, but they are partially preserved. These deformations are called plastic ones.

The magnitude of plastic deformation depends not only on the final magnitude of the acting forces, but also on the order of their application. A tensile diagram is a graph showing the functional relationship between the load and the deformation during a static tension of a specimen until it breaks. This diagram is drawn automatically on a tensile testing machine with a special device (fig. 1).

Fig. 1. Examples of tensile testing machines

§1 PLASTIC PROPERTIES OF METALS

An experimental study of the properties of materials is necessary in order to be able to theoretically estimate the rigidity of a structure, i.e. to evaluate its deformation.

The study of the properties of the material is carried out on samples of a standard shape made from it. Loading (or deformation) of the sample is carried out using special testing machines (fig. 1.1).

Fig. 1.1. Examples of testing machine

Materials in the process of deformation under the load up to the destruction behave differently. Some of them, by the time of the destruction of the sample, undergo significant deformations that do not disappear when the load is removed, due to the destruction, thus demonstrating the plastic behavior.

Other materials at the time of the destruction undergo very small deformations, i.e. the destruction occurs without visible changes to the sample, thus demonstrating the fragile behavior.

Plastic materials include: aluminum, copper, lead. Fragile materials include various inorganic glasses, some types of cast iron (fig. 1.2).

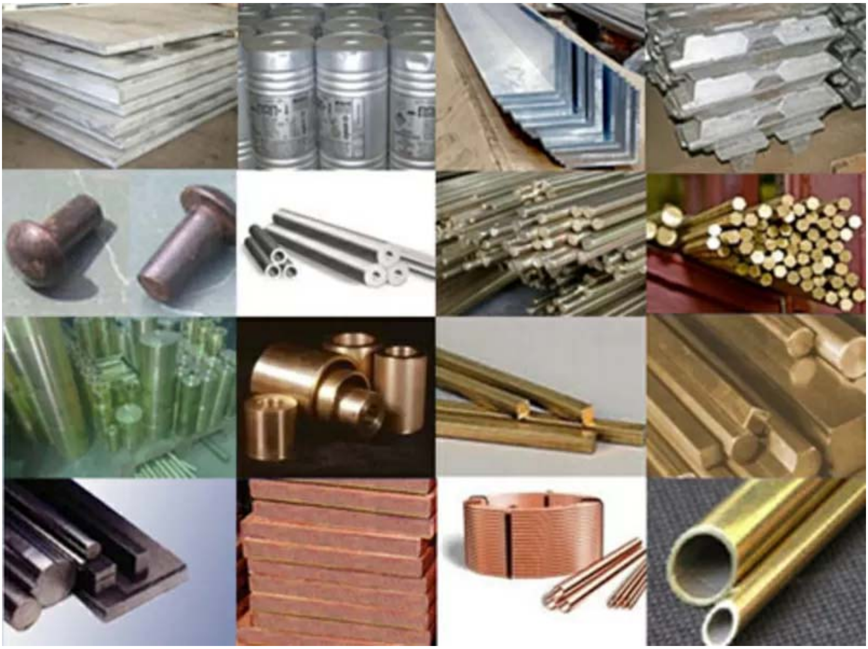

Fig. 1.2. Examples of plastic materials

Many materials under different conditions of deformation behave differently, either as plastic or as brittle ones. Strictly speaking, such materials cannot be called plastic or brittle.

It is more correct to assert that under the considered conditions of the occurrence of the deformation, culminating in destruction, the material is in a plastic state, under other conditions the same material may be in a brittle state.

In this manual, we will get acquainted with the basics of the theory of plasticity, which is a branch of mechanics. This section examines the deformations of solids outside of elasticity. The theory of plasticity studies the macroscopic properties of plastic bodies (fig. 1.3). It deals with the methods for determining the distribution of stresses and strains in plastically deformable bodies.

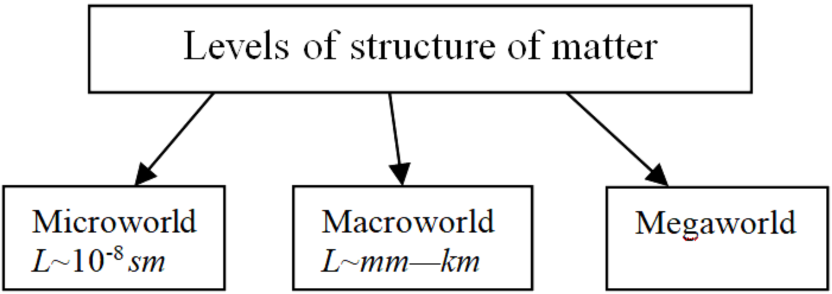

Fig. 1.3. Levels of the structure of matter

To determine the plastic properties of materials, experiments are carried out on tension-compression of a flat or a cylindrical sample and the deformation of a thin-walled cylindrical tube under the action of a tensile force, torque and internal pressure. They are the experiments enabling the independent counting of efforts and deformations.

The theory of plasticity plays an important role in technology, because it is closely related to the most important issues of the design of structures, the study of the technological processes of plastic deformation of the metals.

In a state of equilibrium, for example, under the static loading external forces are balanced by the reaction of the material – the forces of interatomic interaction. These internal forces determine the level of the resistance of the material to deformation. For a quantitative assessment of the resistance of a material to deformation under loading, it is more convenient not to use the magnitude of internal forces, but to use their intensity, or stress.

§ 2 IDEALIZATION OF THE PLASTIC BEHAVIOR OF MATERIALS

Consider a cylindrical body (fig. 2.1). This body is under the action of a tensile force *P*. The height of the cylinder *L*0, *F* is the cross-sectional area.

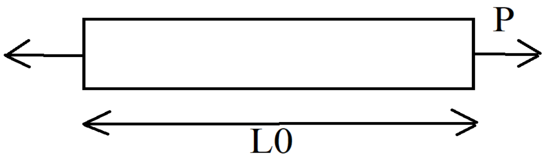

Fig. 2.1. Projection of a cylindrical body

Measurements must be carried out in an area remote from the ends (fig. 2.2).

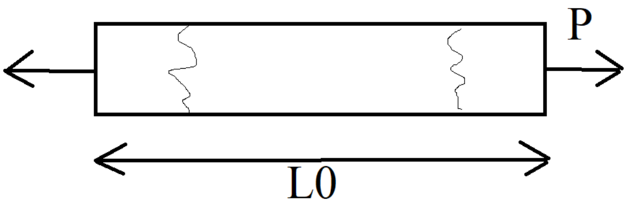

Fig. 2.2. Recommended measurement area

Deformation changes further from the ends of the region. In the literature on the strength of materials, we can often see the result of such measurements in the form of a dimensional quantity Δl. In this case, there is a graph of the dependence of the force *P* on *x*. Note that the most acceptable option would be to calculate the relative amount of the deformation.

As ε, the relative elongation is considered, which is found by the formula:

$$\varepsilon = \Delta l \,/\, l_0,$$

where l_0 – length of the calculated section before deformation;
Δl – specimen elongation.

Instead of the force P, we introduce into consideration the stresses determined by the formula:

$$\sigma = \frac{P}{F},$$

where F – cross-sectional area.

Note that the original cross-section changes under the influence of the tensile force.

The stresses used in mechanics can be true and conditional. It is known that during loading, the size of the area on which stresses act changes. If these changes are not taken into account, then the stress is calculated as the ratio of the load at a given time to the original cross-sectional area. This stress is called conditional. If the force is related to the value of the actual section at a given moment in time, then the value of the true stress is obtained.

Only true stresses have a physical meaning, but in practice it is often more convenient to use conditional stresses. This is especially justified at low degrees of deformation (within the elastic section), when the change in the cross-sectional area is small.

For this case, we construct a diagram of the dependence of σ on ε (fig. 2.3).

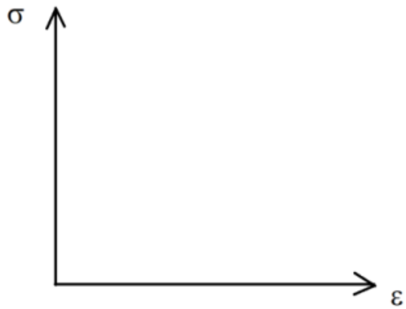

Fig. 2.3. Initial coordinate system

If the force is related to the original cross section, then there will be a falling diagram. And if we refer to the current value of the cross-sectional area, then the diagram will increase until the very moment of rupture.

A stress-strain diagram represents an oblique line extending from the origin in the stress-strain system (fig. 2.4).

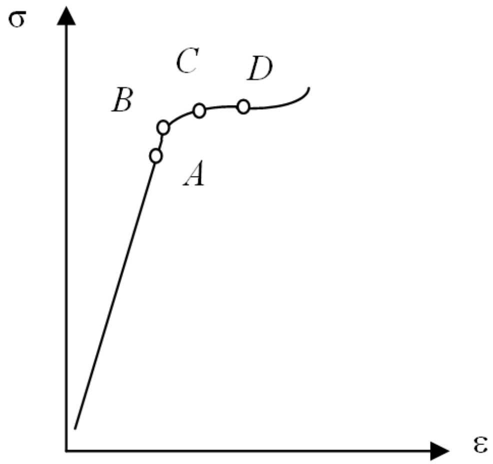

Fig. 2.4. Diagram of elastic-plastic material tension

At point *A*, shown in fig. 2.4, the material proportionality limit is reached, at point *B* – elastic limit, at point *C* – yield stress. After reaching the state corresponding to point *D*, the process of hardening of the material begins.

Let's take a closer look at the stage of building such a diagram. The first section of the curve will continue to point *A*. In this case, the dependence of stress on the deformation is linear. At point *A*, the material proportionality limit is reached (fig. 2.5).

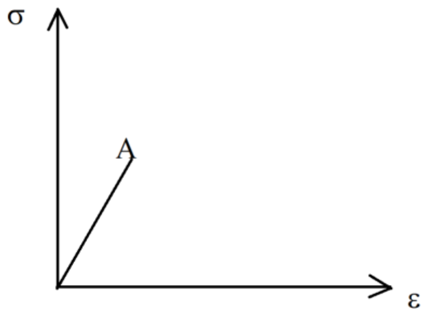

Fig. 2.5. The first section of the diagram

In this area, the ratio σ / ε is a constant value. This quantity is called Young's modulus. It is equal to the tangent of the angle of inclination of this straight line relative to the positive axis *Ox*. Let us denote this angle by the letter φ.

Thus, the proportionality limit is the highest level of a conditional stress at which there is no significant violation of Hooke's law (whatever the elongation is, such is the force). In other words, there is a direct proportional relationship between the stress and the strain.

When passing this section to point *A* (or to any point in this section), we can begin to reduce the load (or unload). In this case, the point will follow the diagram along the same trajectory to the origin.

After loading within this section *OA*, the sample returns to its original state. Such deformation, which completely disappears after unloading, is called elastic deformation. The mechanism of the elastic deformation is a change in the distance between atoms.

With a further increase in the load, a non-linear section *AB* will appear (fig. 2.6).

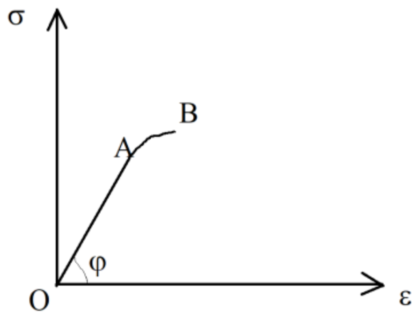

Fig. 2.6. Non-linear section

In this section, the relationship between σ and ε can be represented by some nonlinear function:

$$\sigma = f(\varepsilon).$$

This dependence is different for different materials. When unloading carried out in the section *AB*, the unloading trajectory will repeat the load trajectory.

This section is called a section of non-linear elasticity. Elasticity means having loaded the body, having removed the loads, the body having taken the same shape.

Point *B* corresponds to the presence of permanent deformations. At this point, the elastic limit is reached. This point corresponds to the maximum stress, up to which no plastic deformation occurs in the material. Elastic limit is the highest conditional stress level at which the material exhibits elastic properties, consisting in the fact that the sample almost completely restores its original dimensions after the external load is removed. This section is called a section of non-linear elasticity. With a further increase in the load, many materials have a *BC* section and a *CD* area (fig. 2.7).

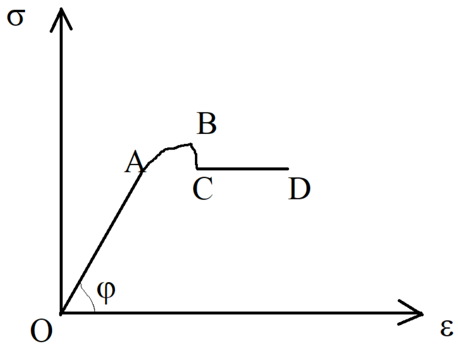

Fig. 2.7. Image of sections *BC* and *CD*

The *CD* section is called the yield pad. Sometimes it occurs, sometimes it doesn't. Its presence depends on the structure of the material. Its presence is influenced by the conditions of the material manufacture and the conditions of turning. The area is called the yield tooth. As a rule, its presence is not taken into account in the calculations. Section *CD* illustrates a material flow. The value of the load in this section is called the yield point. The yield point is the stress at which the deformation grows at a constant stress.

In this region, a new deformation mechanism appears consisting in the shift of atomic layers relative to each other. Because of these shifts after unloading, the sample does not return to its original state.

The plastic deformation is accompanied by heating the sample, changes in its electrical conductivity and magnetic properties, as well as acoustic radiation.

Also, the yield stress is the lowest conditional stress level at which a significant increase in sample deformations is observed under a constant (or slightly decreasing) load.

What is the characteristic of the turnover area?

The yield area is characterized by the following: a constant voltage is applied, and an increase in the deformation is observed. This phenomenon is called a material flow in the theory of plasticity.

The deformation occurs up to the point *D*, and then the graph of the function increases. In this situation, the material becomes stronger (fig. 2.8).

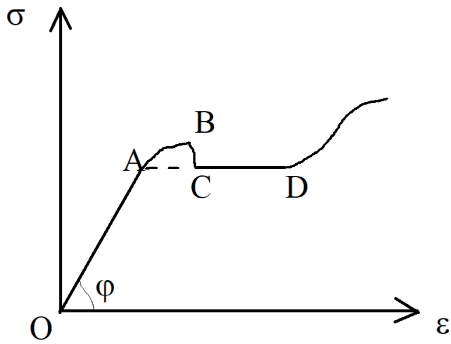

Fig. 2.8. The next section of the diagram

In the *CD* area, the material structure is restructured. After the leveling of all the material occurs, an increase in resistance to the load occurs. What happens when unloading takes place if we start to unload from the point *K* (fig. 2.9)?

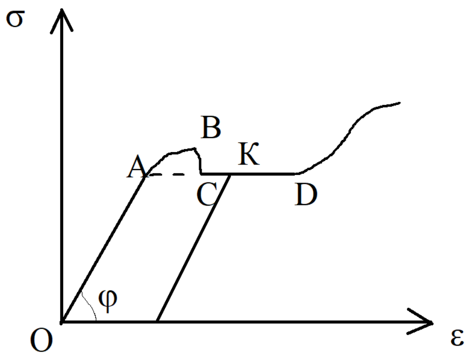

Fig. 2.9. Position of the point *K*

As a rule, the unloading path will run parallel to zero (section *KQ*). Some permanent deformation ε^p turns out. And there is a deformation that took off, that is elastic deformation (ε^e).

During unloading, the plastic deformation remains unchanged, while the elastic one disappears completely (fig. 2.10). Thus, it is possible to formulate a definition of the concept of plasticity. Plasticity is the property of a body to acquire permanent deformations.

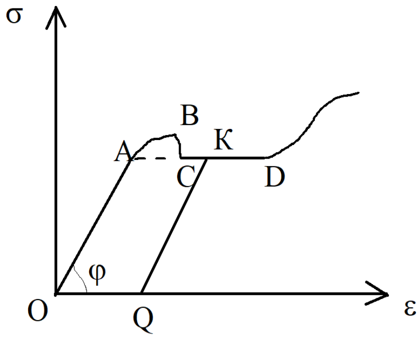

Fig. 2.10. Section *KQ*

At the point *Q*, the voltage is zero. The further movement along this segment will correspond to the compression loading. The following situation is observed (fig. 2.11).

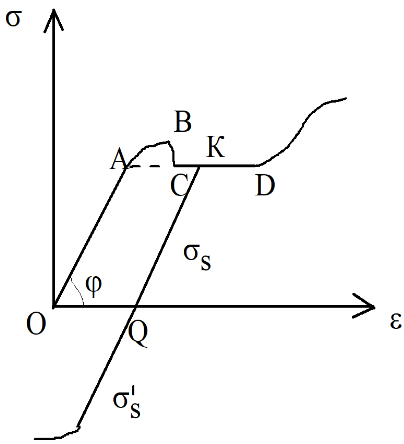

Fig. 2.11. Compression loading

The magnitude of σ_s under tension coincides in absolute value with σ_s under compression (this is true for homogeneous isotropic materials).

Suppose, we have loaded the sample to the yield point, and we begin to unload it. First, a linear section is observed, and then a non-linear section is observed. It turns out that the yield strength in a compression after the deformation in the plastic state is lower than in a tension. This effect is called the Bauschinger effect.

This effect can be described as follows. The application to the hardened sample of stresses of the opposite sign with the transition beyond the elastic limit, entails the softening of the material, and the new elastic limit decreases. One peculiarity should be noted. The fact that we are going along a parallel curve and the fact that the same angle φ is observed is an idealization. In a real experiment, unloading proceeds according to a nonlinear law, and an additional unloading occurs according to a nonlinear law. With this idealization, the reverse load follows the same curve and hits the yield area (fig. 2.12).

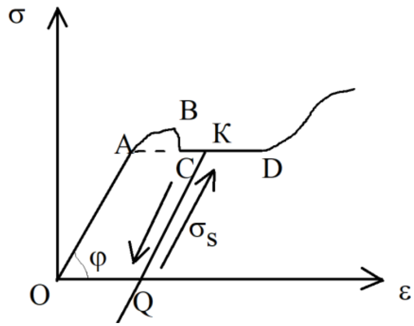

Fig. 2.12. Loading and unloading from the yield pad

In a real experiment, the following picture is observed (fig. 2.13).

Unloading and additional loading are carried out along a certain curve. In this case, a loop is formed. The width of this loop is insignificant, therefore, when solving the problems, this section is taken linear. Taking this loop into account is a very specific task. Its presence leads to great calculation difficulties. This loop is called a hysteresis loop. It is also a characteristic of the material. The phenomenon of hysteresis can be defined as an irreversible loss of the deformation energy as a result of the non-coincidence of the loading curve with the unloading curve.

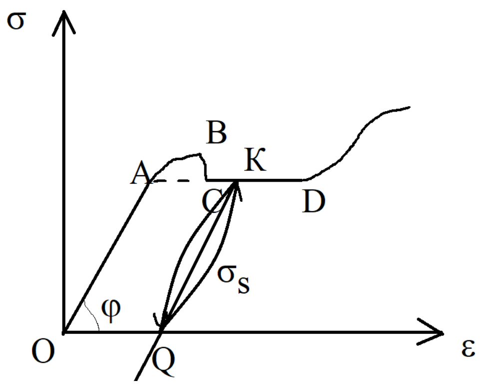

Fig. 2.13. Unloading and loading along a curve

When analyzing the tension and compression diagrams, the phenomenon of hysteresis is neglected. For some materials, such a loop is small, for others it is significant.

Let us continue increasing the loading (fig. 2.14).

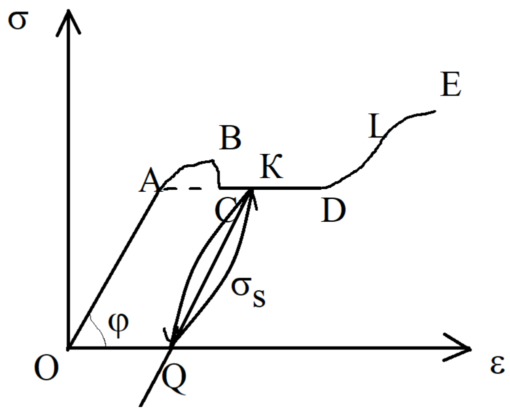

Fig. 2.14. Section *DL*

Consider some point *L* in the section *DE*. Consider unloading from this position. In most cases, this is the same curve described by a linear law (fig. 2.15).

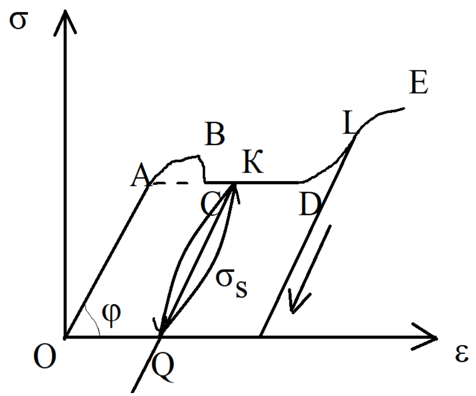

Fig. 2.15. Unloading from the position *L*

If we first unload from this position and then load, it turns out that the elastic component of the material has significantly increased, which means a new yield point has appeared. The strengthening of the material is observed.

During the preliminary deformation, the hardening region leads to the fact that the region of a linear behavior of the material increases. This is the hardening effect. Since the stress σL is greater than the initial elastic limit, we note an increase in the elastic limit as the plastic deformation increases. The material is hardened and therefore, the noted phenomenon is called hardening. The greater the effect will be, the greater the slope of the σ-ε curve will be.

However, there may be a different situation. Unloading can take place along a section located at a different angle φ'. This difference in the angles indicates that the material acquires anisotropy upon the deformation. And it must be taken into account when a break occurs. *DL* – material is hardening the area.

An ultimate strength is the maximum stress that a specimen can withstand without destruction, the conditional stress corresponding to the highest load level perceived by the specimen. A breaking stress is the stress at which the specimen breaks. This limit has no particular practical value and it is used just when studying the process of cracking.

An analytical solution cannot be obtained when using the diagram while solving the problems. Just a numerical solution can be obtained. Consider a situation where the linear section is very small (fig. 2.16).

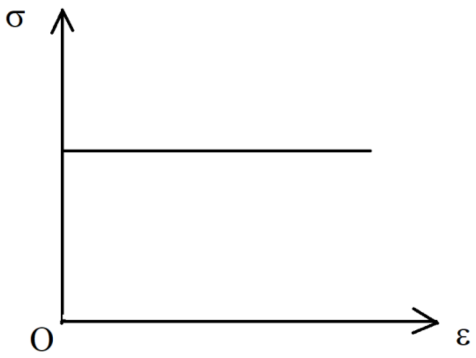

Fig. 2.16. Diagram of a rigidly plastic material

The diagram corresponds to a hard plastic material. We cannot determine ε from the values of σ. Until a certain moment, until the stress σ reaches σ_s, nothing happens to the body. A rigidly plastic body does not deform at stresses below the yield point. The flow in it develops only at stresses that satisfy the fluidity. Since the elastic deformations of most materials are small, they are completely neglected for the sake of simplicity.

In mild steel, titanium alloys, the transition of the elastic section of the tensile diagram to the plastic state occurs abruptly. This corresponds to the case of a perfectly elastic-plastic material. For an ideally elastic-plastic body, the deformation is replaced by a plastic deformation at non-increasing stresses (fig. 2.17).

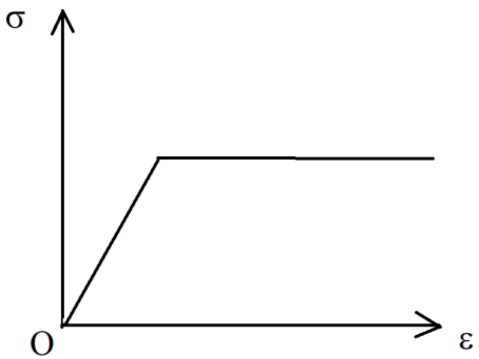

Fig. 2.17. Diagram of an ideal plastic material

When the yield point is reached, the value of the plastic deformation is uncertain, it can grow arbitrarily, but with a decreasing stress (i.e., unloading), it obeys Hooke's law.

The next type of the material is an elastic-plastic material. An elastic-plastic material with nonlinear hardening is shown in fig. 2.18.

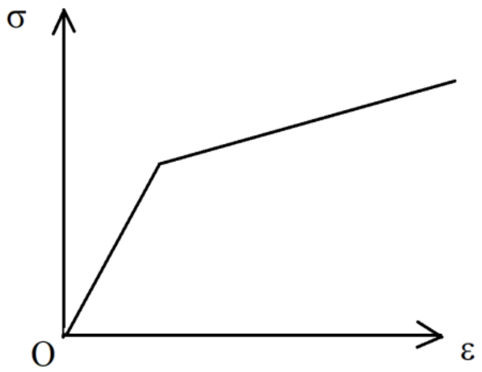

Fig. 2.18. Diagram of an elastic-plastic material

The material is non-linear, since in the process of deformation, it changes its behavior.

The next type of the material is an ideal elastic-plastic material with a yield plate (fig. 2.19).

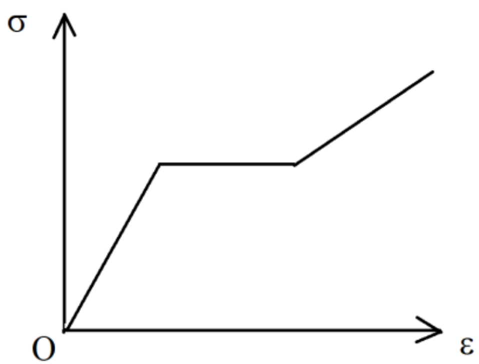

Fig. 2.19. Diagram of an ideal elastic-plastic material with a yield pad

There is a diagram of the compression of a brittle material in fig. 2.20.

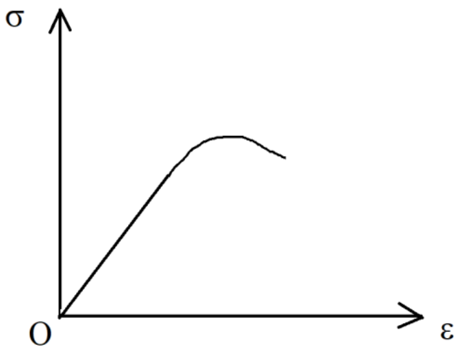

Fig. 2.20. Diagram of a brittle material

Homework 1

1. List the mechanical characteristics determined by tensile tests of the material. Indicate the characteristics of strength and ductility.
2. Give the definition of the proportionality limit.
3. Give the definition of the elastic limit.
4. Give a definition of the yield point.
5. Give the definition of the ultimate strength.
6. How can one determine the yield point in the absence of a yield pad?
7. What deformations are called elastic and residual ones?
8. Is it possible to determine the modulus of elasticity of the material from the stress diagram?

§ 3 PHYSICAL FOUNDATIONS OF PLASTIC DEFORMATION

Metals are characterized by the presence of a metallic bond when positively charged ions are located at the sites of the atomic crystal lattice. These ions are surrounded by electron gas. The presence of such a metallic bond gives the metal the ability to undergo a plastic deformation.

Let us recall that plasticity is a property of a solid under the action of external forces or internal stresses, without collapsing, irreversibly changing its shape and size. The change in the shape and size of a metal body is called a plastic deformation.

The deformation is a change in the shape and size of a solid under the influence of applied external forces.

The deformation can be elastic, disappearing after removing the load, and plastic, remaining after removing the load.

Let us dwell in more details on the mechanisms of the plastic deformation. Even small forces applied to the metal cause its deformation. Initial deformations are always elastic. Their magnitude is in a direct proportion to the load (Hooke's law).

During the elastic deformation, under the action of external forces, the distances between the atoms change. After removing the load, the atoms return to their original position under the action of interatomic forces. The metal is restored to its original size and shape.

During the plastic deformation, such a phenomenon as sliding can be observed. During the plastic deformation, one part of the crystal is irreversibly shifted relative to the other by an integer number of lattice periods. It is shifted along the so-called shear (slip) planes. It should be noted that they are the planes in which the largest number of atoms is located.

Iron and tungsten have six shear planes (fig. 3.1). In this case, copper and aluminum have four shear planes with three sliding directions in each. Zinc and magnesium have one plane with three sliding directions.

The more shear the elements in the lattice are, the higher the ductility of the metal is.

The easiest shift along certain planes and directions is explained by the fact that with a movement of atoms from one stable equilibrium position to another, the values of the expended efforts will be minimal. This means that energy costs will be the least.

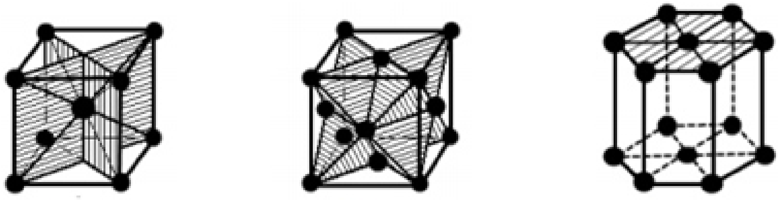

Fig. 3.1. Planes and directions of shear in the crystal lattice

If the load is removed, the displaced part of the crystal will not return to its old place and the deformation will remain.

Slip or shear along certain planes is the main mechanism, but not the only one in the plastic deformation of metals. Under some conditions, the plastic deformation can also occur through twinning. At low temperatures, metals undergo a transition from the sliding mechanism to the twinning mechanism. The essence of the twinning lies in the fact that under the action of stresses, one part of the grain is displaced with respect to the other part, occupying a symmetrical position and being, as it were, its mirror image (fig. 3.2).

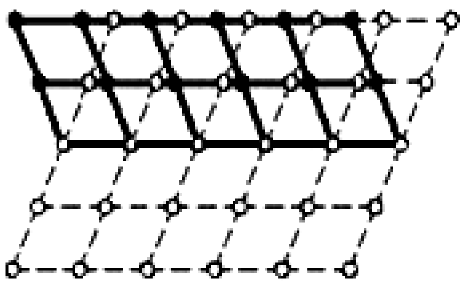

Fig. 3.2. Scheme of the twinning process

The sliding process should not be thought of as the simultaneous movement of one part of the crystal relative to another one. Such a rigid or synchronous shear would require stresses hundreds or even thousands of times higher than those at which the plastic deformation process actually proceeds.

In the dislocation mechanism of the plastic deformation, a slip occurs as a result of a dislocation movement in the crystal, when the shear occurs sequentially from atom to atom near the dislocation core. In this case, the shear force is much less than with the simultaneous shear of all atoms. The process can be explained by the models of a caterpillar movement and a carpet movement (fig. 3.3).

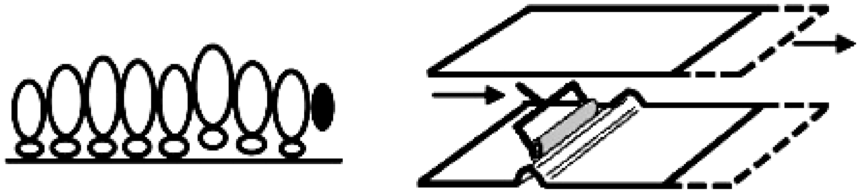

Fig. 3.3. A dislocation mechanism of a plastic deformation

The caterpillar moves by sequentially lifting one pair of legs and rearranging them to a new location, rather than by lifting all legs at the same time and moving one step. When all the legs of the caterpillar consistently perform this operation, it will move one step. This mode of the movement requires much less effort from a caterpillar. Similarly, the carpet moves across the floor when a fold is rolled on it, which requires significantly less effort than transporting the entire carpet.

The dislocation mechanism of a plastic deformation is explained as follows. The atoms located in the dislocation field are excited. Their energy is increased. And they are brought out of a stable equilibrium position with a minimal free energy. Therefore, for a limited group of atoms in the dislocation region in order to shift and take a new stable equilibrium position, it is sufficient to apply a substantially lower stress than during their synchronous shift, i.e. do little work and spend a minimum of energy.

Thus, the movement of a dislocation is a process of successive breaking and restoration of bonds in the crystal lattice. As a result of the path of a dislocation from one crystal boundary to another, a part of the crystal is displaced by one interatomic state.

The defects of the crystal structure in the form of planar group intrusions are dislocations. As a cause of a plastic deformation, dislocations can be the cause of a crack initiation.

Under the action of shear stresses, dislocations can move in the crystal, causing its plastic deformation. If a large number of dislocations are involved in the motion, then the rate of the plastic deformation is directly proportional to the density of moving dislocations.

What is the localization of plastic deformations? This is a pronounced unevenness of the plastic deformation in the body in the form of areas where the degree of the deformation is significantly higher than in the rest of the body. The development of the localization of deformations during the processing of metals by pressure is associated with the formation of zones of a hindered deformation with geometry of workpieces and tools that are unfavorable for a uniform flow of the metal.

In the general case, the localization of deformations causes a significant unevenness of the structure and contributes to their premature destruction.

§ 4 MATERIAL TECHNOLOGY AND TECHNOLOGICAL PROPERTIES

The technology of materials is a collection of modern knowledge about the methods of production of materials and means of their processing in order to manufacture products for various purposes (fig. 4.1). Metals and alloys are produced by smelting at high temperatures from various metal ores. The branch of industry involved in the production of metals and alloys is called metallurgy. Polymers (plastics, rubber, synthetic fibers) are most often produced using organic synthesis processes. The initial raw materials are oil, gas, coal.

Fig. 4.1. The process of production of materials

Finished products and blanks for further processing from metals and alloys are produced by casting or pressure processing. Foundry is engaged in the manufacture of products by pouring molten metal into a special mold, the internal cavity of which has the configuration of the product. A distinction is made between sand casting (into the ground) and special casting methods. Sand casting molds are made by compacting molding sands based on quartz sand.

The processing of metals by pressure is called the change in the shape of the workpiece under the influence of external forces (fig. 4.2). The types of metal forming include rolling, pressing, drawing, forging and stamping.

Fig. 4.2. The processing of metals by pressure

Rolling consists in squeezing the workpiece between rotating rolls.

During pressing, the metal is squeezed out of the closed volume through the hole.

Drawing consists in pulling a workpiece through a hole.

Forging is the process of a free deformation of metal by hammer blows or press pressure.

Parts are obtained by stamping using a special tool – a stamp, which is a metal split form, inside which a cavity is located, corresponding to the configuration of the part.

Alloys designed to obtain parts by pressure treatment are called wrought.

A relatively new direction in the production of metal parts is powder metallurgy, which is engaged in the production of parts from metal powders by pressing and sintering. Plastic products are produced by pressing, casting or extrusion. Rubber products are obtained by processing between rolls, extrusion, pressing or casting, followed by processing.

Articles made of ceramic materials are obtained by molding and firing or pressing and sintering.

Welding is a technological process of obtaining permanent joints of materials by establishing interatomic bonds between the parts to be welded during their heating or plastic deformation, or the joint action of both (fig. 4.3).

Fig. 4.3. Welding

Welding joins homogeneous and dissimilar metals and their alloys, metals with some non-metallic materials (ceramics, graphite, glass), as well as plastics. The final stage in the manufacture of products is often cutting, which consists in removing a layer of material in the form of chips from the workpiece with a cutting tool. As a result, the workpiece acquires the correct shape, exact dimensions, and the required surface quality.

Technological properties determine the ability of materials to undergo various types of processing. Foundry properties are characterized by the ability of metals and alloys in the molten state to fill well the cavity of the casting mold and accurately reproduce its shape (fluidity), the amount of volume reduction during solidification (shrinkage), the tendency to form cracks and pores, and the tendency to absorb gases in the molten state.

Malleability is the ability of metals and alloys to undergo various types of pressure treatment without destruction. Weldability is determined by the ability of materials to form strong welded joints. The machinability is determined by the ability of the materials to be machinable with a cutting tool.

Physical, chemical and performance properties of the materials.

The physical properties of the materials include density, melting point, electrical conductivity, thermal conductivity, magnetic properties, a coefficient of thermal expansion, etc. Density is the ratio of the mass of a homogeneous material to a unit of its volume. This property is important when using materials in aviation and rocket technology, where the structures created must be lightweight and durable. The melting point is the temperature at which a metal changes from a solid to a liquid state.

The lower the melting point of the metal is, the easier the processes of its melting, welding, and the cheaper are. Electrical conductivity is the ability of a material to conduct electric current well and without loss of heat generation. Metals and their alloys, especially copper and aluminum, have good electrical conductivity.

Most non-metallic materials are not capable of conducting electrical current, which is also an important property used in electrical insulating materials. Thermal conductivity is the ability of a material to transfer heat from warmer parts of bodies to less heated ones (fig. 4.4). Metallic materials are characterized by a good thermal conductivity. Magnetic properties i.e. only iron, nickel, cobalt and their alloys have the ability to be well magnetized. The coefficients of a linear and a volumetric expansion characterize the ability of a material to expand when heated. This property is important to take into account when building bridges, laying railway and tram tracks, etc (fig. 4.5).

Fig. 4.4. Thermal conductivity

Fig. 4.5. The bridge of Minsk

Chemical properties characterize the tendency of the materials to interact with various substances. They are associated with the ability of the materials to withstand the harmful effects of these substances. The ability of metals and alloys to resist the action of various aggressive media is called corrosion resistance, and the similar ability of non-metallic materials is called chemical resistance. The operational (service) properties include heat resistance, wear resistance, radiation resistance, corrosion and chemical resistance, etc.

Heat resistance characterizes the ability of a metallic material to resist oxidation in a gaseous environment at high temperatures (fig. 4.6).

Heat resistance characterizes the ability of a material to maintain mechanical properties at high temperatures. Wear resistance is the ability of a material to resist the destruction of its surface layers by friction.

Radiation resistance characterizes the ability of a material to resist the effects of nuclear radiation (fig. 4.7). Strength has two meanings. Strength is considered as resistance to a plastic deformation (that is, to change in shape) and as resistance to fracture, but both are resistance to the action of an external load.

Fig. 4.6. Heat resistance

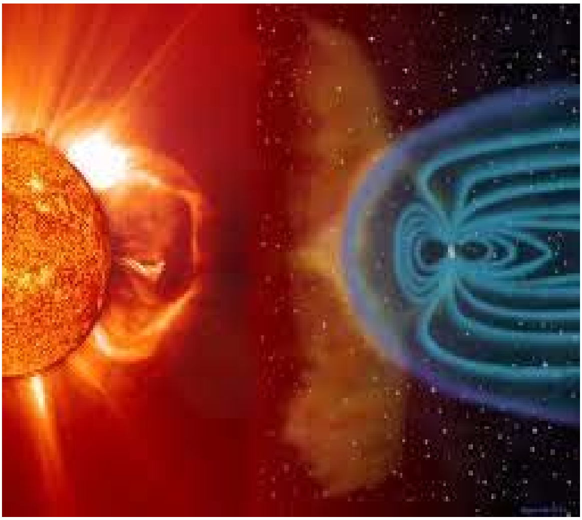

Fig. 4.7. Radiation resistance

The main iron alloys are steel and cast iron, as well as the alloys based on aluminum, titanium, magnesium, copper, nickel. They allow one to obtain a set of properties – strength, plasticity, toughness – for machine parts and structures; hardness, wear resistance – for tools, as well as special properties: heat resistance, high electrical conductivity, magnetic and non-magnetic, corrosion resistance in aggressive environments, etc.

Physicochemical properties are classified as structurally insensitive and structurally sensitive. The first properties of the melting and boiling points, thermal conductivity, temperature expansion, they are determined mainly by the strength of the atomic bond and therefore are in correlation with each other.

The second properties depend both on the nature and strength of atomic bonds and on the structure that is formed during different types of processing.

The metallurgical industry always strives to create high-strength metal materials without loss of ductility, selecting the optimal chemical compositions of steel, improving production technologies. To achieve high mechanical properties, while maintaining the same composition and volume of the product, unique modes of smelting, mechanical, thermal, chemical-thermal treatment are selected to create a homogeneous, fine-grained, clean and defect-free structure of steel.

Laboratory work 1

TENSILE TESTING OF PLASTIC MATERIALS

The purpose of the work: study of the tensile testing technique for plastic materials; determination of tensile strength characteristics of carbon steel.

Statement of work. Mechanical tests of carbon steel specimens were carried out on a tensile testing machine by automatic recording of tensile diagrams P, kN – Δl + δm, mm, where δm – elastic deformations of stressed parts of the testing machine.

The scales of loads μ_p = 0.6 and deformations $\mu_{\Delta l}$, sizes of samples before and after the tests are indicated.

Determine the characteristics of tensile strength and ductility of steel.

Initial data: μ, Δl, d_1, l_1, initial diameter d_0 = 15 mm, length l = 170 mm of the working part of the sample.

Table 4.1

№	M	d_1, mm	l_1
1	0.62	8.41	195
2	0.61	8.35	197
3	0.59	8.28	196
4	0.60	8.31	198
5	0.60	8.40	194
6	0.61	8.39	192
7	0.60	8.37	193
8	0.61	8.38	194.5
9	0.62	8.36	195.5
10	0.62	8.29	191
0	0.61	8.32	190

Progress.

We calculate the cross-sectional area:

$$F_0 = \pi r^2 = \pi\,(d_0\,/\,2)^2 = 176.71\ \text{mm}^2 = 176.71 \cdot 10^{-6}\ \text{m}^2.$$

The calculated sample length is determined by the following relationship:

$$l_0 = 10d_0 = 150 \text{ mm}.$$

The sample is installed in the grips of the testing machine and stretched until fracture. After destruction, we determine the characteristic dimensions of the sample:

Let the neck diameter d_1 = 8.3 mm. Let the calculated length after breaking l_1 = 196 mm.

We calculate the cross-sectional area in the neck:

$$F_1 = \pi r^2 = \pi 8.3^2 / 4 = 54.11 \text{ mm}^2 = 54.11 \cdot 10^{-6} \text{ m}^2.$$

A tensile failure of a steel specimen after necking. In the center of the minimum section of the neck, a crack originates, which then develops along the conical surface at approximately an angle of 45 degrees, where the force acts, until a final destruction.

It is known that the load P_1 corresponds to the proportional limit. For steel, it corresponds to h_1 = 47.5 (this value is the same for all variants). Then we find

$$P_1 = h_1\mu = 47.5 \cdot 0.6 = 28.5 \text{ kN}.$$

The load P_2 corresponding to the elastic limit (h_2 = 52.5 for steel) is determined by:

$$P_2 = h_2\mu = 52.5 \cdot 0.6 = 31.5 \text{ kN}.$$

The load corresponding to the yield point is determined by the ordinate of the yield point h_3 = 57.5.

$$P_3 = h_3\mu = 57.5 \cdot 0.6 = 34.0 \text{ kN}.$$

The load corresponding to the ultimate strength (h_4 = 95) is the highest load at break of the specimen

$$P_4 = h_4\mu = 95 \cdot 0.6 = 57 \text{ kN}.$$

The load P_5 corresponding to the destruction of the specimen (h_5 = 89) is equal to

$$P_5 = h_5\mu = 89 \cdot 0.6 = 53.4 \text{ kN}.$$

Determine the strength characteristics:

a) stress, the excess of which leads to a violation of the linear relationship between the load P and the elongation Δl

$$\sigma_P = P_1 10^3 / F_0 = 28.5 \cdot 10^3 / (176.71 \cdot 10^{-6}) = 161.281 \cdot 10^6 \text{ Pa};$$

b) elastic limit (stress, which corresponds to a relative permanent deformation of 0.005 %

$$\sigma_e = P_2 10^3 / F_0 = 34.5 \cdot 10^3 / (176.71 \cdot 10^{-6}) = 195.235 \cdot 10^6 \text{ Pa};$$

c) tensile strength or ultimate strength (voltage that corresponds to the greatest load after fracture of the sample)

$$\sigma_s = P_4 10^3 / F_0 = 57 \cdot 10^3 / (176.71 \cdot 10^{-6}) = 322.562 \cdot 10^6 \text{ Pa}.$$

Pull-off resistance in the sample neck

$$s_k = P_5 10^3 / F_{sh} = 53.4 \cdot 10^3 / (54.11 \cdot 10^{-6}) = 986.879 \cdot 10^6 \text{ Pa}.$$

The higher the characteristics are, the stronger the material is.

Finding the tensile characteristics of steel:

a) relative residual elongation after a rupture of the sample:

$$\delta = (l_1 - l_0) / l_0 100 = (196 - 150) / 150 \cdot 100 = 30.667 \text{ \%};$$

b) relative residual narrowing in the neck after stretching:

$$\psi = (F_0 - F_1) / F_0 = (176.71 \cdot 10^{-6} - 54.11 \cdot 10^{-6}) / (176.71 \cdot 10^{-6}) \cdot 100 = \\ = 69.379 \text{ \%}.$$

Conclusions. The method of tensile testing of plastic materials has been studied. The characteristics of tensile strength and ductility of carbon steel have been determined.

§ 5 TRUSSES WITH DIFFERENT MATERIAL PROPERTIES

Deformations and stability of steel structures depend on the geometric and physical nonlinearity of their behavior. At large displacements of the structure, the equilibrium conditions and the "displacement-deformation" relationship are nonlinear. If the material in some parts of the structure reaches the yield point, then the stress-strain ratios change, as well as the stiffness ratios of the structural elements.

The various characteristics that determine the behavior and final state of a metal sample, depending on the type and intensity of forces, are called the mechanical properties of the metal.

From the point of view of a civil engineer, early 21st century can be characterized as an era of spatial bar structures. In recent years, reinforced concrete shells have been replaced by mesh shells, whose own weight per unit of an overlapping area is very small.

Shell-type rods are used to bridge large-span buildings without the need for additional supports, for example, for sports stadiums, assembly halls and exhibition halls. Such structures are made of straight-line elements that are connected using mechanical systems or welding. The curvature obtained after the assembly ensures the spatial work of such structures. Illustrations of spatial bar structures built in Moscow in recent years are shown in fig. 5.1, 5.2, 5.3.

Fig. 5.1. A spatial bar structure

Steel trusses are widely used in many areas of construction: in the coatings and floors of industrial and civil buildings, bridges, power transmission towers, communication facilities, television and radio broadcasting, transport overpasses, lifting cranes.

Calculation of spatial bar structures is a laborious and complex task. It should be performed taking into account all possible limiting states in various operating conditions of structures. In the calculation of steel structures, it is customary to distinguish the following types of limit states of the first group: plastic, brittle and fatigue failure; loss of stability of shape or position; transition to a variable system; material fluidity; inelastic shear in joints; a qualitative change in the configuration.

Fig. 5.2. Dome of the depot of the Moscow monorail transport system

Fig. 5.3. Dome of the building of the Museum
of the Great Patriotic War in Minsk

The simultaneous consideration of geometric and physical nonlinearities is still an extremely difficult task, especially in the calculations of spatial structures for stability.

At present, it is generally accepted that it is possible to reliably determine the critical load and the cause of the loss of stability of a spatial rod structure just by taking into account its geometric and physical nonlinearity, as well as additional factors – the design of nodes and the influence of imperfections.

A truss is a system of rods interconnected at nodes, forming a geometrically unchangeable structure with hinged nodes (fig. 5.4).

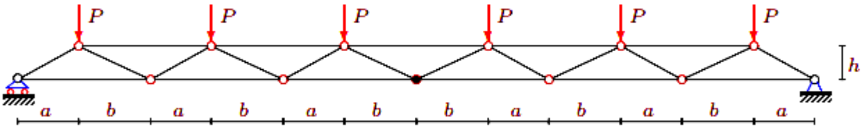

Fig. 5.4. Illustration of a truss

We will assume that all truss rods experience only axial forces (tension or compression), which makes it possible to use the material more fully than in a solid beam.

Trusses are more economical than beams in terms of a steel consumption, but more laborious to manufacture. Trusses are flat (all the rods are in the same plane) and spatial.

Flat trusses can perceive the load applied only in their plane, and need to be fixed from their plane with ties or other elements. Spatial trusses form a rigid spatial beam capable of absorbing a load acting in any direction (fig. 5.5).

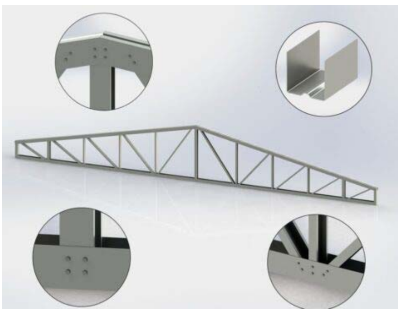

Fig. 5.5. Spatial trusses

Fig. 5.6 shows the main elements of the truss.

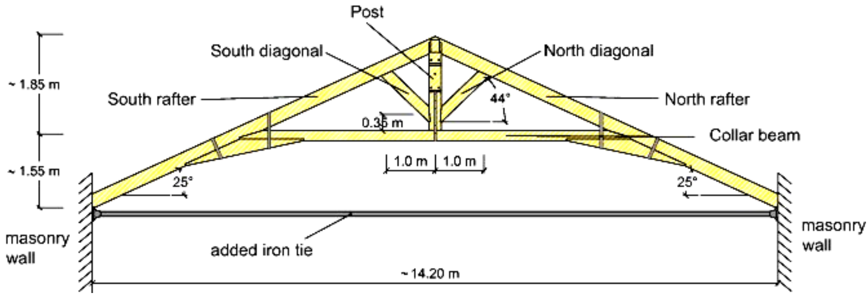

Fig. 5.6. The main elements of the truss

Truss belts are elements that limit its contour from above (upper belt) and from below (lower belt).

Racks – vertical bars of the lattice.

Braces – inclined bars of the lattice.

Racks supporting just the element of the lower chord in the nodes (without braces) are called suspensions. The suspensions work only for the perception of the load suspended from the lower chord node. In the absence of such a load, the forces in the suspensions are equal to zero.

The distance between the nearest chord nodes is called the panel length.

The distance between the axes of the truss supports is called the span of the truss. The points at which the bars converge are called nodes. The nodes on which the truss rests are called support nodes. The upper knot is ridge, and all others are intermediate.

The connection of the elements in the nodes is carried out by direct adjoining of some elements to others.

In some trusses, there are "zero" rods, the force in which is equal to zero. Their role is to ensure the geometric immutability of the truss.

The rules for defining zero bars are as follows:

– in a two-rod unloaded node, both rods are zero (fig. 5.7);

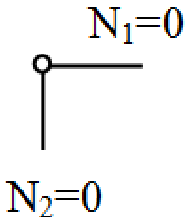

Fig. 5.7. The case when both rods are zero

– in a three-rod unloaded node, the forces in the rods located on one straight line are equal to each other, the third rod is zero (fig. 5.8);

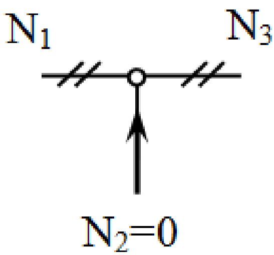

Fig. 5.8. The case of a three-rod unloaded node

– if in a two-rod loaded node the external force coincides with the direction of one of the rods, the force in this rod is equal to the external force, the second rod is zero (fig. 5.9);

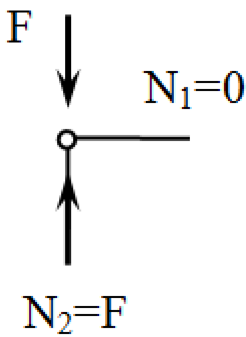

Fig. 5.9. The case of a three-rod loaded node

Also in a truss, if in a three-rod node the external force coincides with the direction of one of the stitches, the force in this rod is equal to the external force, and the forces in the rods located on one straight line are equal to each other (fig. 5.10).

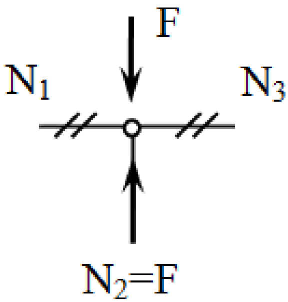

Fig. 5.10. The case of a three-rod knot, for which the external force coincides with the direction of one of the stitches

Homework 2

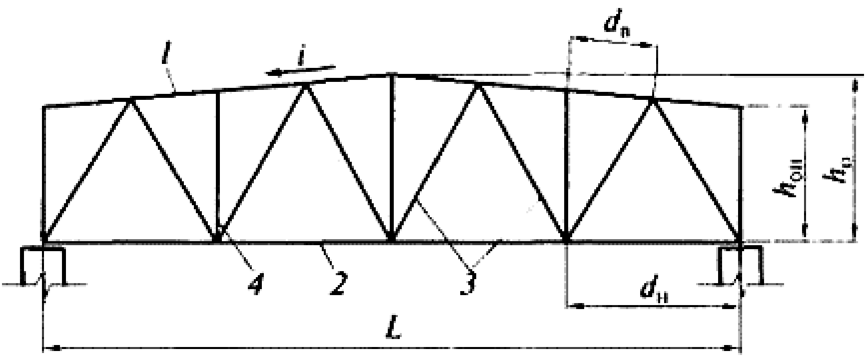

Fig. 5.11

Sign the truss items:
1 –
2 –
3 –
4 –
Answer: top chord, bottom chord, braces, strut.

Laboratory work 2

DETERMINATION OF FORCES IN THE BARS OF A THREE-BAR TRUSS

The truss works in bending from an external vertical load, as a rule, applied at the nodes. Due to this, axial tensile forces arise in the elements of the truss, which ensures the most complete use of the bearing capacity of the material than in bending elements.

Triangular trusses have the greatest height, therefore, according to the conditions of manufacture and transportation, they are used with spans of no more than 36 m. The most common lattice is triangular, since the total length of its zigzag and the number of nodes with it is less than in trusses with other types of lattices. The rational angle of inclination of the grating to the lower chord is 45–50 degrees.

One of the main indicators of the effectiveness of coverings from trusses is the mass of the structure, reduced to a square meter of an overlapped area. A metal consumption for a truss depends on the size of the span to be covered, the design load and the bearing capacity of the material.

The basis for designing nodes and trusses is the intersection of the axes of all rods converging at a node in the center of the node. In this case, a balance of forces in the node is achieved for the preservation of axial forces for all bar elements.

Fig. 5.12 shows the basic reactions of the supports that will be used to determine the forces of the rods.

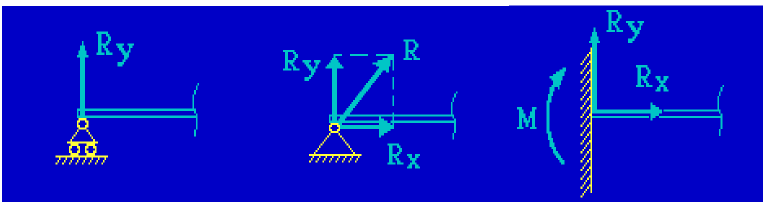

Fig. 5.12. Basic reactions of supports

A task. It is necessary to determine the forces in the truss rods. The length of the upper rod is 1 m, the length of the lower rods is $\frac{1}{\sqrt{2}}$ (fig. 5.13).

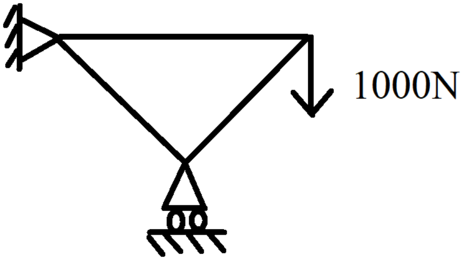

Fig. 5.13. Initial position of the structure

Progress.

Let us write down the conditions for the balance of forces and moments (fig. 5.14).

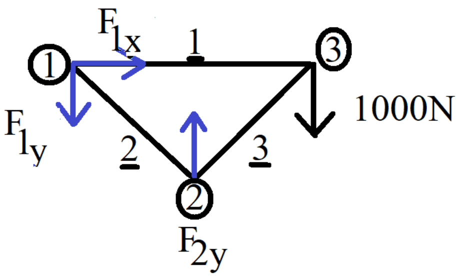

Fig. 5.14. Schematic representation of the structure with designations

Force projections on the *Ox* axis:

$$\sum F_x:\ F_{1x} = 0.$$

Force projections on the *Oy* axis:

$$\sum F_y:\ F_{1y} + 1000 - F_{2y} = 0.$$

Sum of moments relative to node 1:

$$\sum M_1:\ F_{2y} \cdot 0.5 - 1000 \cdot 1 = 0.$$

Then the solution to the resulting system:

$$F_{2y} = 2000\ N; \quad F_{1y} = 1000\ N.$$

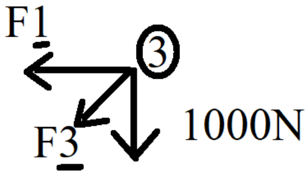

Fig. 5.15. Application of forces at node 3

Force projections on the *Ox* axis:

$$\sum F_x : -F_1 - F_3 \cos 45^{\circ} = 0.$$

Force projections on the *Oy* axis:

$$\sum F_y : -1000 - F_3 \sin 45^{\circ} = 0.$$

Then the solution to the resulting system:

$$F_3 = -1000\sqrt{2} = -1414.2\ \text{N}.$$

The minus sign shows that the real direction of the action of this force is the opposite.

$$F_1 = -F_3 / \sqrt{2} = 1000\ \text{N}.$$

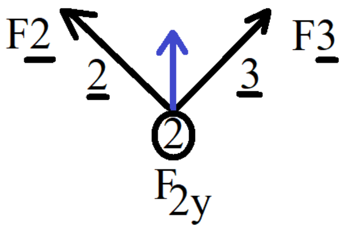

Fig. 5.16. Application of forces at node 2

Force projections on the Ox axis:

$$\sum F_x : -F_2 \cos 45^\text{o} + F_3 \cos 45^\text{o} = 0.$$

Then the solution to the resulting system: $F_2 = F_3 = 1414.2$ N.

Task. It is necessary to determine the forces in the truss rods. The lengths of the lower rods are equal to *a* meters, the angles of inclination in relation to the upper rod coincide and they are equal to the angle α (fig. 5.17).

Table 5.1

№	*a, m*	α, °
1	10	55
2	11	47
3	21	46
4	15	50
5	16	49
6	13	48
7	14	48
8	17	47
9	15	52
10	11	53

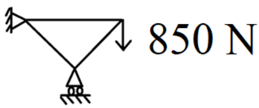

Fig. 5.17. Construction diagram

§ 6 CALCULATION OF STEEL STRUCTURES TAKING INTO ACCOUNT PLASTIC DEFORMATIONS

Calculation of steel structures taking into account the plastic deformations of steel enables the use of an additional material resource, and leads to a more economical design. The design codes for steel structures in different countries are focused to varying degrees on the use of the plastic properties of steel in design practice. Individual elements of steel structures, as a rule, are calculated taking into account inelastic deformations of steel.

The application of the theory of plasticity to the design of steel structures served as a prerequisite for the creation of a method for calculating structures by limiting states.

Existing theories of plasticity can be divided into two main groups:

– deformation theories;

– the theory of a plastic flow.

Deformation theories establish relationships between stresses and deformations. In the theories of a plastic flow of a material, stresses are associated with small increments of deformations.

Deformation theories include the theory of small elastic-plastic deformations widely used in the calculations of building structures.

The following hypotheses are accepted in the theory:

1) the law of volume change. The volumetric deformation is directly proportional to the average normal stress with the same coefficient as in the theory of elasticity. No change in volume occurs during plastic deformations;

2) the law of form change. The quantities describing the strains are proportional to the components describing the stresses;

3) the law of a single deformation curve. For all types of stress state (uniaxial, biaxial, and triaxial), a single relationship between stresses and deformations is valid, which is a mathematical record of the experimental deformation diagram under an uniaxial tension of the sample.

The theory of a plastic flow is based on the following hypotheses:

1) the law of volume change. The volumetric deformation is directly proportional to the average normal stress with the same coefficient as in the theory of elasticity. No change in volume occurs due to plastic deformations;

2) the law of form change. The components describing the plastic strain increments are proportional to the components describing the stresses. The aspect ratio is a function of stresses and strains;

3) the stress intensity is a function of the integral on the intensity of the increments of plastic deformations and does not depend on the type of a stress state.

At a simple stress state of a solid elastic-plastic body, under the condition of simple loading, the results obtained by the two theories are in agreement. However, in a complex stress state, deformation theories give a significant error. The theory of plastic flow is in good agreement with experimental data.

Spatial trusses are steel structures. Their behavior is therefore highly dependent on the properties of the steel. Especially important is the state of tension at a material point, which occurs from a given state of displacements at the same point. The dependence for the uniaxial stress state is much simpler than that for the three-dimensional stress state.

Fig. 6.1 shows stress as a function of the strain ε.

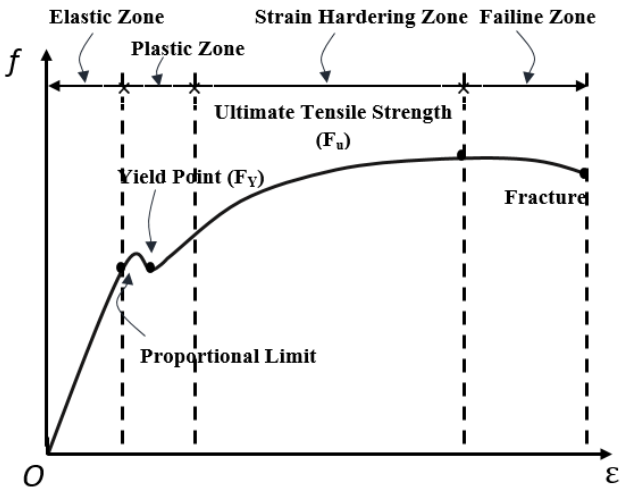

Fig. 6.1. Deformation diagram of mild steel

The stress-strain diagram is divided into three zones: elastic zone; plastic zone; strain hardening.

§ 7 TYPES OF LOADING OF ROD SYSTEMS

There are several simple and complex types of loading.

Tension (compression) occurs when the bar is loaded with forces that coincide in direction with its axis (fig. 7.1).

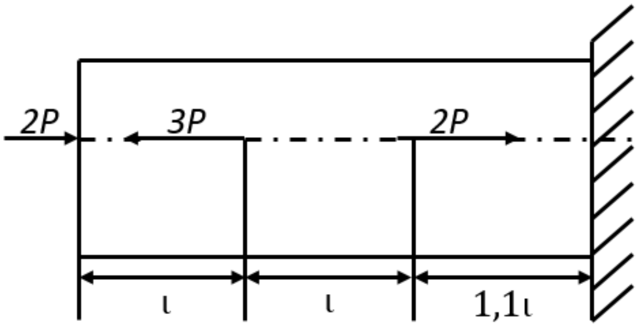

Fig. 7.1. Scheme of the action of tensile forces in the sample

Many structural elements work in tension, compression: truss rods, columns, rods of steam engines and piston pumps, tie screws and other parts.

A shear or shear occurs when external forces displace two parallel sections relative to one another, with a constant distance between them (fig. 7.2).

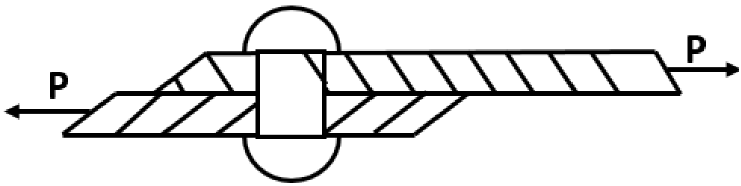

Fig. 7.2. Shear forces

Torsion occurs when external forces act on the bar, which generate moments relative to the longitudinal axis of the bar. Shafts work for torsion, spindles of lathes and boring machines, etc. (fig. 7.3).

Bending is a type of loading when external forces cause moments about the symmetry axis (or principal axis) located in the cross-sectional plane. The moment is called bending. The simplest case is a plane bend, when all external forces lie in one plane, which coincides in all cases we are considering with the plane of symmetry (or the main plane) of the beam.

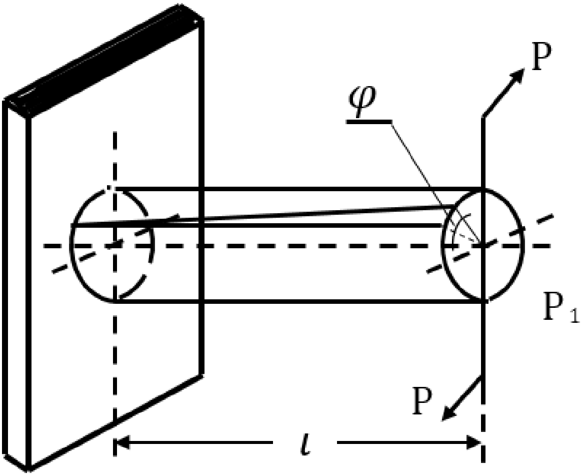

Fig. 7.3. Torsion of the cylinder

§ 8 CALCULATION OF STEEL STRUCTURES TAKING INTO ACCOUNT MECHANICAL AND DEFORMATION CHARACTERISTICS

The set of phenomena associated with a change in the mechanical, physical and other properties of metals in the process of plastic deformation is called strain hardening or work hardening. Strengthening upon work-hardening is explained by an increase in the density of dislocations by several orders of magnitude. With an increase in the degree of deformation, the plasticity characteristics (relative elongation, relative contraction) decrease, and the strength characteristics (elastic limit, yield strength, tensile strength and hardness) increase.

The deformed metal is in a nonequilibrium state. The transition to an equilibrium state is associated with a decrease in distortions in the crystal lattice, stress relief, which is determined by the possibility of moving atoms. At low temperatures, the mobility of atoms is low; therefore, the work-hardening state can persist indefinitely.

The quality of a material is the combination of its properties that satisfy certain needs in accordance with the purpose. The quality level is determined by the corresponding indicators, which are a quantitative characteristic of one or more properties of materials that determine their quality in relation to specific conditions of manufacture and use.

According to the number of characterized properties, quality indicators are divided into single and complex ones.

A single quality index is characterized by just one property (for example, the hardness of steel).

A complex indicator is characterized by several product properties. At the same time, the products are considered to be of high quality only if the entire complex of the assessed properties meets the established quality requirements.

A case of a complex indicator of the quality of steel can serve as an assessment of the chemical composition, mechanical properties, micro- and macrostructure. Comprehensive quality indicators are established by state standards. Quality control methods can be very diverse: visual inspection, organoleptic analysis and instrumental control.

According to the stage of determining the quality, the control is distinguished to be preliminary, intermediate and final.

During the preliminary control, the quality of the feedstock is assessed, with the intermediate control the compliance with the established technological process should be checked. The final control determines the quality of the finished product, its suitability and compliance with the standards. Products that fully meet the requirements of the standards and specifications are regarded as suitable ones. Products that have defects and deviations from the standards are considered to be defective ones.

The quality of a material is mainly determined by its properties, chemical composition and structure.

Moreover, the properties of the material depend on the structure, which, in turn, depends on the chemical composition. Therefore, when assessing the quality, the properties, composition and structure of the material can be determined.

There are various methods for studying the structure of the materials. Macroanalysis studies the structure visible with the naked eye or at low magnification with a magnifying glass. Macroanalysis reveals various structural features and defects (cracks, porosity, cavities, etc.).

Microanalysis is the study of a structure with an optical microscope at a magnification of up to 3,000 times. The electron microscope enables one to study the structure at magnifications up to 25,000 times. X-ray analysis is used to detect internal defects. It is based on the fact that X-rays passing through a material and through defects are attenuated to varying degrees. The penetration depth of X-rays into steel is 80 mm. Transmission with gamma rays has the same physical basis, but they are able to penetrate to great depths.

Transillumination with radio beams of the centimeter and millimeter range makes it possible to detect defects in the surface layer of non-metallic materials, since the penetrating ability of radio waves in metallic materials is low. Magnetic flaw detection enables detecting the defects in the surface layer (up to 2 mm) of metallic materials with magnetic properties. It is based on the distortion of the magnetic field at the sites of defects.

Ultrasonic flaw detection is possible for efficient quality control on a large bunk. It is based on the fact that in the presence of a defect, the intensity of ultrasound passing through the material changes. Capillary flaw detection is used to detect fine cracks invisible to the eye. It uses the effect of filling these cracks with fluids that easily wet the material.

§ 9 MECHANICAL PROPERTIES OF MATERIALS

Mechanical properties characterize the ability of materials to resist external forces. The main mechanical properties are: strength, hardness, impact strength, elasticity, plasticity, fragility.

Strength is the ability of a material to resist the destructive effects of external forces.

Hardness is the ability of a material to resist the penetration of another, harder body into it under the action of a load.

Viscosity is the property of a material to resist destruction under dynamic loads.

Elasticity is the property of materials to regain their size and shape after the termination of the load.

Plasticity is the ability of materials to change their size and shape under the influence of external forces, without collapsing.

Brittleness is the property of materials to collapse under the influence of external forces without residual deformation.

The hardness of metals is measured by pressing a hard tip of various shapes into the test piece.

The impact strength is determined by the work of the spent on the destruction of the sample, referred to its cross-sectional area F (J / m^2). Tests are carried out by blowing a special pendulum impact device. The test is carried out using a standard notched specimen mounted on the pillars of the pile driver. A pendulum of a certain mass strikes the side opposite to the notch.

§ 10 BASIC HYPOTHESES OF CONTINUUM MECHANICS

The modern theory of metal forming is based on the fundamental science тире- continuum mechanics. In continuum mechanics, equations are found out characterizing the kinematic properties (i.e., the deformed state) and the force properties (stressed state) of a deformable body, as well as equations for the relationship between the stressed and the deformed state (the theory of elasticity and plasticity). Thus, the theory of the stress state, the theory of the deformed state and the theory of plasticity are branches of continuum mechanics.

The main hypotheses of this fundamental science are as follows: hypothesis of body continuity; hypothesis about the natural stress state; hypothesis of isotropy of material properties; the hypothesis of the homogeneity of material properties.

It is known that metals are a collection of atoms arranged in an orderly manner in a crystal lattice, i.e. they have a discrete structure. Atoms act on each other by forces that do not obey the laws of classical mechanics. There are more than 1,020 atoms in 1 cubic centimeter of a solid metal. To describe their interaction with each other, a huge number of equations are needed, which cannot be solved by a computer.

From a practical point of view, it is not the behavior of individual atoms that is important, but the whole body as a whole. It enables one to make a theory at a macroscopic level but not at an atomic one. For this, the continuity hypothesis is introduced: the volume occupied by the body is continuously filled with the matter. An infinitely small volume of the material is called a material particle.

In the process of loading by external forces, the material particles can change their position. A material environment is considered deformable if the distance between them changes during the movement of the material particles. In the theory of continuous media, it is believed that the movement of the material particles under the influence of external forces is continuous. From a mathematical point of view, this means that all quantities that determine the deformable medium are continuous functions of coordinates.

The natural unstressed state hypothesis assumes that there is no stress in the body prior to the application of loads. In reality, such tensions exist. These can be residual stresses associated with the inhomogeneity of plastic deformation at the previous stages of the technological process, casting stresses that rise in the process of uneven crystallization of the material, etc.

In continuum mechanics, it is usually assumed that a body is homogeneous and isotropic. The first statement means that the mechanical characteristics of the material are unchanged in the area under consideration, and the second statement means that the properties of the material are equal in any direction. The inhomogeneity of the material can be physically associated with various foreign inclusions. A rolled metal sheet is a case of a non-isotropic material. In the direction of rolling, its characteristics differ significantly from those in the direction perpendicular to rolling.

§ 11 STRESS TENSOR

A tense state is a state of the body caused by the action of external forces. External forces are of the two main types: surface and volumetric.

Surface forces are applied to the surface of the body. Volume (mass) forces include forces acting on all material points of the body and proportional to their masses.

Mechanical stress is a measure of internal forces arising in a body deformed under the action of external forces.

We will assume that the displacements of the points of the deformable body are very small in comparison with the dimensions of the body under consideration. In this case, the relationship between deformations and displacements of points of the body is expressed by the Cauchy relations to be considered later.

In the theory of plasticity, it is assumed that deformations and stresses are independent of time. With prolonged loading of the power elements of machines and structures, accompanied by elevated temperatures, additional deformations occur, as a result of which they can change over time and stresses. These phenomena are considered in another theory – in the theory of creep.

Consider a deformable body that is in equilibrium under load (fig. 11.1).

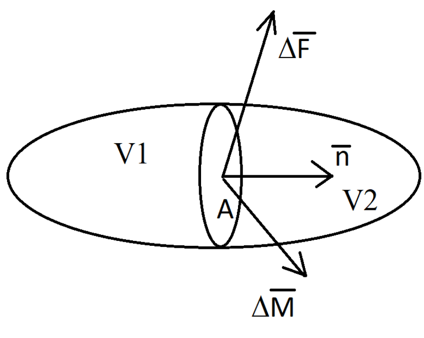

Fig. 11.1. Initial loaded platform

ΔA – surface element centered at a point P, $\bar{n}$ – unit normal vector V_1.

The actions exerted by V_1 at point P on V_2 can be represented by the force vector ΔF and the vector of the moment ΔM.

Then the limit is:

$$\lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \sigma^n; \qquad \lim_{\Delta A \to 0} \frac{\Delta M}{\Delta A} = 0,$$

where σ^n – stress vector at point *P*.

The combination of all stresses (*P*) for all directions $\overline{n}$ determines the stress state at point *P* (fig. 11.2).

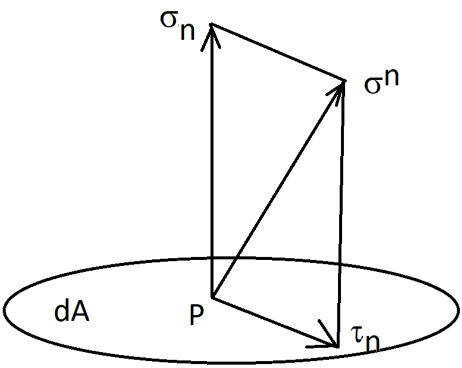

Fig. 11.2. Voltage vector

σ_n – normal stress (normal component directed along the normal $\overline{n}$).

τ_n – shear stress. If the voltage vector is directed in the same direction as the normal, then $\sigma_n = \sigma^n$ and $\tau = 0$ – main normal stress (fig. 11.3).

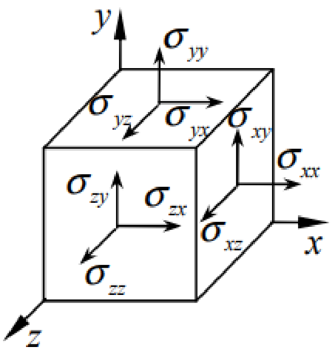

Fig. 11.3. Decomposition of the stress tensor

The stress state at a point of a body is a combination of normal and tangential stresses on a set of planes passing through a given point of the body. For a complete analysis of the stress state at a point, it is sufficient to know the normal and shear stresses on the three arbitrary mutually perpendicular areas. Let us cut out an infinitesimal parallelepiped in three pairs of mutually perpendicular planes in the vicinity of the considered point of the body and show the stresses acting on its faces (fig. 11.4).

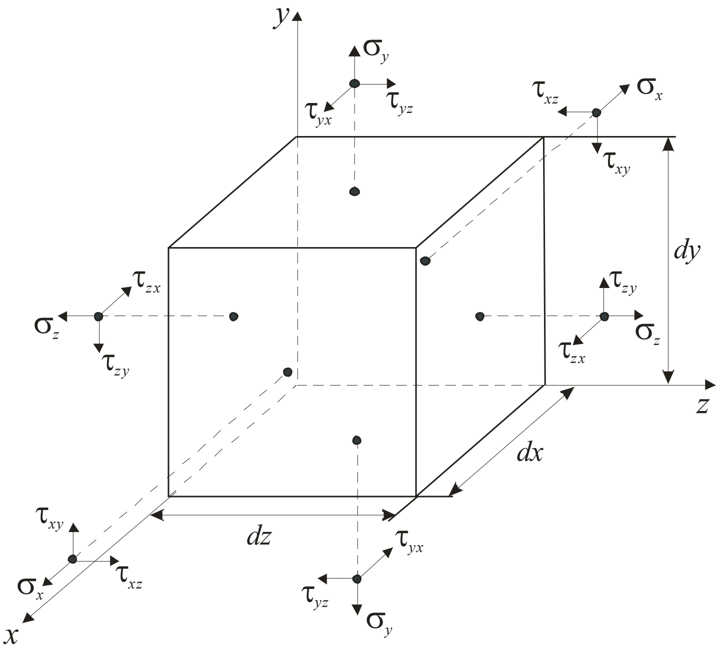

Fig. 11.4. Infinitesimal parallelepiped

Normal stresses σ have an axis index that coincides with the normal to the site, tangents τ have two indices: the first is the index of the normal to the site, the second is the index showing which coordinate axis the tangential stress is acting on. Normal stresses σ are considered positive if they are directed towards the outer normal to the site they act on. Shear stresses τ are considered positive if they give on the corresponding projection axis the same sign as the positive normal stresses acting on the site. The total number of stress components describing the stress state at a point on the selected sites is six, since the shear stresses obey the law of pairing shear stresses:

$$\tau_{xy} = \tau_{yx};$$

$$\tau_{yx} = \tau_{zy};$$

$$\tau_{zx} = \tau_{xz}.$$

The stress state at a point will be considered known if the total stress vector in any area passing through this point is known.

If we know the stress state at every point of the body, then we know the stress state of the whole body.

Consider the stress components acting on the faces of a cubic element.

Nine stress components in three mutually perpendicular areas passing through the point under consideration fully characterize the stress state at this point and form a symmetric second-rank tensor, called the stress tensor:

$$T_\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}.$$

With these values, it is possible to find the projections on the coordinate axes of the total stress vector $\overline{\sigma}_n$ in an arbitrarily oriented area with a unit normal $\overline{n}$ (Cauchy's formulas):

$$\sigma_{nx} = \sigma_{xx}n_x + \sigma_{xy}n_y + \sigma_{xz}n_z;$$

$$\sigma_{ny} = \sigma_{xy}n_x + \sigma_{yy}n_y + \sigma_{yz}n_z;$$

$$\sigma_{nz} = \sigma_{xz}n_x + \sigma_{yz}n_y + \sigma_{zz}n_z,$$

where n_x, n_y, n_z – normal vector components $\overline{n}$, equal to direction cosines.

Cartesian coordinates x, y, z can be denoted as x_1, x_2, x_3 or in general form x_i.

By σ_{ij} we mean the totality of all nine stress components (i.e., the stress tensor), and by n_i, the components of the unit normal vector.

An abbreviated notation for summation in tensor analysis is that the sign of the sum is omitted, and over any index twice repeated in a monomial, summation is performed over the values 1, 2, 3:

$$\sigma_{nj} = \sigma_{ij} n_i.$$

A repeated index is called dumb or dummy, and a non-repeating j is called free.

It is known that at each point of a deformable body there are at least three mutually perpendicular areas, in which the shear stresses are equal to zero. These areas are called principal, and the normal stresses in them are called principal stresses. The directions of the normal to the main areas form the main directions (or axes) of the stress tensor and do not depend on the original coordinate system x, y, z.

Then the total stress can be found by the formula:

$$\sigma = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}.$$

Normal voltage:

$$\sigma_{nx} = \sigma_{xx} n_x + \sigma_{yy} n_y + \sigma_{zz} n_z.$$

Shear stress:

$$\tau = \sqrt{\sigma^2 - \sigma_n^2}.$$

Principal stresses are the roots of the cubic equation

$$\sigma^3 - I_1(T_\sigma)\sigma^2 - I_2(T_\sigma)\sigma - I_3(T_\sigma) = 0.$$

Here the coefficients are determined by the formulas

$$I_1(T_\sigma) = \sigma_{xx} + \sigma_{yy} + \sigma_{zz};$$

$$I_2(T_\sigma) = -\sigma_{xx}\sigma_{yy} - \sigma_{yy}\sigma_{zz} - \sigma_{zz}\sigma_{xx} + \sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2;$$

$$I_3(T_\sigma) = \begin{vmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{vmatrix}.$$

The roots of this equation are denoted as $\sigma_1, \sigma_2, \sigma_3$.

The coefficients $I_1(T_\sigma), I_2(T_\sigma), I_3(T_\sigma)$ are called stress tensor invariants.

The main shear stresses are determined by the formulas:

$$\tau_1 = \frac{\sigma_2 - \sigma_3}{2}; \qquad \tau_2 = \frac{\sigma_3 - \sigma_1}{2}; \qquad \tau_3 = \frac{\sigma_1 - \sigma_2}{2}.$$

The general case of a stress state can be represented as the sum of two stress states (fig. 11.5).

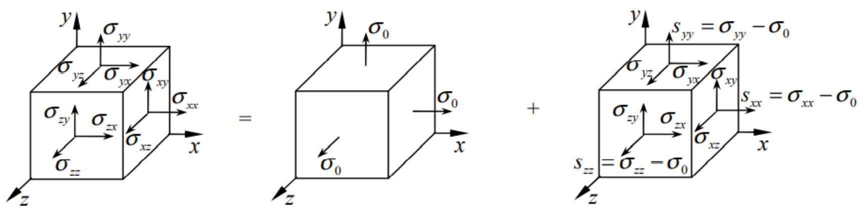

Fig. 11.5. Stress tensor

The stress tensor of the first stress state is called spherical. It is denoted by:

$$T_{\sigma_0} = \begin{pmatrix} \sigma_0 & 0 & 0 \\ 0 & \sigma_0 & 0 \\ 0 & 0 & \sigma_0 \end{pmatrix}.$$

Here $\sigma_0 = (\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) / 3$ – average (hydrostatic) pressure.

The stress tensor of the second stress state is called the stress deviator:

$$D_\sigma = \begin{pmatrix} s_{xx} & s_{xy} & s_{xz} \\ s_{yx} & s_{yy} & s_{yz} \\ s_{zx} & s_{zy} & s_{zz} \end{pmatrix},$$

moreover

$$s_{ij} = \sigma_{ij} - \delta_{ij}\sigma_0,$$

where δ_{ij} – Kronecker symbol:

$$\delta_{ij} = \begin{cases} 1, if\ i = j \\ 0, if\ i \neq j \end{cases}.$$

In the first described stress state, the shape of the element does not change, but the volume alone changes. In the second described stress state, the change in the volume is zero and the shape of the element alone is distorted.

The stress deviator shows how a given stress state differs from a uniform extension or compression by stresses σ_0 .

The invariants of the stress deviator through its principal stresses are expressed by the following formulas:

$$I_1(D_\sigma) = s_1 + s_2 + s_3 = 0;$$

$$I_2(T_\sigma) = -s_1 s_2 - s_2 s_3 - s_3 s_1;$$

$$I_3(T_\sigma) = s_1 s_2 s_3 .$$

For the complex characteristics of stresses, two quantities are used: the intensity of normal stresses and the intensity of shear stresses.

The stress intensity is a quantity proportional to the square root of the second invariant of the stress deviator. Depending on the adopted proportionality coefficient, the concepts of the intensity of normal stresses or simply the intensity of stresses are distinguished

$$\sigma_i = \sqrt{3I_2(D_\sigma)},$$

the shear stress intensity

$$T = \sqrt{I_2(D_\sigma)}.$$

There are 3 types of stress at a point:

1) uniaxial (linear) stress state;

2) biaxial (flat) stress state. The two main voltages are nonzero;

3) triaxial (volumetric) stress state. All three main voltages are nonzero.

A linear stress state is called a simple stress state, a flat and a volumetric state are called a complex stress state.

§ 12 PLANE DEFORMED AND PLANE STRESS STATE

To assess the stresses created in a material by the action of an external load (force, effort), it is necessary to know the magnitude, and the direction of the force, and the magnitude of the area the force acts on, as well as its orientation to the force vector.

The stress state at some point of the body is considered definite if the stresses on the three mutually perpendicular areas at this point are known.

In pressure treatment, there are often cases where deformations in one direction are negligible compared to deformations in other directions. This phenomenon usually occurs when stamping blanks with an elongated axis, when the main flow of the metal occurs in directions perpendicular to the axis, i.e. in transverse directions. In these cases, it is believed that there is a plane deformation of the metal in the cross-sections of the workpiece. The stress state itself is called a flat deformed state.

When solving many other practical problems, it can be assumed that there are no tensions in one of the main directions. Sheet metal stamping is just the case. In most sheet stamping processes, the stresses normal to the surface of the sheet blanks are fractions of a percent of the stresses arising in the cross sections of the blanks. In these cases, there is a plane stress state.

The normal stress in the direction of the z-axis is:

1. For a flat deformed state:

a) with elastic deformations:

$$\sigma_z = \mu(\sigma_x + \sigma_y),$$

where μ – Poisson's ratio;

b) with plastic deformations:

$$\sigma_z = (\sigma_x + \sigma_y) / 2.$$

2. For a plane stress state $\sigma_z = 0$.

3. For a plane stress state in the *z* direction, there are no stresses, but there is a deformation. For a flat deformed one, on the contrary, there is no deformation, but there are stresses (fig. 12.1, 12.2).

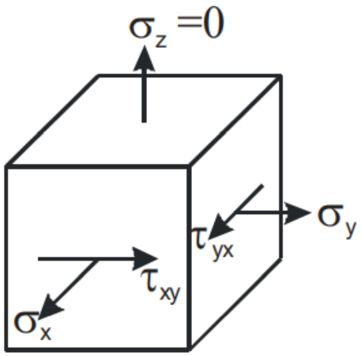

Fig. 12.1. Plane stress state

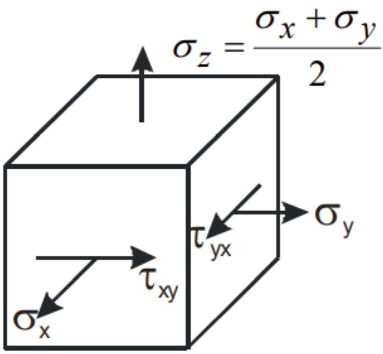

Fig. 12.2. Stress state in a plane deformed state

The equilibrium equations for the plane problem can be obtained from the main system of equilibrium equations and have the form

$$\begin{cases} \dfrac{\partial \sigma_x}{\partial x} + \dfrac{\partial \tau_{xy}}{\partial y} = 0; \\ \dfrac{\partial \tau_{xy}}{\partial x} + \dfrac{\partial \sigma_y}{\partial y} = 0. \end{cases}$$

§13 DESCRIPTION OF THE MOTION OF A CONTINUOUS MEDIUM

Under the action of external forces, each particle of the deformable body receives a certain speed. There are two equivalent approaches to describing the motion of material particles of a continuous medium – Lagrange's approach and Euler's approach.

According to Lagrange's approach, material particles are an object of study, in particular, the change in its kinematic characteristics (position in space, velocity and acceleration). To describe the motion in this form, it is necessary to individualize each particle. The parameters that individualize each particle are their coordinates X, Y, Z at the initial moment of time $t = t_0$. The coordinates of a particle in motionless space (global coordinate system $Oxyz$) depend on the initial coordinates of the particle and time:

$$x = f_x(X, Y, Z, t);$$

$$y = f_y(X, Y, Z, t);$$

$$z = f_z(X, Y, Z, t).$$

Fixing the initial coordinates and considering just the time to be variable, we obtain the law of motion of each particle. If we fix the time, then we get the distribution of the material particles in space at a particular moment in time. If we consider the initial coordinates and time as variables, then we will obtain a description of the motion of a continuous medium.

The initial X, Y, Z coordinates that define each particle and the time t are called Lagrange variables.

The displacement of the material points is defined as the difference between their current and initial position

$$u_x = x - X;$$

$$u_y = y - Y;$$

$$u_z = z - Z.$$

Let us introduce the concept of an accompanying coordinate system. It is a movable deformable coordinate system, the coordinate lines of which are always associated with the same material particles. At the initial moment of time, the coordinate lines are rectilinear to coincide with the coordinate lines of the Cartesian coordinate system. In the future, the accompanying coordinate system moves and deforms together with the material environment. The coordinate lines of such a system in the general case, when the medium moves, become curvilinear (fig. 13.1).

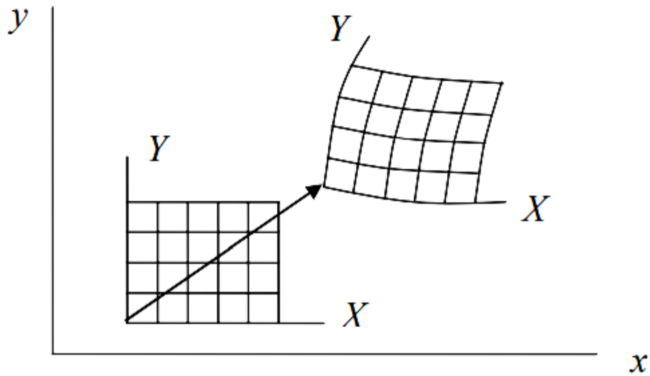

Fig. 13.1. The movement of the environment and the accompanying coordinate system

According to Euler's approach, a stationary space of the observer, filled with a moving medium is an object of study. The quantities characterizing the motion are considered to be functions of the coordinates of a point in the fixed space of the observer x, y, z and time t.

These variables are called Euler variables. Thus, the object of study in Euler's approach is various fields (i.e., the distribution of quantities in space) that characterize the motion of a continuous medium. The motion of a continuous medium, from the point of view of Euler, can be considered given if the distribution of displacements or velocities of the continuous medium in the stationary space of the observer is known, depending on the Euler variables:

$$u_x = u_x(x, y, z, t); \quad v_x = v_x(x, y, z, t);$$

$$u_y = u_y(x, y, z, t); \quad v_y = v_y(x, y, z, t);$$

$$u_z = u_z(x, y, z, t); \quad v_z = v_z(x, y, z, t).$$

You can go from the Lagrange variables to the Euler variables and vice versa. Resolving the system with respect to the Lagrange variables, we get:

$$X = F_x(x, y, z, t);$$

$$Y = F_y(x, y, z, t);$$

$$Z = F_z(x, y, z, t).$$

The description of the motion according to Euler and Lagrange is mechanically equivalent. In the theory of metal working by pressure for analytical calculations, the method of describing the motion according to Euler has found greater application. In numerical calculations, both Euler and Lagrange descriptions of the motion of a continuous medium are used.

§ 14 MATERIAL DESTRUCTION

According to experts, destruction in the engineering aspect of the concept described is the process of dividing the body into two or more parts, as a result of which the body ceases to perform the function assigned to it. The development of ideas on the process of destruction can be represented in the form of diagrams shown in fig. 14.1.

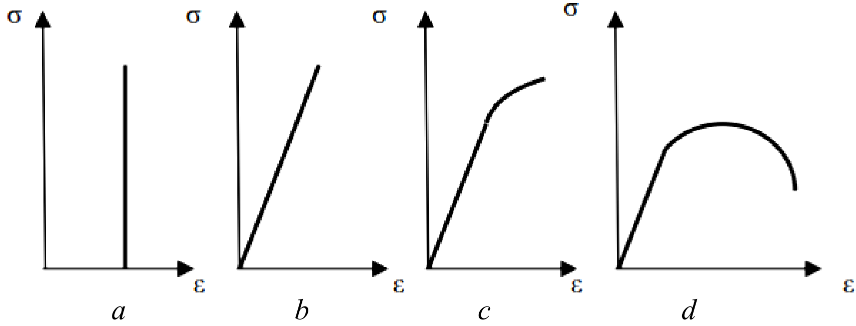

Fig. 14.1. Development of ideas on the process of destruction

Until the 17th century the destruction was believed to occur suddenly with nothing to have preceded it (fig. 14.1, *a*).

In the 17th century, R. Hooke found out that the destruction had been preceded by an elastic deformation (fig. 14.1, *b*).

In the 19th century, when steel began to be widely used as a structural material, it was found out that the fracture process was preceded not only by elastic, but by plastic deformation as well (fig. 14.1, *c*).

In the 20th century after World War II, it became clear that the destruction was preceded by both the elastic and the plastic deformation, and the destruction itself should be considered as a process of initiation and subsequent growth of a crack (fig. 14.1, *d*). But only some especially highly plastic metals such as aluminum can be dismembered exclusively by the plastic deformation, i. e. without cracking.

According to the magnitude of the previous plastic deformation fracture is usually divided into two main types – brittle and ductile ones. If the magnitude of the plastic deformation does not exceed 1–2 %, then the fracture is brittle, but if the plastic deformation exceeds 2 %, then the fracture is considered tough.

CONTENT

Учебное издание

ГУНДИНА Мария Анатольевна
ЮХНОВСКАЯ Ольга Витальевна

ПРИКЛАДНАЯ МАТЕМАТИКА ПЛАСТИЧЕСКОЙ ДЕФОРМАЦИИ МЕТАЛЛОВ

Учебно-методическое пособие
для студентов специальности
1-38 80 01 «Приборостроение»

На английском языке

Редактор *Е. И. Беницевич*
Компьютерная верстка *Н. А. Школьниковой*

Подписано в печать 31.03.2022. Формат 60×84 $^1/_{16}$. Бумага офсетная. Ризография.
Усл. печ. л. 3,95. Уч.-изд. л. 3,09. Тираж 100. Заказ 107.

Издатель и полиграфическое исполнение: Белорусский национальный технический университет.
Свидетельство о государственной регистрации издателя, изготовителя, распространителя
печатных изданий № 1/173 от 12.02.2014. Пр. Независимости, 65. 220013, г. Минск.