MINISTRY OF EDUCATION OF THE REPUBLIC OF BELARUS
Belarusian National Technical University

Department "Engineering graphics of construction"

Y. I. SADOVSKI
I. M. SHUBERT
E. A. SADOVSKAYA
E. A. TELESH

SPECIAL SECTIONS OF DESCRIPTIVE GEOMETRY FOR BUILDERS AND ARCHITECTS

Educational-methodical guide
for students of specialties
7-07-0732-01 "Construction of buildings and structures",
7-07-0732-02 "Engineering networks, equipment of buildings and structures",
6-05-0718-01 "Engineering economics",
6-05-0732-02 "Expertise and property management",
6-05-0719-01 "Engineering and pedagogical activities"

Recommended by the Educational and Methodological Association of education in the field of construction and architecture

Minsk
BNTU
2024

УДК 515.18(075.8)
ББК 22.151.3я7
С14

R e v i e w e r s:
Head of the Department of Standardization, Metrology
and Engineering Graphics of
Belarusian State Agrarian Technical University
Ph. D., Associate professor *P. V. Avramenko;*
Chief Project Engineer of the
Limited Liability Company «TAPAS» *D. G. Pakhomchik*

Sadovski Y. I.
С14 Special sections of descriptive geometry for builders and architects: for students of specialities 7-07-0732-01 "Construction of buildings and structures", 7-07-0732-02 "Engineering networks, equipment of buildings and structures", 6-05-0718-01 "Engineering economics", 6-05-0732-02 "Expertise and property management", 6-05-0719-01 "Engineering and pedagogical activities" / Y. I. Sadovski, I. M. Shubert, E. A. Sadovskaya, E. A. Telesh. – Minsk : BNTU, 2024. – 58 p.
ISBN 978-985-31-0114-0.

This textbook is designed for first-year students of architectural and construction specialties in order to improve the skills of independent work and enrich the educational process with additional material.

The manual contains educational material on special sections of descriptive geometry – perspective, projections with numerical marks and the basics of shadow geometry.

This publication can also be intended for distance learning, since it contains theoretical material on descriptive geometry as the fundamental basis of the course of all graphic disciplines.

УДК 515.18(075.8)
ББК 22.151.3я7

ISBN 978-985-31-0114-0

CONTENTS

INTRODUCTION

Descriptive geometry is aimed at studying a subject that develops spatial representations and imagination, constructive geometric, abstract logical thinking, the ability to analyze and synthesize spatial forms and relationships based on graphical models of space, which are practically implemented in the form of drawings of specific spatial objects and dependencies.

The main purpose of teaching the subject "Descriptive Geometry" is to provide students with the knowledge and skills necessary to perform and read drawings for various purposes and make decisions on drawings of geometric and technical problems.

The proposed textbook is prepared for students of technical specialties of the architectural and construction profile and includes specialized sections related to the construction of single-picture images – perspective, projections with digital markings and the basics of shadow geometry. The publication is intended for international students, as well as all those who are interested in technical literature on this topic in English.

1. PERSPECTIVE

1.1. Central projection

To get the central projection of a geometric shape on the projection plane *K* (fig. 1.1), you need to:

1. Through the points of the figure, draw the projecting rays so that they pass through the center of the projection – the point *S*.

2. Determine the intersection points of the projecting rays with the projection plane *K*.

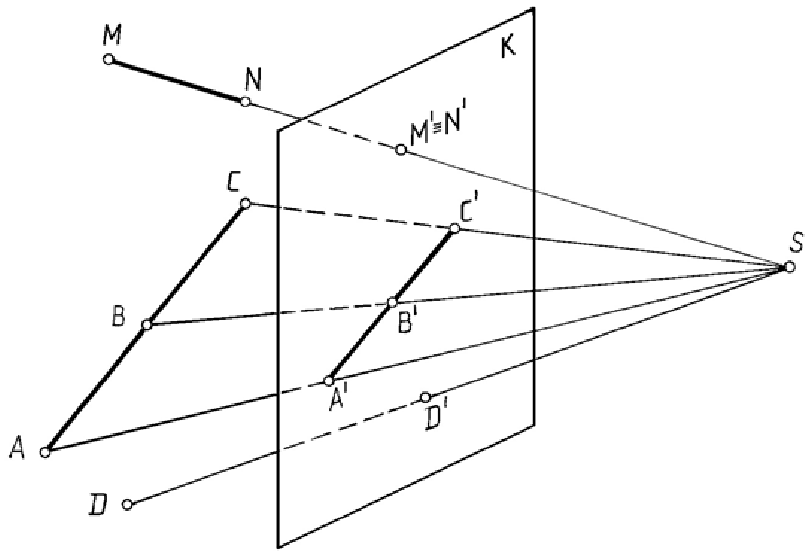


Fig. 1.1

Properties of the central projection (fig. 1.1):

1. The projection of a point is a point.

2. The projection of a straight line is a straight line or a point.

3. If a point belongs to a straight line, then the projection of the point belongs to the projection of the straight line.

A perspective is an image constructed according to the method of central projection with a certain center of projections *S* and a plane of projections *K* (the picture), which meets the conditions of visual perception. The advantage of perspective is visibility; the disadvantages are projecting projections onto one plane (obtaining a reversible drawing).

A perspective constructed on a plane is called linear, on a cylindrical surface – panoramic, on a spherical surface – domed.

1.2. Linear perspective device

Linear perspective is used in architecture and construction.

The linear perspective device is shown in fig. 1.2.

Π_1 – the horizontal plane, on which the object of projection is located, is called the object plane.

K – the plane perpendicular to the object plane, on which the perspective image is carried out, is called the picture plane or picture, $K \perp \Pi_1$.

S – the center of the projections, i. e. the point at which the observer's eye is located, is called the point of view.

The *N*-plane passing through the point of view *S* and parallel to the picture is called the neutral plane, $N \parallel K$.

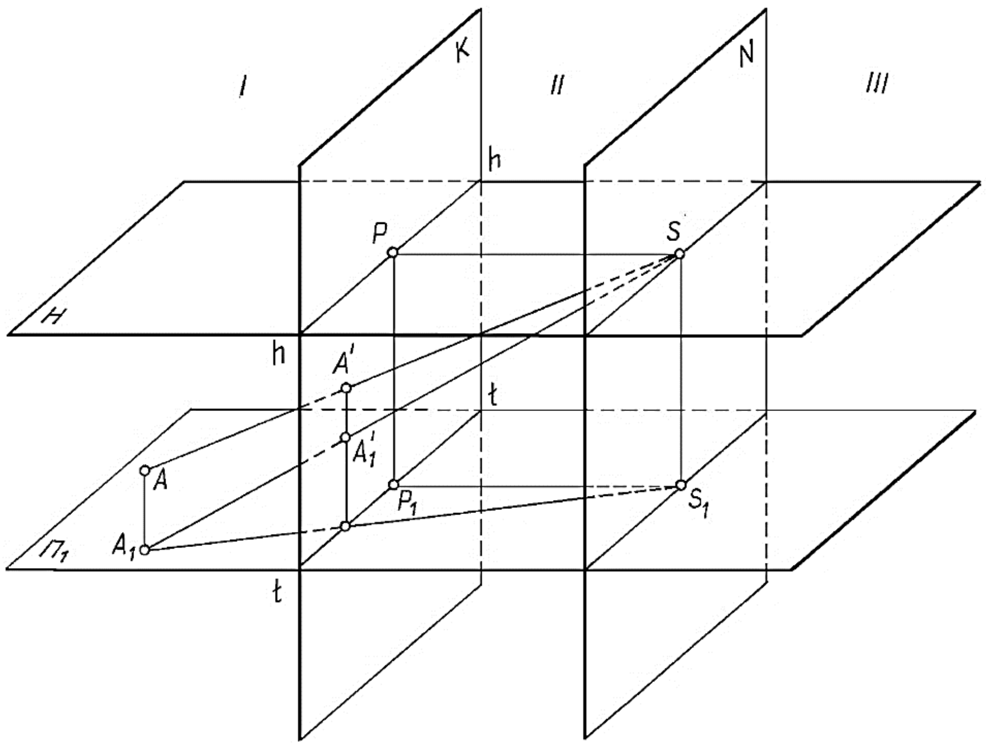


Fig. 1.2

The picture and neutral planes divide the entire space into three parts:

– the object space I (which is located behind the picture from the observer and in which the projected object (object) is located));

– intermediate space II (enclosed between the picture and the neutral plane);

– imaginary space III (located on the other side of the neutral plane).

H – the horizontal plane passing through the point of view is called the horizon plane, $H \parallel \Pi_1$.

$hh = H \cap K$ – horizon line.

$tt = \Pi_1 \cap K$ – base of the picture (ground line).

$hh \parallel tt$.

SP – the perpendicular lowered from the point of view S to the picture plane *K* is called the main ray, $SP \perp K$.

$P = SP \cap K$ – the main point of the picture.

PP_1 – the main line of the picture, $PP_1 \perp tt$ and $PP_1 \perp hh$.

Orthogonal projections of points on the object plane Π_1, called the bases of these points:

– A_1 – the base of the point A, located in the object space;

– P_1 – the base of the main point of the picture;

– S_1 – the base of the point of view or the point of standing.

– SS_1 – the distance from the point of view to the object plane, called the height of the point of view (the height of the horizon), $|SS_1| = |PP_1|$.

PS – the distance from the point of view to the picture, called the main distance.

$A' = AS \cap K$ – the perspective of point A.

$A_1' = A_1S \cap K$ – the perspective of the base or the secondary projection of point A.

One of the requirements for a drawing is its reversibility. To obtain a reversible drawing when projecting onto a single projection plane, a secondary projection is required. Thus, the perspective of a point and its secondary projection uniquely determine the position of the point in space. $A'A_1'$ is the connection line between the perspective of a point and its secondary projection. $A'A_1' \perp hh$ and $A'A_1' \parallel PP_1$.

The construction of the perspective (fig. 1.3) begins with the assignment of the main elements of the linear perspective apparatus that belong to the picture. First, set the horizontal lines of the earth tt and the horizon hh, the distance between which is equal to the height of the point of view SS_1. In any place (usually in the center), set the main line of the picture, drawing PP_1 perpendicular to the horizon line.

Sometimes the perspective image shows the distance circle. This is a circle centered at point P with radius $PD = PS$. The point D is called the distance point.

According to the position of the secondary projection of the point (the perspective of the base of the point or the horizontal projection of the point) relative to the lines hh and tt, one can judge the position of the point in space, as can be seen from the scheme of the perspective device shown in fig. 1.2, 1.3, 1.4.

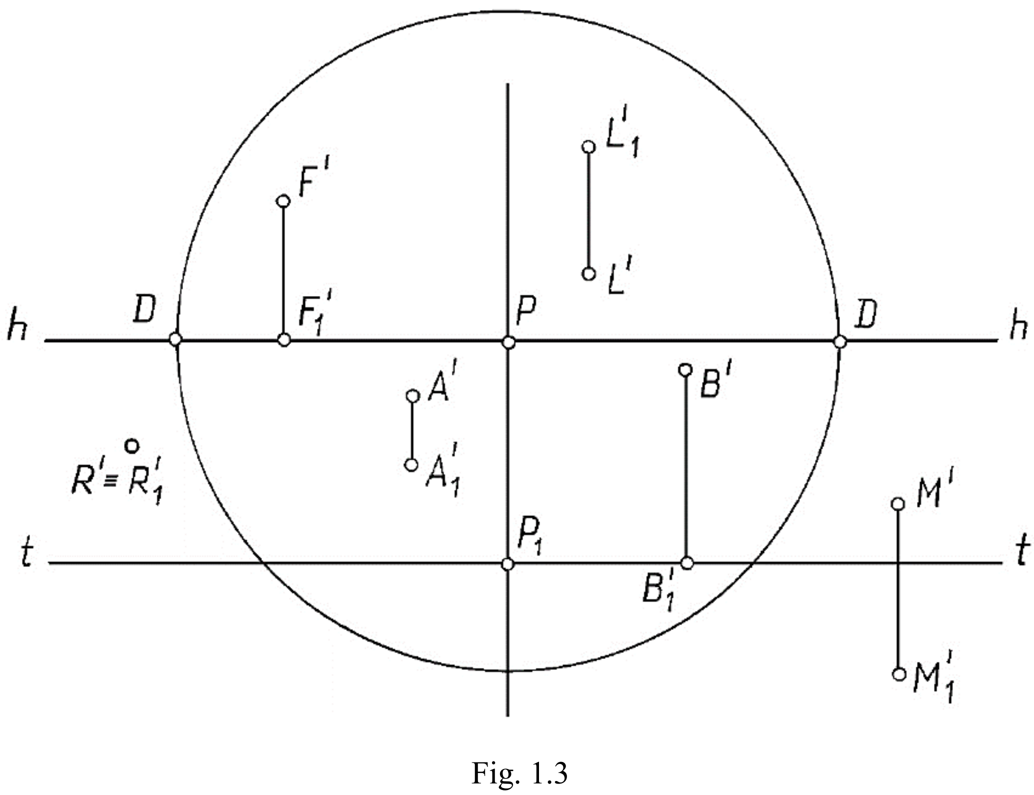


Fig. 1.3

If A_1' (fig. 1.3) is located between the lines *hh* and *tt*, then the point *A* is located in the object space.

If M_1' is located below *tt*, then the point *M* is in the intermediate space.

If L_1' is located above the line *hh*, then the point *L* is in imaginary space.

If B_1' is located on the *tt* line, then the point *B* belongs to the picture.

If $R' \equiv R_1'$, then the point *R* lies on the plane Π_1.

If F_1' is located on the line *hh*, then the point *F* is at infinity, but it is impossible to show this in the drawing.

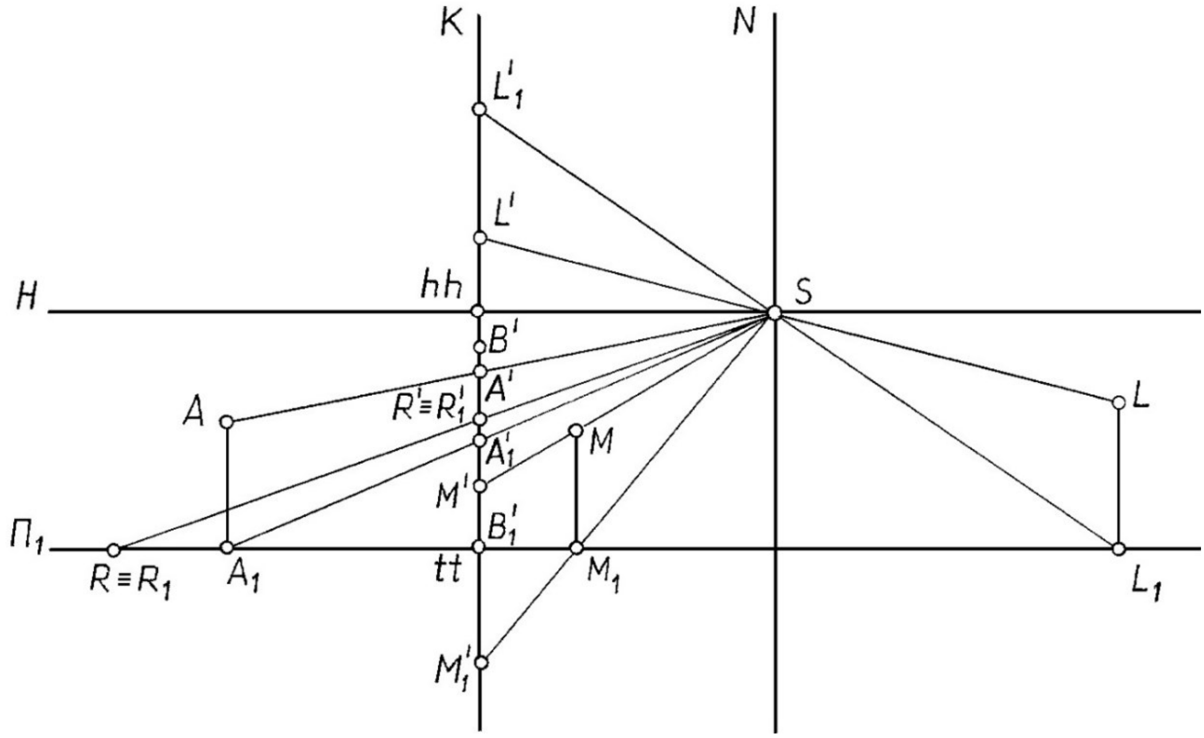


Fig. 1.4

1.3. Straight line perspective

Two points define a straight line in space. To build the perspective of a straight line, usually build the perspective of its two points – the picture trace and the vanishing point of the straight line (fig. 1.5).

A picture trail is the point where a straight line intersects with a picture.

In fig. 1.5:

1' – the picture trace of a straight line *n*;

2' – picture trace of a straight line *m*.

The vanishing point is the perspective of an incongruous (infinitely distant) point of a straight line. To build the vanishing point of a straight line, you need to draw a ray parallel to the straight line through the point of view and find the point of intersection of this ray with the picture. Denote the vanishing point with the letter *F*.

If the lines are parallel in space and not parallel to the picture, then in perspective they intersect at the common vanishing point *F* (fig. 1.5).

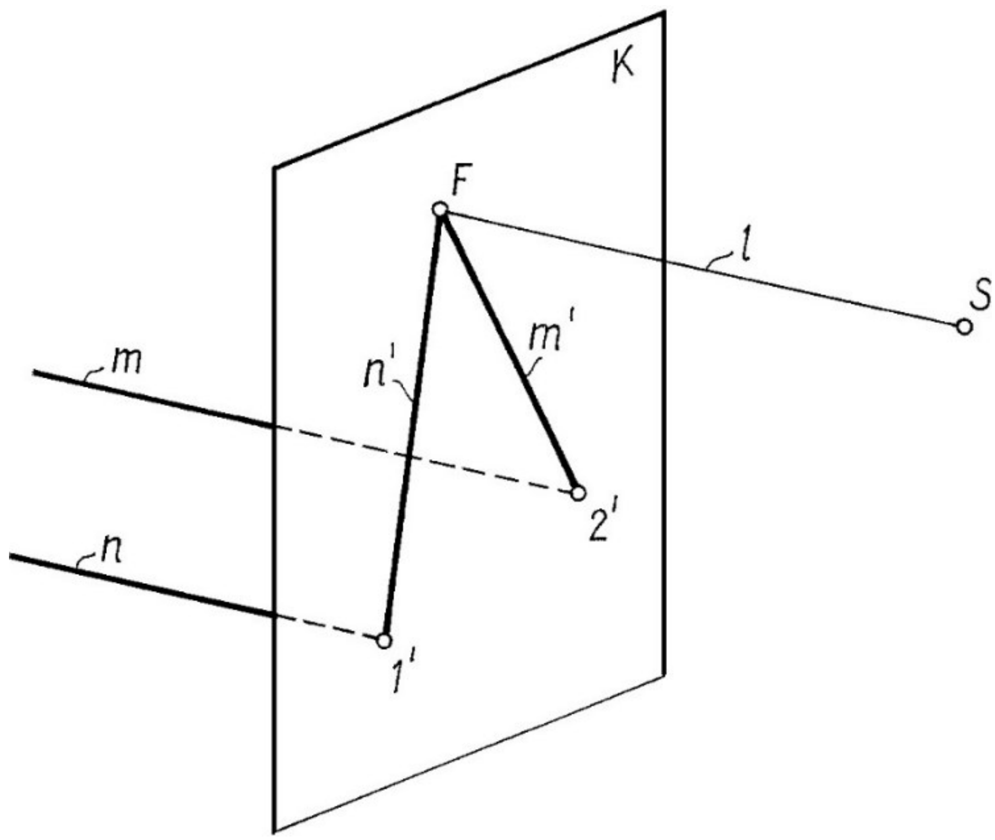


Fig. 1.5

1.4. Building the perspective of a straight line belonging to the subject plane

To build the perspective of a straight line based on its orthogonal projections, you need to find the vanishing point and the picture trace on the orthogonal drawing, and then transfer them to the perspective image. If necessary, set the remote point *D*.

Example.

Construct the perspective of a straight line *l* belonging to the object plane (fig. 1.6).

***Solution*:**

1. In the perspective image set the horizontal lines *tt* and *hh*, the distance between which is equal to the height of the point of view Z_S. In any place (usually in the center), we set the main line PP_1.

2. Determine the picture trace (point *1*) on the orthogonal drawing and build it in perspective on *tt*, putting $P_1 1' = P_1 1_1$.

3. Determine the vanishing point *F* on the orthogonal drawing:

$S_1F_1 \parallel 1_1$; $F_1 = F_1\ S_1 \cap K_1$.

4. Build the perspective of the vanishing point of the straight line, putting on *hh* $PF' = P_1F_1$.

5. Construct the perspective of the line *1'* by connecting the points *1'* and *F'*. In this case, the secondary projection of the line and its perspective coincide $l_1' \equiv l'$.

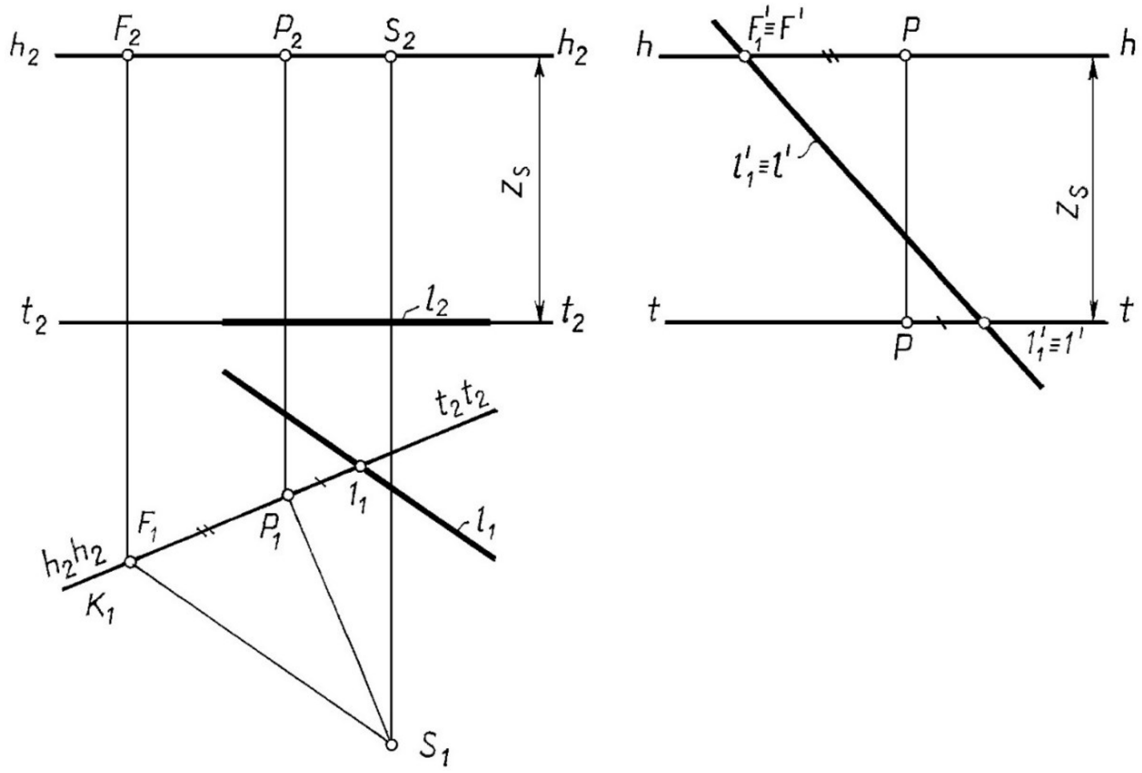


Fig. 1.6

Example.

Build the perspective of a straight line *n* that belongs to the object plane and passes through the point of standing S_1 (fig. 1.7).

If the straight line belongs to the object plane and passes through the point of standing S_1, then its perspective is parallel to the main line of the picture PP_1 (fig. 1.7).

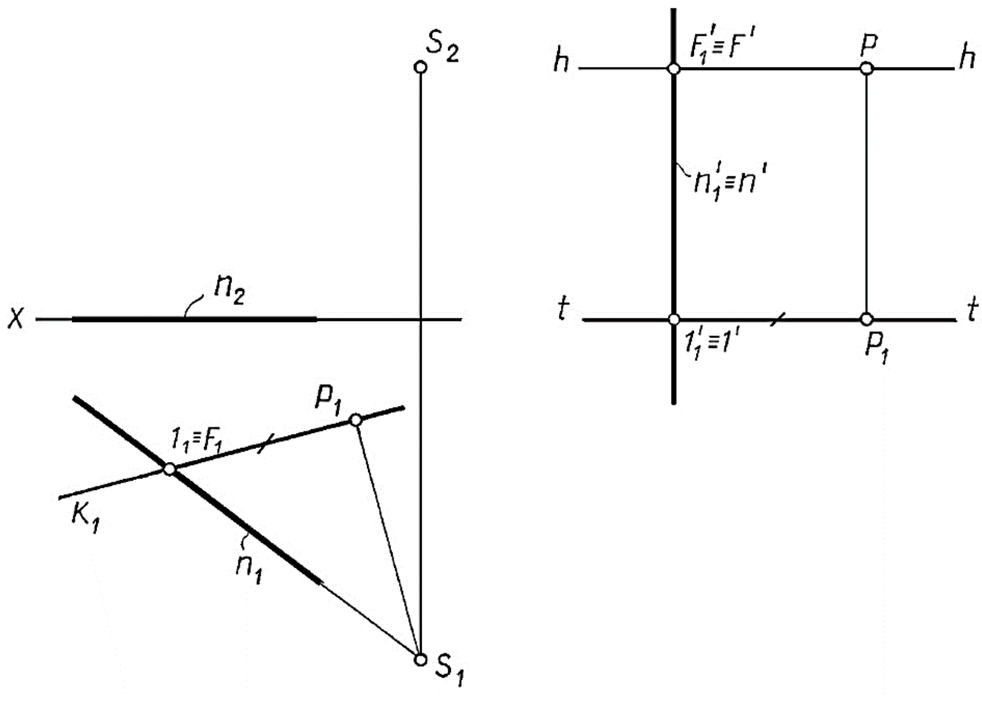


Fig. 1.7

Example.

Construct the perspective of a straight line *m* that belongs to the object plane and is perpendicular to the picture (fig. 1.8)

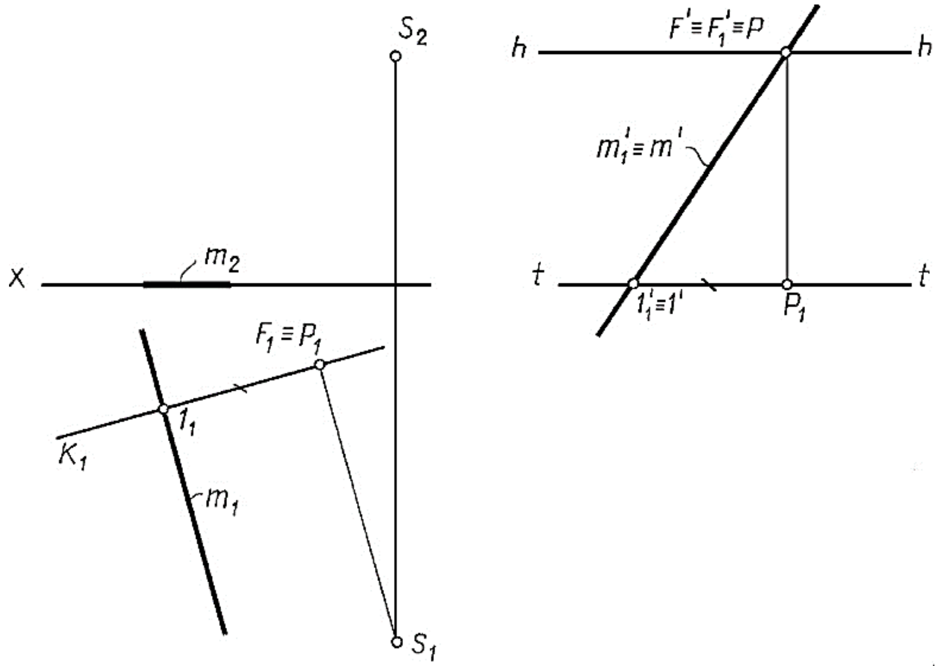


Fig. 1.8

If a straight line is perpendicular to the picture $m \perp K$, then its vanishing point coincides with the main point of the picture *P* (fig. 1.8).

Example.

Build the perspective of the straight line *b*, which belongs to the object plane and forms an angle of 45° with the picture (fig. 1.9)

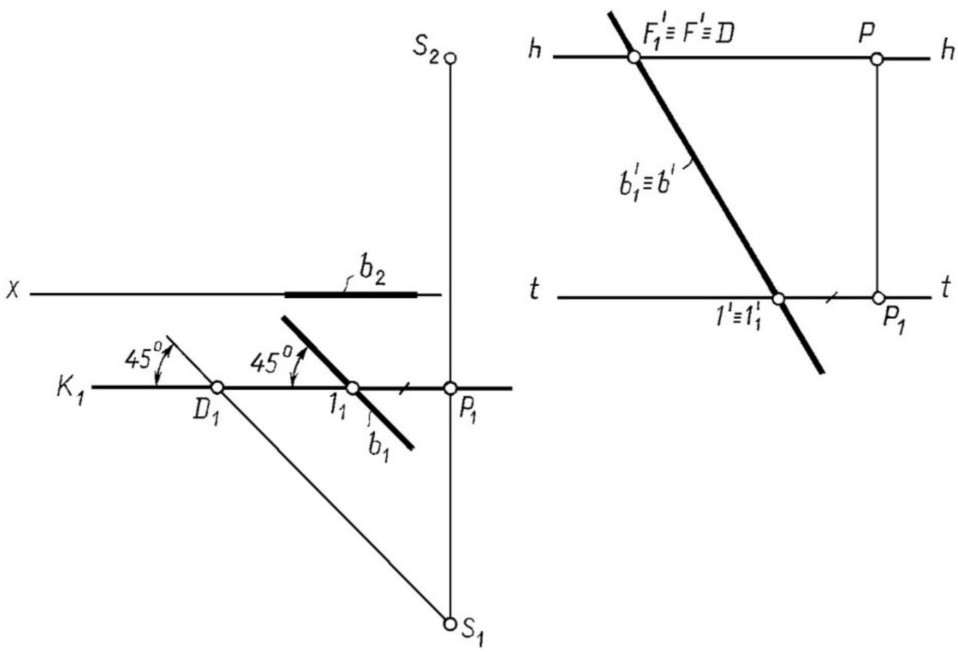


Fig. 1.9

If the straight line belongs to the object plane or is parallel to the object plane and forms an angle of 45° with the picture, then its vanishing point coincides with the distance point D. In figure 1.9, the triangle $D_1P_1S_1$ is isosceles, $|S_1P_1|=|\ D_1P_1|$, so $F\equiv D$.

The perspective of a vertical segment can not be built on the picture trail and the vanishing point. To build vertical segments, you can use the method of removal in the picture or the side wall, they will be described below.

1.5. Building the perspective of a point belonging to the subject plane

Example.

Build the perspective of the point A, which belongs to the object plane (fig. 1.10).

***Solution*:**

Note that the perspective of a point is constructed as the intersection point of the perspectives of two straight lines passing through this point.

1. In the orthogonal drawing we draw a straight line $m \perp K$ through the point A and a straight line n passing through the standing point S_1.
2. We construct the perspective of these lines m', n'.
3. Mark the perspective of the point A' at the intersection of the perspectives of the constructed lines m' and n' .
4. In this case, the secondary projection of the point and the perspective of the point coincide $A_1' \equiv A'$.

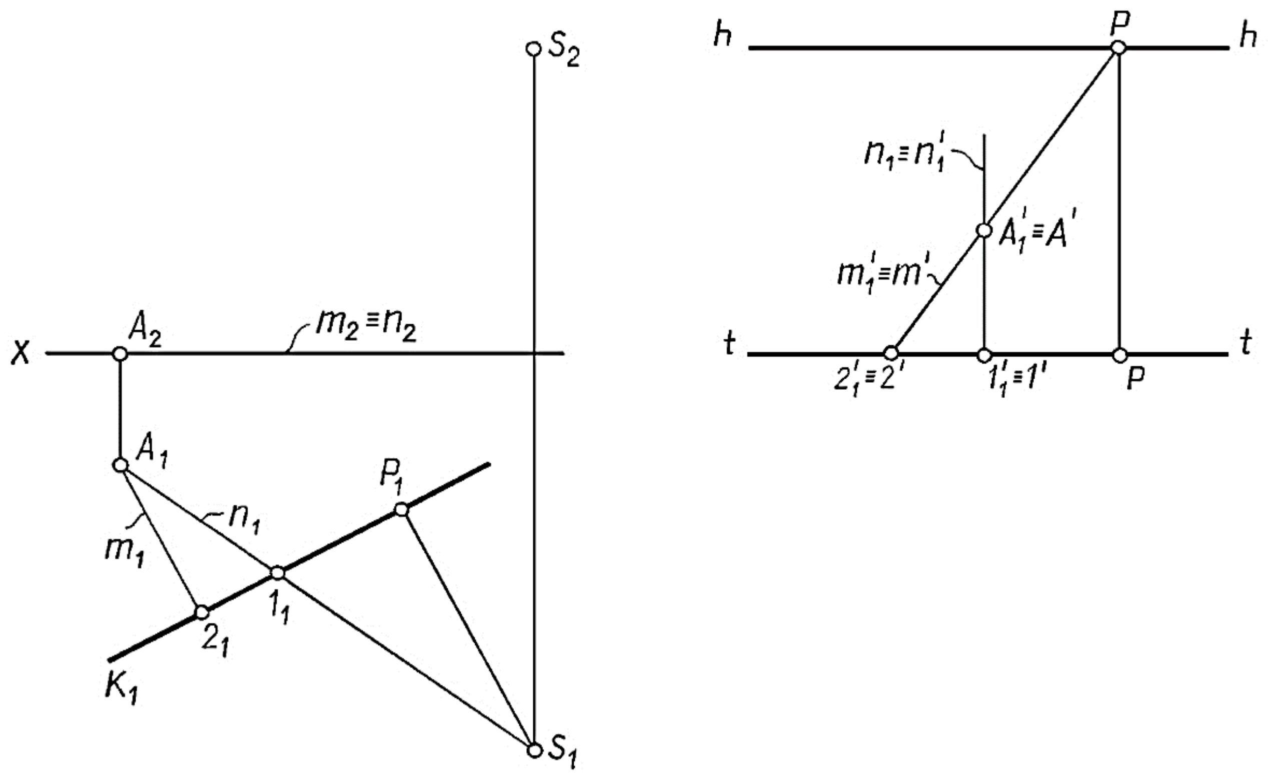


Fig. 1.10

1.6. Building the perspective of a straight line segment belonging to the object plane

Example.

Build the perspective of the segment *AB* (fig. 1.11).

***Solution*:**

1. We build the perspective of the straight line that belongs to the segment *AB* (at points *2* and *F*).
2. We define the points *A* and *B* on this line using auxiliary lines (*SA* and *SB*) passing through the standing point.

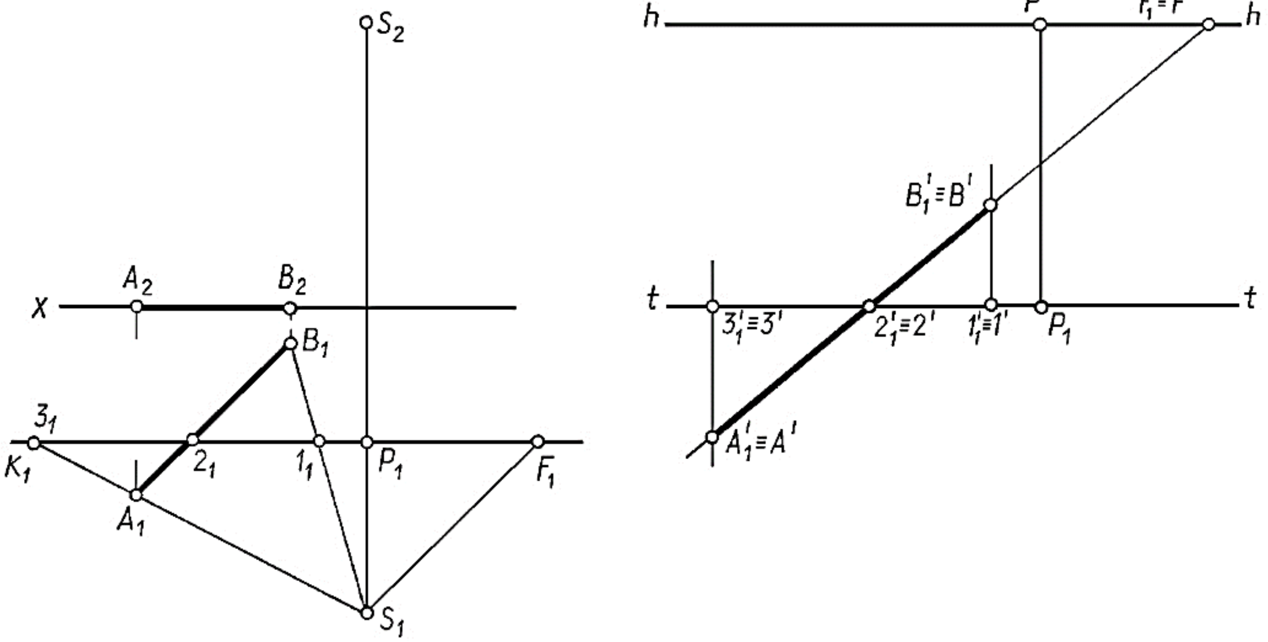


Fig. 1.11

1.7. Building the perspective of a shape based on their orthogonal projections

Building the perspective of a flat shape belonging to the object plane.

Example.

Build the perspective of a flat figure that belongs to the object plane (fig. 1.12).

***Solution*:**

1. We construct the perspective of the lines bounding the plane figure: *AB*, *NM*, and *NA*, *MB* (pairwise parallel with the vanishing points F_1 and F_2).
2. The points of intersection of the perspectives of these lines define the vertices of the plane figure *A*', *B*', *M*', *N*'. Secondary projection and perspective of the lamina coincide $A_1'B_1'M_1'N_1' \equiv A'B'M'N'$.

This method of constructing a perspective using two vanishing points is called the architects method.

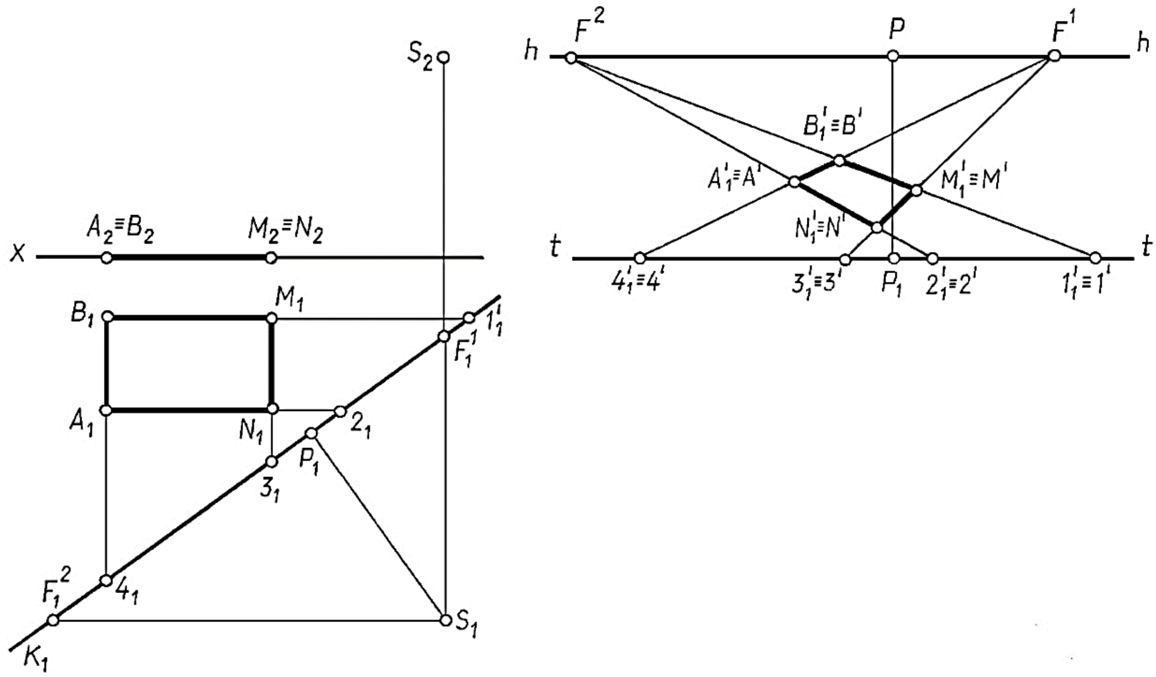


Fig. 1.12

1.8. Building the perspective of a vertical segment using the take-out in the picture, the side wall, the radial method

Since it is impossible to build a picture trace of a straight line and a vanishing point for a vertical segment of a straight line, in this case it is necessary to use other methods of constructing a perspective, namely: taking it out into the picture or using a side wall.

Example.

Build the perspective of the vertical segment *AB* (fig. 1.13).

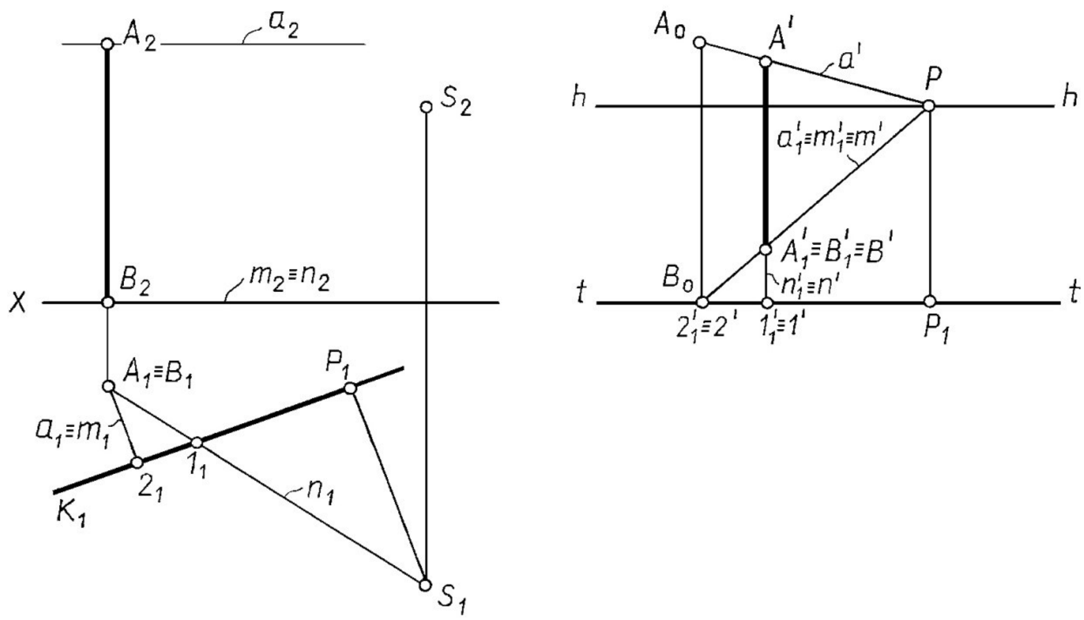


Fig. 1.13

***Solution*:**

The construction of the perspective of a vertical segment is based on the fact that the full size of such a segment can be postponed only in the picture ($A_2\,B_2 = A_0B_0$, see fig. 1.13) and then, knowing the law of change in the projection value of the depicted segment, build its perspective $A'B'$. At the same time, as the vertical segment is removed from the picture, the image of the segment decreases in the object space (and increases in the intermediate space).

1. We build the perspective of the base of the segment AB of the point B (fig. 1.13). The secondary projection of the segment $B' \equiv B_1' \equiv A_1'$ coincides with it.

2. We build the perspective of the point A. To do this, you can use the extension to the picture (fig. 1.14) or the side vertical plane (side wall), as shown in fig. 1.14. The same problem can be solved using the side wall – fig. 1.14.

In this case, the picture and object trace is set to a vertical plane that is convenient for performing constructions – the side wall, fig. 1.14

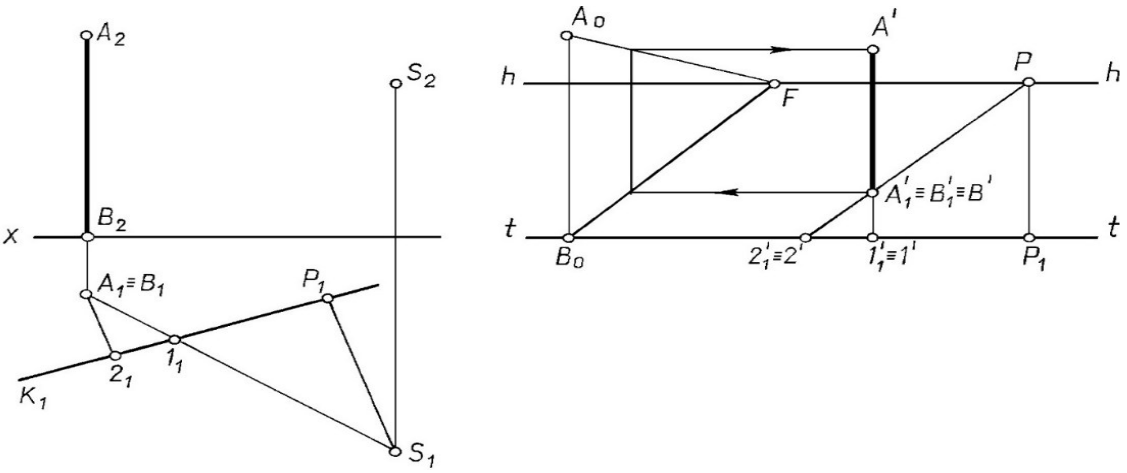


Fig. 1.14

Also, this problem can be solved in a radical way fig. 1.15, i.e., find the intersection points of the projecting rays SA and SB with the picture K.

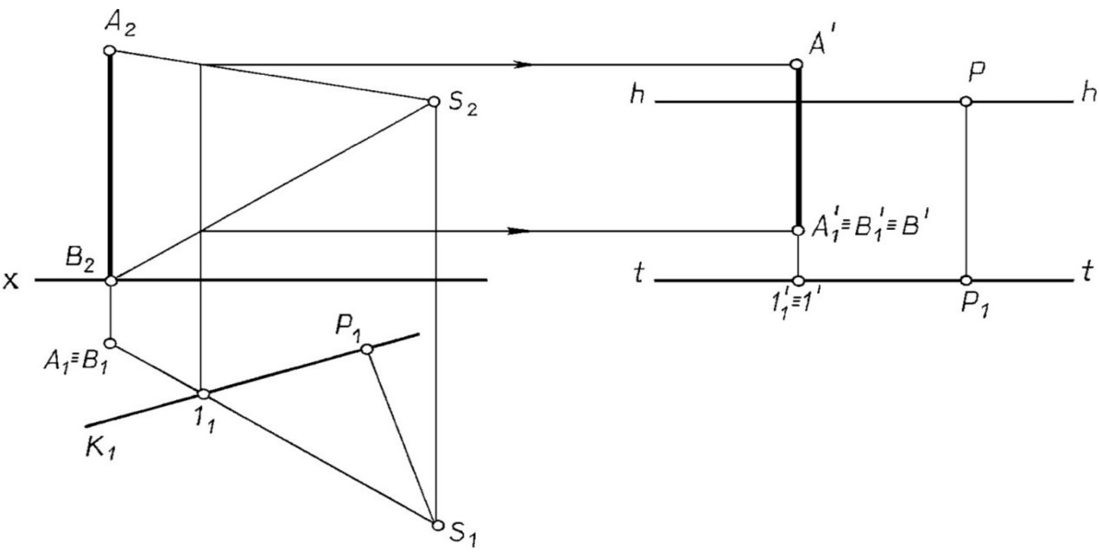


Fig. 1.15

1.9. Building a straight line perspective of the general position

Example.

Plot the perspective of the *AB* line segment of the general position (fig. 1.16)

***Solution*:**

1. Determine the picture trace of the line that the segment *AB* belongs to – the point *R* on the orthogonal drawing – and build it in perspective.

2. Find the vanishing point of the straight line – the point *F* on the orthogonal drawing – and build it in perspective.

3. *F*'*R*' – the perspective of the line to which the segment *AB* belongs;

$F_1'R_1'$ – secondary projection.

4. Construct the perspective of point *A* and point *B* with the help of auxiliary lines *SA* and *SB* going to the standing point S_1.

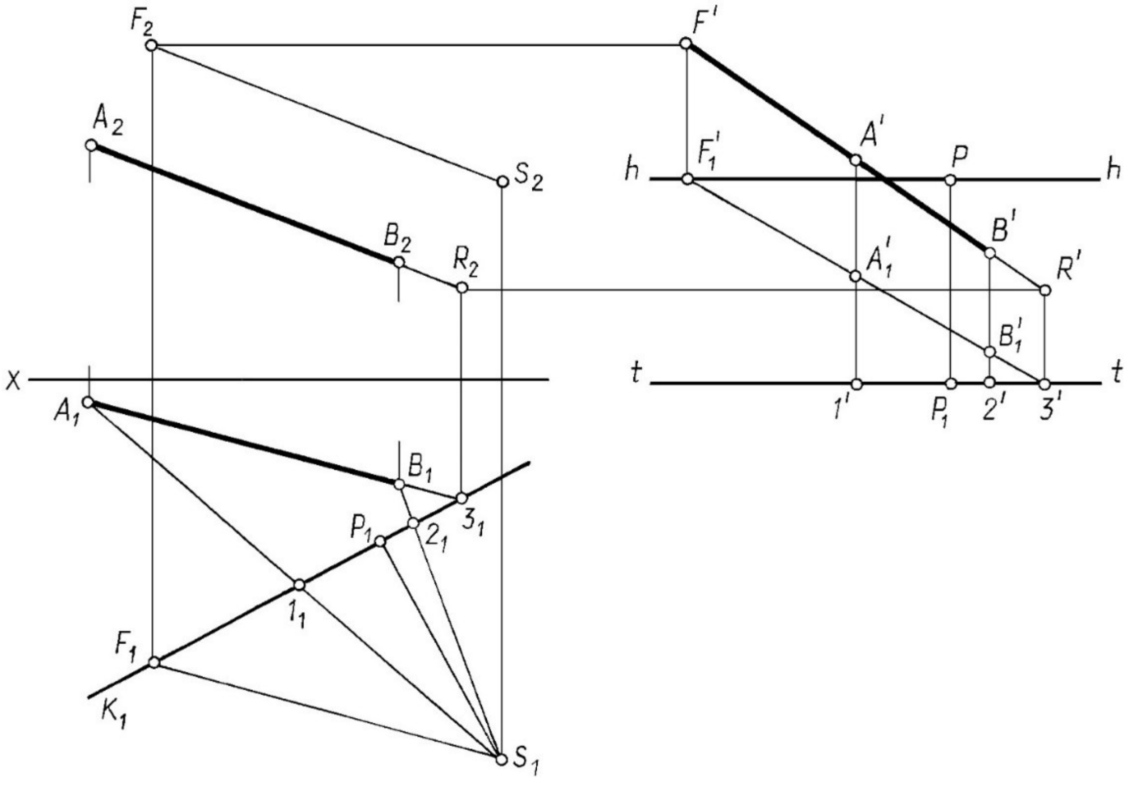


Fig. 1.16

1.10. Ways to build a perspective

When building a perspective, use the following methods:

– radial or ray trace method, which is reduced to determining the points of intersection of the rays with the picture plane (fig. 1.15);

– architects, based on the use of vanishing points of parallel lines of two or more families (fig. 1.17);

– scales, based on the patterns of distortion of segments in the direction of the *X*, *Y*, *Z* axes (scales of latitudes, depths, heights) (fig. 1.18), and etc.

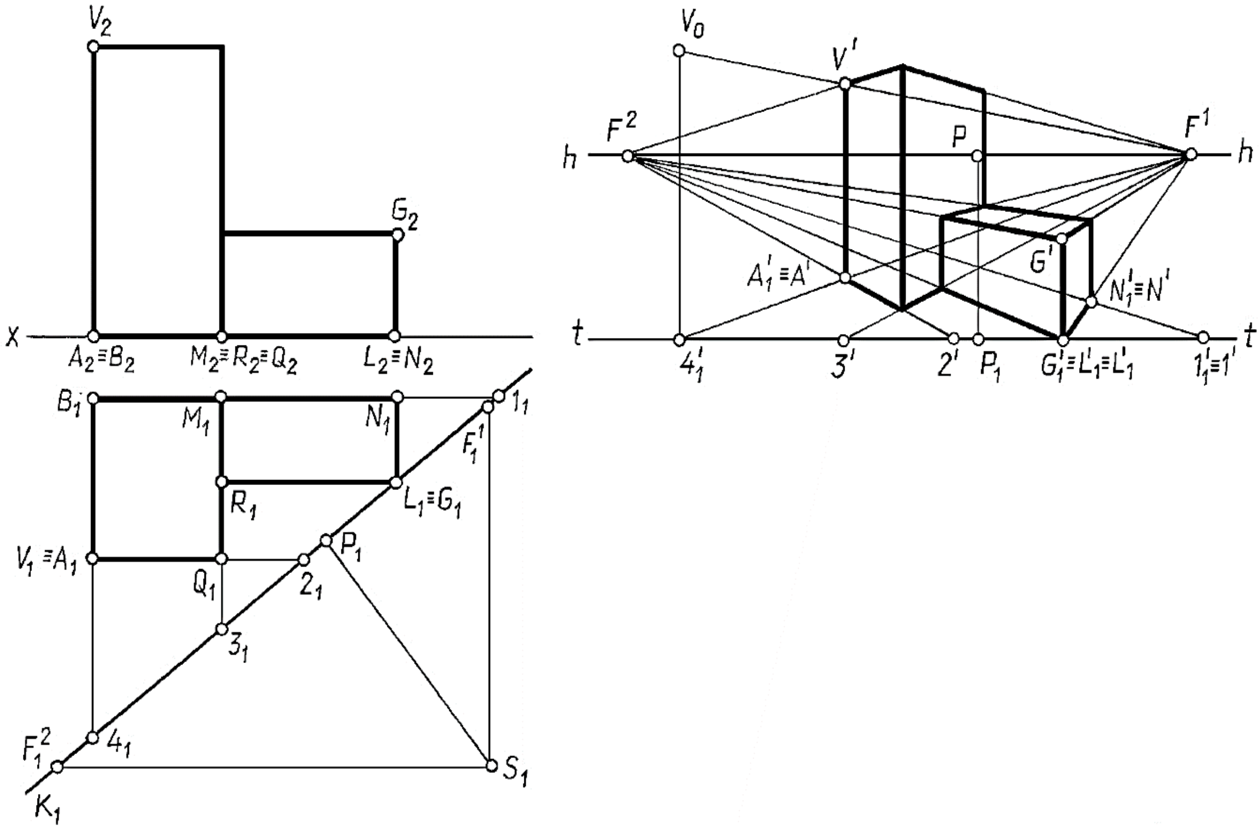


Fig. 1.17

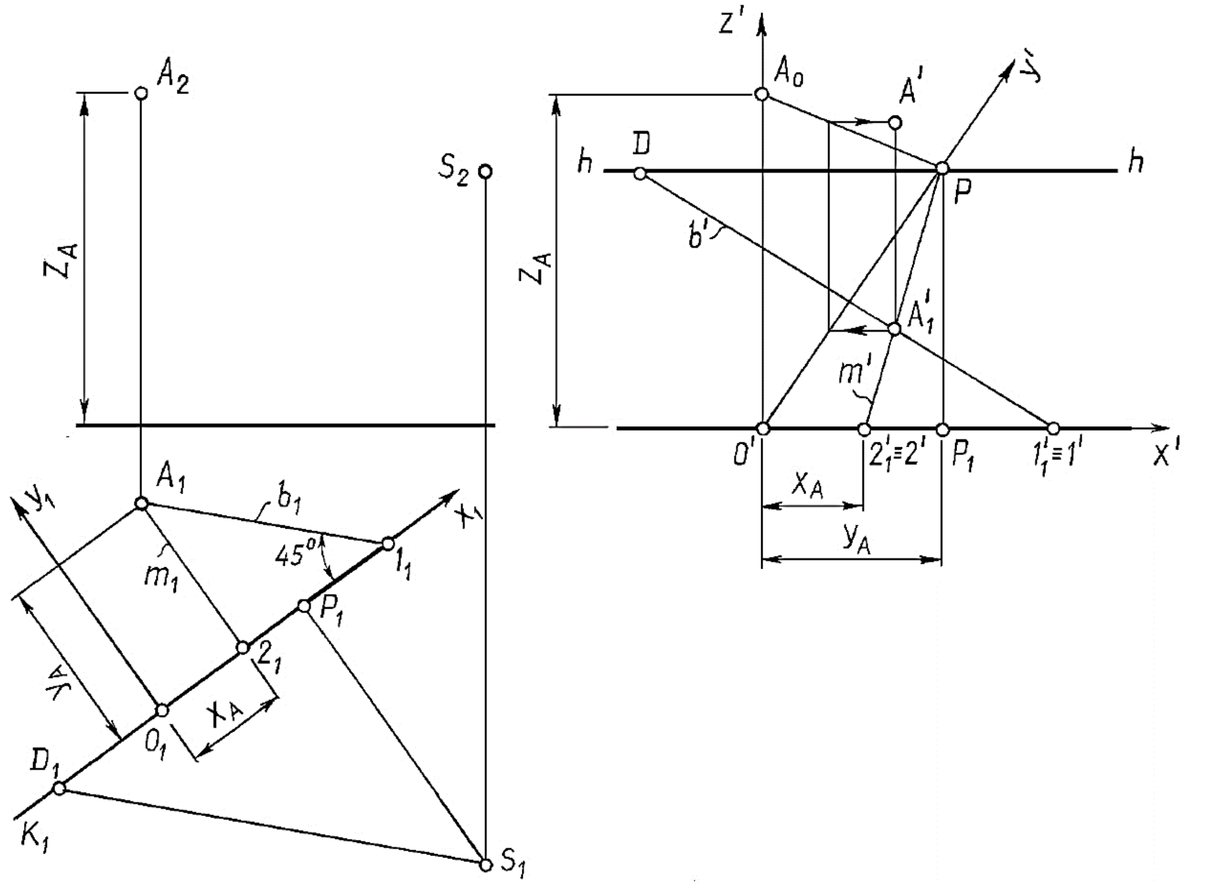


Fig. 1.18

1.11. Choosing a point of view

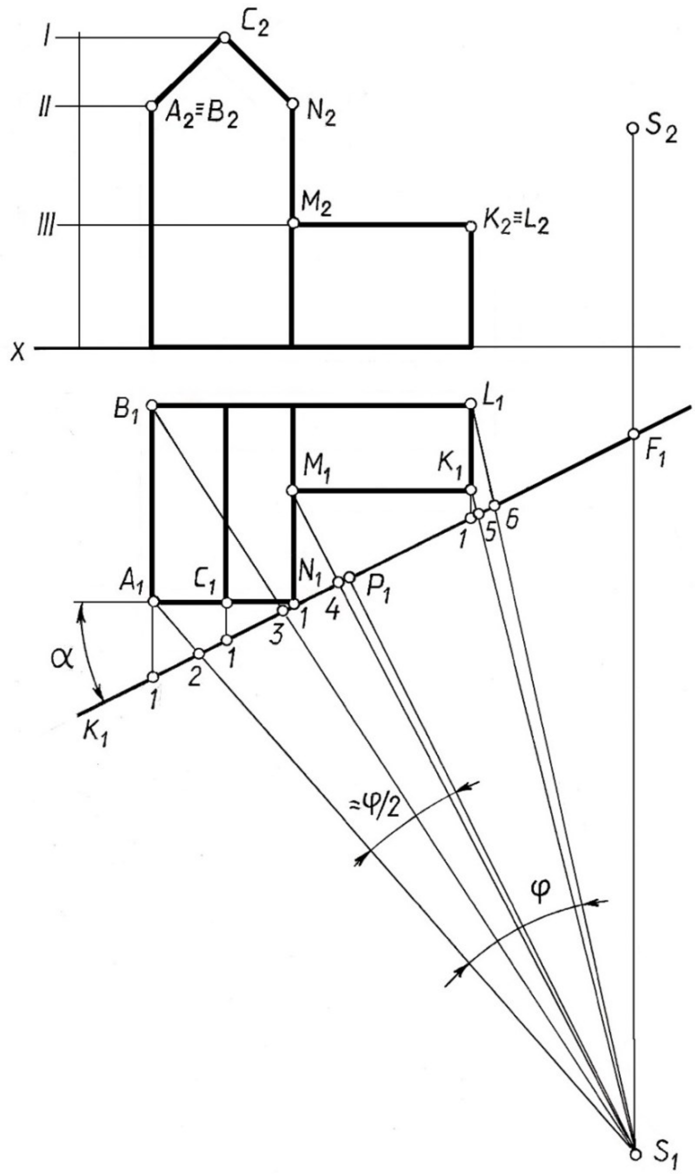


Fig. 1.19

The choice of a point of view includes three main elements that are closely related and set together:

a) the value of the angle of view φ;

b) the distance of the point of view from the object as the position of the main beam S_1P_1 ;

c) the position of the horizon line *hh*.

Let's choose a point of view on a specific example (fig. 1.19), taking into account the following recommendations:

1. The picture is set so that it passes through at least one vertical edge of the geometric shape.

2. The angle of inclination of the picture to the facade α, which should be more reflected in the perspective, is 20°–30°.

3. It is desirable that the main beam coincides with the bisector of the angle of view – the angle enclosed between the extreme points of the object.

4. The angle of view φ is allowed within 18°–53°. The optimal value is $\varphi = 28°$.

The type of the perspective image also depends on the height of the point of view, i.e. the height of the horizon.

The perspective obtained from the point of view, located at the height of human height (about two meters), is called a perspective from a normal horizon.

Sometimes the point of view is placed above the object being depicted, at an altitude of 100 m or higher, then the perspective is called a bird's-eye perspective.

A perspective from the zero horizon is called the perspective when the point of view is located on the subject plane.

If the height of the horizon is small or equal to zero (fig. 1.20), the so-called "omitted plan" is used to build the perspective.

In this case, the secondary projection of the object (plan) is constructed not on the object plane, but on some horizontal plane t_0t_0, shifted from the object plane by an arbitrary distance. In this regard, a new line t_0t_0 appears on the perspective image – the line of the omitted plan.

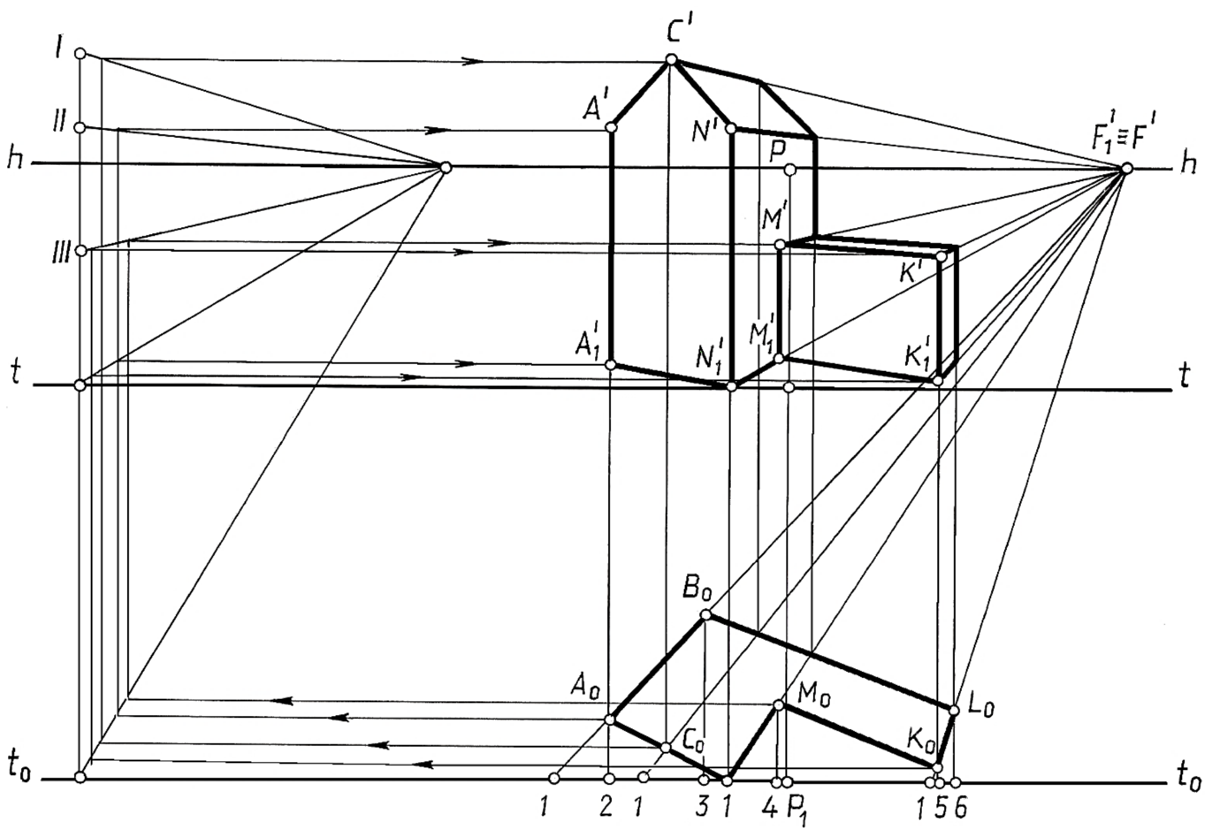


Fig. 1.20

The transition from the omitted plan to the construction of the perspective of the volume is performed using the side wall (the natural values of the height of levels I, II and III are set aside in the picture, and then moved to the original position; in fig. 1.20 the constructions are shown by arrows).

1.12. Plotting the tracks and vanishing points of a straight line from the perspective and secondary projection of the straight line

Example.

Construct the picture trace M, the object trace B, and the vanishing point F of the given line l (fig. 1.21).

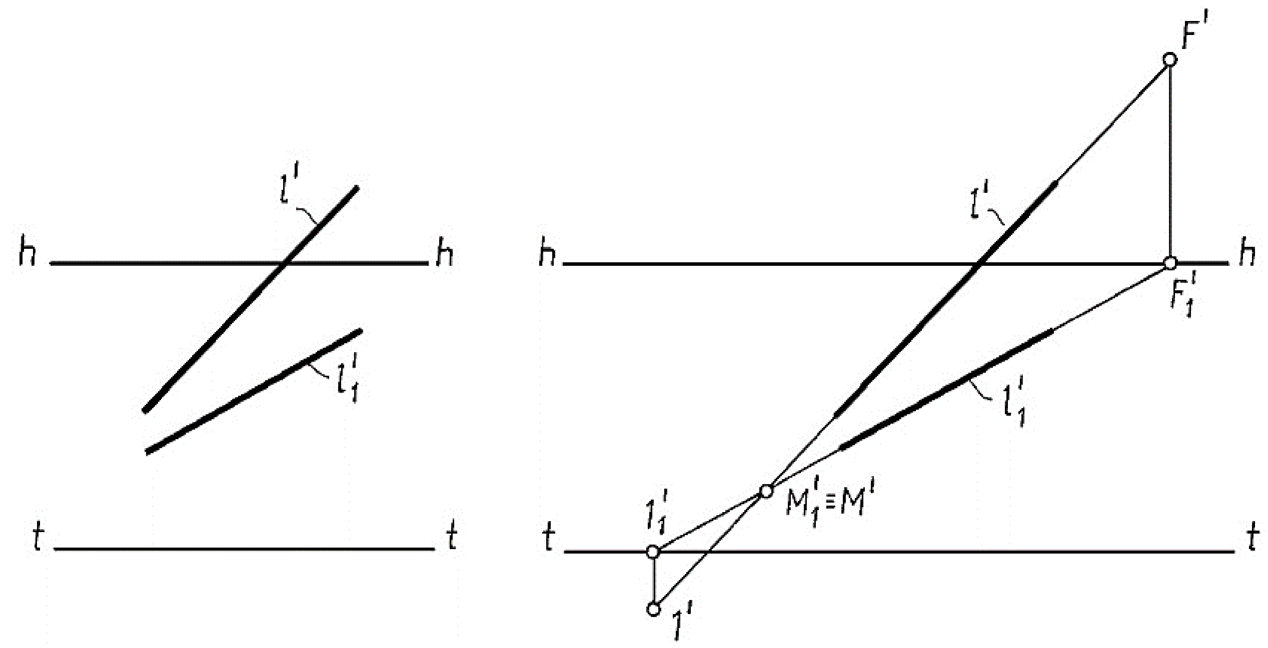


Fig. 1.21

***Solution*:**

1. We build a picture trail – the point of intersection of a straight line with the picture. This is the point 1_1, where the secondary projection $1_1' = l_1' \cap tt$, and the perspective $1'$ belongs to the perspective of the line l'.

2. Mark the object trace of the line – the point of intersection of the line with the object plane: $M' \equiv M_1'$, $M' = l' \cap l_1'$.

3. We build a vanishing point – the perspective of an infinitely distant point of a straight line. This is the point F, where the secondary projection is on the horizon line and belongs to the secondary projection of the line $F_1' = l_1' \cap hh$, and the perspective of the point F' belongs to the perspective of the line l'.

1.13 Dividing segments into equal and proportional parts

The division of segments parallel to the picture ($AB \parallel K$, $MN \parallel K$, $PE \parallel K$) is performed in the same way as in orthogonal projections, since the proportionality of the parts of these segments is preserved.

Example.

Divide the specified segments in a ratio of 1:2 (fig. 1.22).

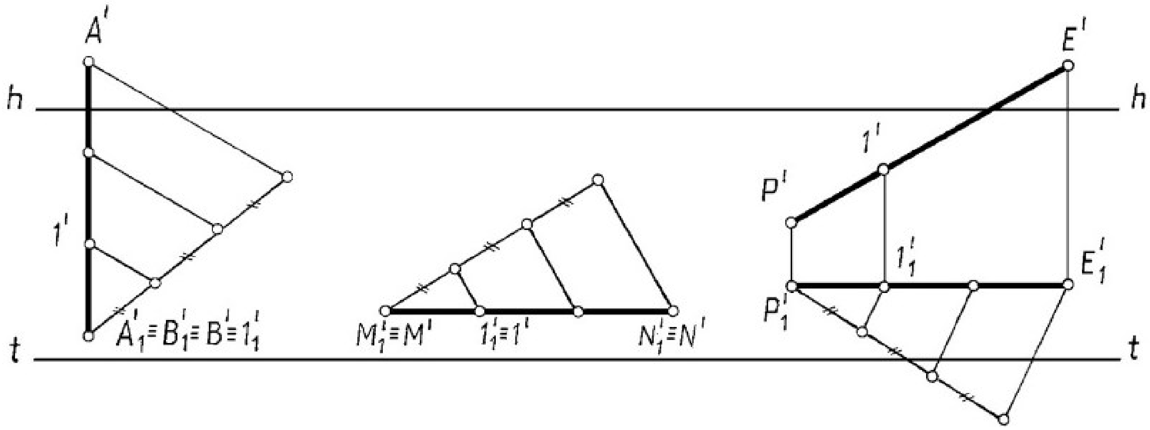


Fig. 1.22

***Solution*:**

For the solution we use Thales theorem. The ray on which we plot the given ratio is drawn at an arbitrary angle (however, this auxiliary ray is parallel to the picture). The completed constructions are clear from the drawing.

The division of segments that are not parallel to the picture is performed using straight lines that belong to the object plane and are parallel to each other, and therefore have a common vanishing point.

Example.

Divide the given segments into three equal parts (fig. 1.23).

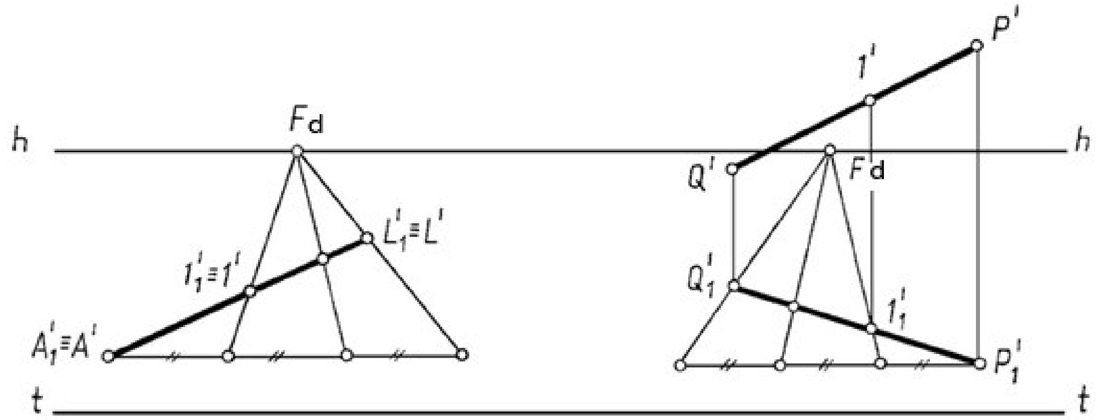


Fig. 1.23

***Solution*:**

The ray on which we lay down three equal segments is drawn parallel to *tt*. This ray is parallel to the picture and belongs to the object plane. Then, connecting the end of the given segment with the last notch on the auxiliary ray, we find the point of intersection of the resulting line with the horizon line (or the point of convergence of the auxiliary lines of division) – we determine F_d. We draw lines through the F_d and the serifs on the auxiliary beam, which divide the secondary projection of the given segments into the required number of parts.

When dividing a line segment *QP* of the general position, the division begins with the secondary projection $Q_1'P_1'$.

2. PROJECTIONS WITH NUMERICAL MARKS

When designing engineering and building structures, one has to resort to the image of the earth's surface. The shape of the surface of the earth and earthen structures is complex, and their vertical dimensions in relation to horizontal ones are very small, for example: roads, bridges, airfields, construction sites, hydraulic facilities, etc. To display them on construction drawings, there is a special method – projection with numerical marks.

2.1. Point projections

The essence of the projection method with numerical elevations is that the points of an object are projected orthogonally onto one horizontal plane. Since one parallel (orthogonal) projection does not determine the position of an object in space, to obtain a reversible drawing, not only the horizontal projection of a point is indicated, but also its distance from the horizontal projection plane, that is the *Z* coordinate, which is called the numerical marks (or simply marks) of this point (fig. 2.1).

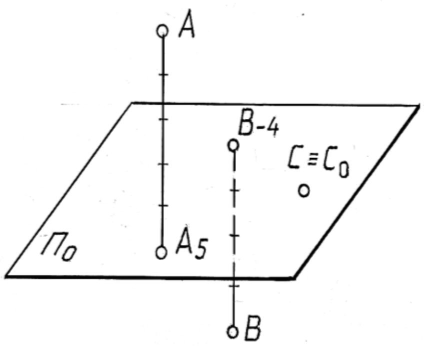


Fig. 2.1

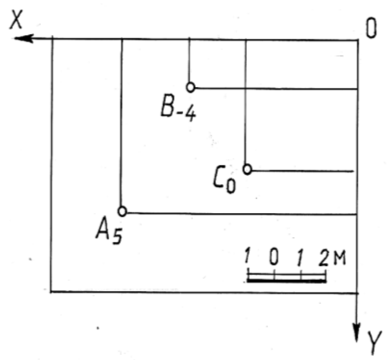


Fig. 2.2

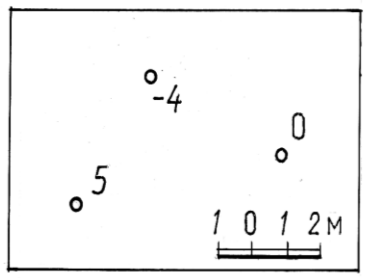


Fig. 2.3

The horizontal projection plane Π_0, onto which geometric objects are projected, is called the main or zero level plane. The position of the projections of points on the plan is determined by the *x* and *y* coordinates, and the numerical marks indicate the value of the *z* coordinate (fig. 2.1). If the points are located above the projection plane, then their elevations are considered positive, if below the projection plane – negative. The elevations of points belonging to the projection plane are called zero (fig. 2.2).

In some cases, when the name of the point does not matter, for simplicity the letter designation of the points is not indicated, but only their numerical marks are left (fig. 2.3).

In drawings made in projections with numerical marks, the coordinate axes, the origin of coordinates and the index of the projection plane are not indicated. Let us agree to call such drawings plans. On the plans, it is necessary to draw a linear scale, which has to be used when solving various metric problems, dimensions are usually indicated in meters (fig. 2.3).

On the territory of the CIS, the level of the Baltic Sea (zero of the Krondstadt Foodstock) is taken as the zero-level plane. When designing engineering structures, any horizontal plane (intermediate level plane) can be taken as the horizontal projection plane, provided that the distance to the Baltic Sea level is known. If the plan is executed on the plane of the zero level (fig. 2.4), then the numerical marks have absolute values (A_7, B_{-2}). If the plan is executed on the projection plane of the intermediate level, then the numerical marks of the points have relative values (A_2, B_{-7}) (distance from the plane of the intermediate level).

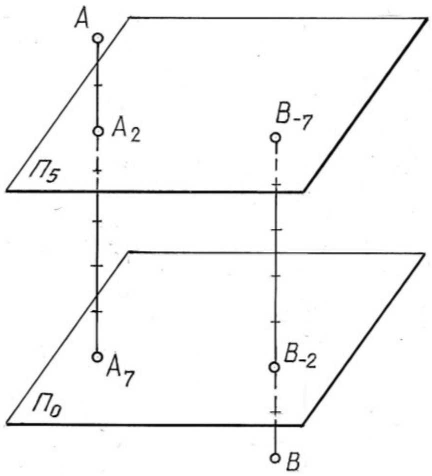


Fig. 2.4

2.2. Line projections

In geometric operations on straight lines the concepts are used: laying a straight line segment, interval and slope of a straight line. Fig. 2.5 shows a line segment *AB* and its projection A_1 $B_{3.5}$ on the plane Π_0. The magnitude of the horizontal projection of the segment is called the inception of the segment and is denoted by the letter *L*. The difference in the elevations of the ends of the segment of a straight line (the vertical distance between the ends of the segment) is called the elevation of the segment and is denoted by the letter *H*.

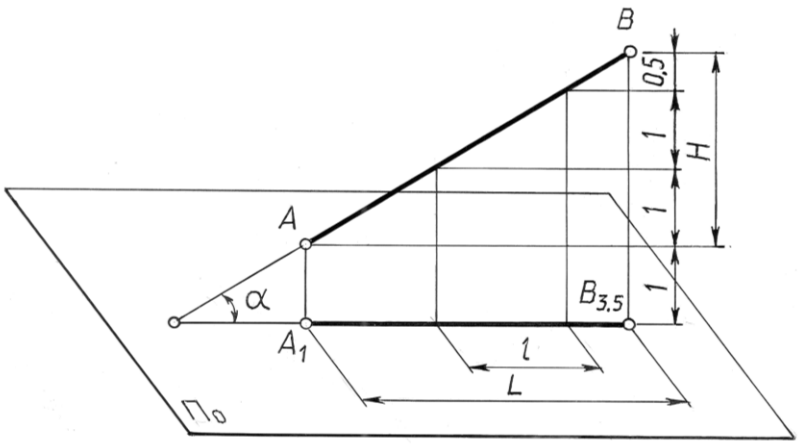


Fig. 2.5

The slope of a straight line is the ratio of the elevation of a straight line segment to its inception. The slope is denoted by the letter *i* and is equal to the tangent of the angle of inclination of the straight line to the plane Π_0 (fig. 2.5).

$$i = \frac{H}{L} = tq\,\alpha\,.$$

The placement of a straight line corresponding to a unit of excess is called an **interval** of a straight line and is denoted by the letter *l* (see fig. 2.5). It is easy to see that the interval of a straight line is the reciprocal of its slope

$$i = \frac{1}{l}\,.$$

In projections with numerical marks, the straight line in general position can be specified:

1) projections of two points of a straight line and their marks (fig. 2.6, *a*);

2) a horizontal projection, marking one of the points of the straight line and the angle of inclination of the straight line to the projection plane (fig. 2.6, *b*);

3) a projection onto the main plane, marking one of its points and the slope of a straight line (fig. 2.6, *c*).

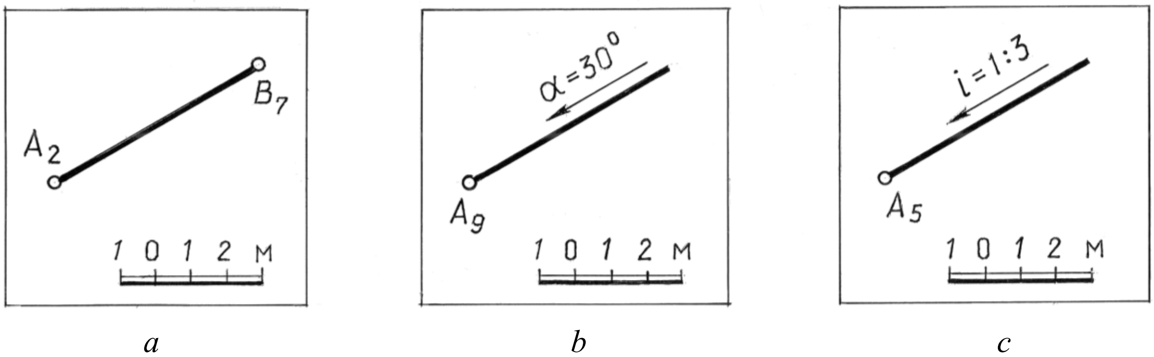


Fig. 2.6

The designation of the angle of inclination or slope of the straight line must be supplemented with an arrow indicating the direction of descent of the straight line. The horizontal line will be denoted by the letter *h* with an indication of the numerical elevation, for example, h_5, or only by elevation *5*. The segment of the vertical line is set by end points with indication of their marks.

2.3. Graduation straight

Graphical actions for determining the interval of a straight line are called grading a straight line. To graduate a straight line means to determine on its horizontal projection points, the difference in elevation of which is equal to one.

There are several ways to graduate a straight line. All of them represent different options for solving the problem of dividing a segment in this respect.

Let's consider the most common ways to solve this problem. The 1st method (fig. 2.7) – using proportional division of the segment. An auxiliary straight line of any direction is drawn through one of the ends of the segment (for example, $A_{2,3}$), and values corresponding to the excess between the end and desired points of the straight segment are plotted on this straight line in an arbitrary scale. The constructed last point on the auxiliary straight line is connected to the second end of the segment, and straight lines are drawn through the division points parallel to the closing straight line. These lines define the required points on a given projection of the segment.

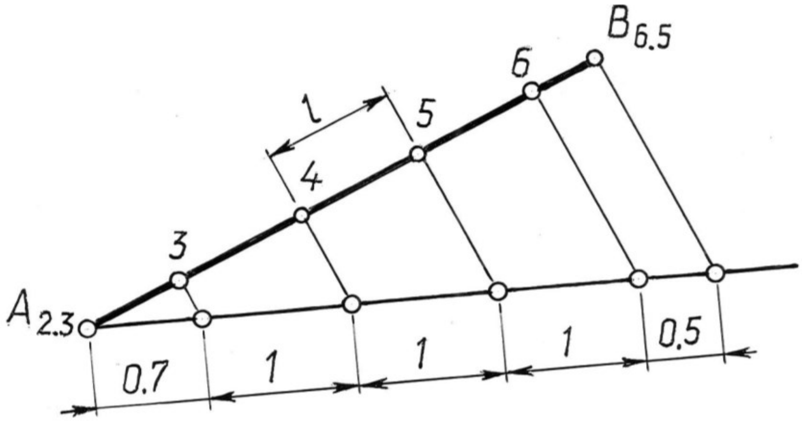


Fig. 2.7

Note that the position between two points, the difference in elevation of which is equal to one, is the interval of a straight line. In fig. 2.7 – the size of the interval is shown between points with marks *4* and *5*.

The second method (fig. 2.8) – the use of an additional horizontally projecting (vertical) plane Π', parallel to a given segment (or passing through it) and then aligned with the projection plane Π_0 by turning around the axis Π'/ Π_0.

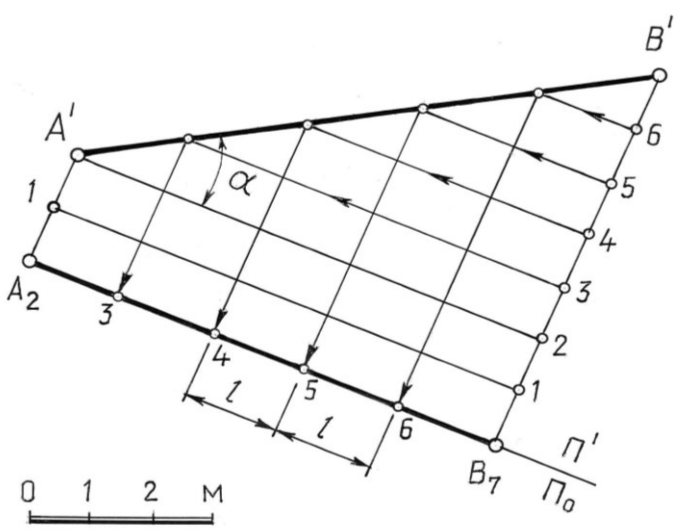


Fig. 2.8

In fig. 2.6, the auxiliary plane Π′ is drawn through a given segment *AB* (A_2B_7), so the axis Π' / Π_0 coincides with the projection of the segment. Having restored the perpendiculars to the projection of the segment (communication line) at the points that are the projections of the ends of the segment, and putting on them segments equal to the heights of these points, we get *A'B'* – the natural value of the segment *AB* and the angle α – the angle of inclination of the straight line to the plane Π_0. Then, using straight lines parallel to the projection of the segment, points with integers are determined on *A'B'* marks. Then the projections of these points are built on the given projection of the segment.

The third method is to graphically define the interval of a straight line using a slope graph of a straight line, called a slope scale.

This method can be used, if the line is specified by a projection, one point with an integer numerical elevation and the slope of the line or the angle of inclination to the main plane is known.

In fig. 2.9 shows the graduation of the straight line, which is set by the horizontal projection, point A_9 and the angle of inclination to the projection plane. The graph of the slope of a straight line is carried out on the scale of the drawing: on one (horizontal) axis, the positions are laid, and on the other (vertical) elevation *H*. From the origin, a straight line is drawn at a given angle α to the axis *L*. Interval *l* of a given straight line.

The found value of the interval *l* is plotted on a given straight line from a given point *A*.

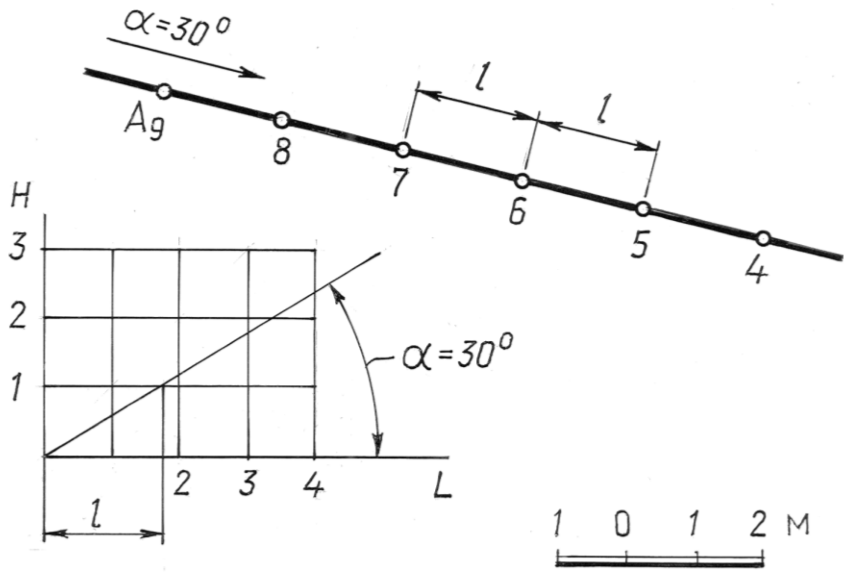


Fig. 2.9

2.4. The relative position of two straight lines

Two straight lines in space can intersect, intersect and be parallel. However, the absence of a second projection does not make it possible to determine the relative position of straight lines directly from the drawing, without preliminary auxiliary constructions. So the relative position of straight lines can be determined by graduating straight lines and comparing intervals, slopes and elevations of points of intersection of projections of straight lines. Let us note signs characteristic of various cases of the arrangement of straight lines.

Parallel straight lines – projections of straight lines are parallel, slopes (or intervals) are equal, and numerical marks increase (or decrease) in one direction (fig. 2.10). In this case, lines connecting points with the same marks are parallel. They are contours of the plane passing through the given lines.

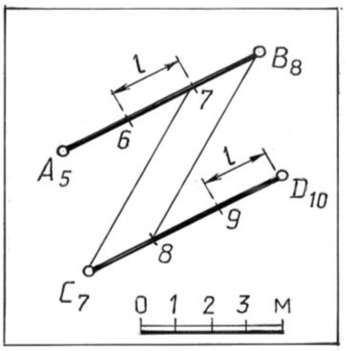


Fig. 2.10

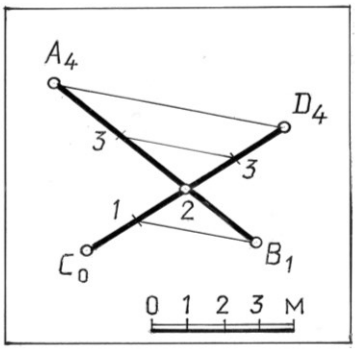


Fig. 2.11

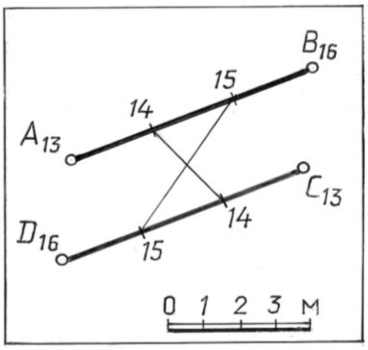


Fig. 2.12

Intersecting straight lines – projections of straight lines intersect at a point that, being referred to each of the intersecting straight lines, has the same mark (fig. 2.11). It is easy to check if lines are graduated. Note that lines connecting points with the same elevation are parallel. They are contours of the plane passing through the given intersecting lines.

Crossed straight lines – straight lines that have no signs of intersection and parallelism (fig. 2.12). In this case, the lines connecting points with the same elevation are not parallel.

Example.

Draw a horizontal line through point *A* (A_3), intersecting the given straight line *CD* (C_1D_7) (fig. 2.13).

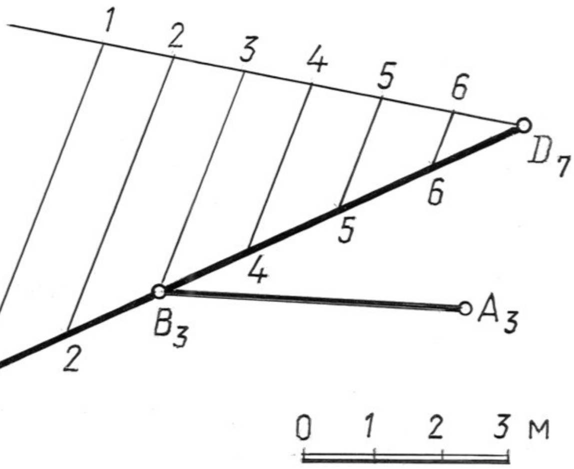


Fig. 2.13

***Solution*:**

The desired horizontal line is determined by point A (A_3) and point B (B_3) on the line CD, which has the same elevation. Graduate the straight CD, using proportional division of the segment. We connect the constructed projection B_3 with projection A_3. Straight line AB (A_3B_3) is the desired one.

2.5. Plane

A plane in projections with numerical elevations can be specified by projections with numerical elevations of the following geometric elements: three points that do not lie on one straight line (fig. 2.14, *a*); straight line and points outside this straight line (fig. 2.14, *b*); parallel straight lines (fig. 2.10); intersecting lines (fig. 2.11); a flat figure (fig. 2.14, *b*). But the most convenient and visual representation of the plane in projections with numerical elevations is the setting using the scale of the slope of the plane.

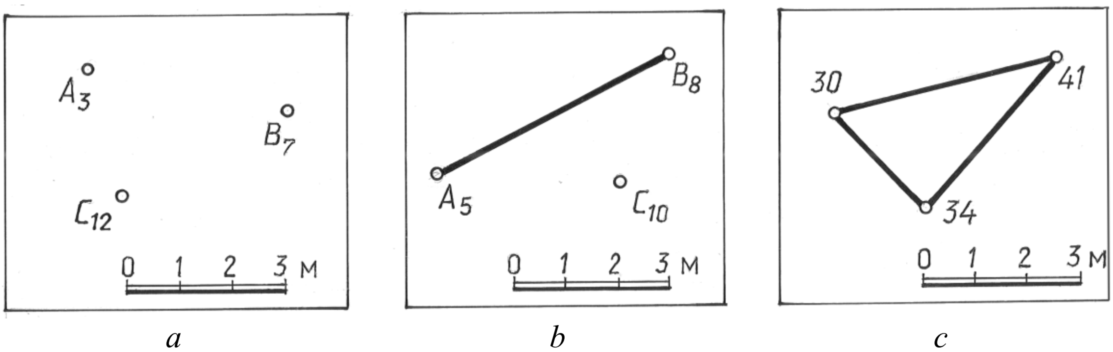


Fig. 2.14

The scale of the slope of the plane, or the scale of the dip, is a graduated projection of the line of the greatest slope of the plane. In fig. 2.15 is a visual representation of the plane Γ in general position. Let's give definitions of the basic elements of this plane, which are used in projections with numerical elevations.

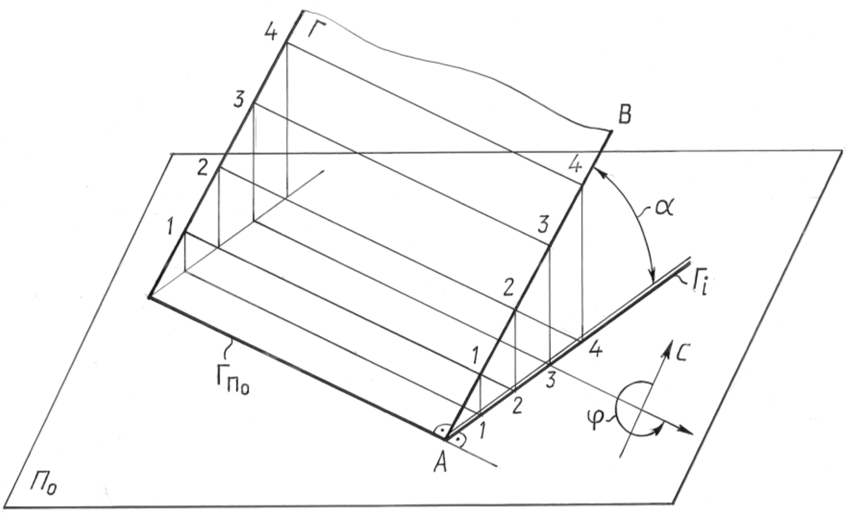


Fig. 2.15

Horizontal elevation – the height of the horizontal above the projection plane (in fig. 2.15, the horizontal lines are drawn respectively with marks *1*, *2*, *3* scale units). The trace of the $\Gamma_{\Pi 0}$ plane is a horizontal line with a zero mark.

The line of the greatest slope of the plane is otherwise called the line of fall *AB* (fig. 2.15) – a straight line belonging to the plane and perpendicular to its horizontals ($AB \perp \Gamma_{\Pi 0}$). It determines the angle of inclination and the angle of incidence of the plane. Since the line of the greatest slope is perpendicular to the contours, the scale of the slope of the plane (the projection of the line of the greatest slope) is also perpendicular to the projections of the contours (the theorem on orthogonal projection of a right angle).

The image of the Γ plane with the scale of the slope of the plane is shown in fig. 2.16. The scale of the slope of the plane is depicted by two parallel straight lines (thick and thin) and is denoted by the same letter as the plane, with a subscript *i*–Γ*i*.

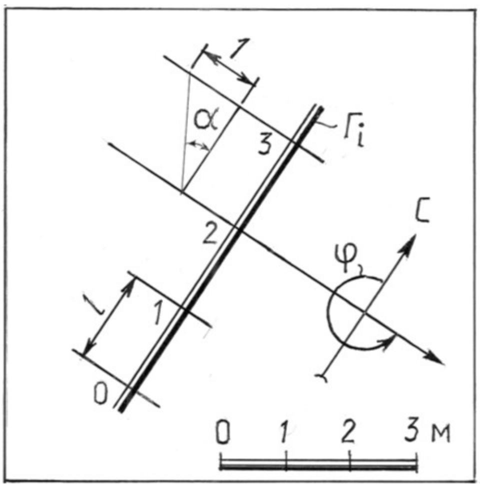


Fig. 2.16

Projections of contour lines are drawn perpendicular to the scale of the slope of the plane. The elevations of these contour lines are indicated along the scale of the slope of the plane (from the side of the thin line). The numbers of numerical marks are put down so that their top is oriented towards the rise of the plane.

The distances between adjacent divisions of the slope scale *l*, corresponding to the elevation unit, are the interval of the line of the greatest slope, and, therefore, the interval of the plane.

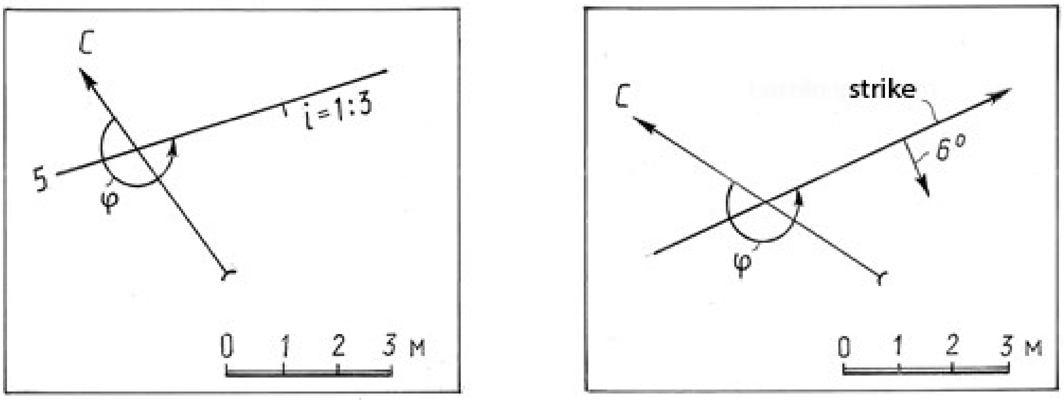
The angle of incidence of the plane α0 is the angle of inclination of the plane to the plane of projections (the angle of inclination of the line of the greatest slope to the plane of projections). In the drawing, in projections with numerical marks, the angle of incidence α is determined from a right-angled triangle, in which one leg is equal to the interval the line of the greatest slope, and the second leg is equal to the unit of height on the scale of the drawing (fig. 2.16).

The slope of the plane is the tangent of the angle of incidence of the plane. The slope of the plane is equal to the slope of the line of the greatest slope. The slope of the plane is the reciprocal of the plane interval. To solve engineering problems on the earth's surface, it is necessary to orient a given plane relative to the Earth's meridian. For this, the concepts are introduced:

– the direction of the plane's strike – the right direction of its contours, if you look at the plane in the direction of increasing marks;

– the strike angle of the plane is the angle between the meridian of the earth and the direction of strike (fig. 2.15, 2.16). The strike angle is measured from the north end of the meridian counterclockwise to the strike direction of the plane.

The plane is defined by the horizontal line *5*, the slope i = 1:3 and the direction of descent, which is indicated by a stroke in the direction of descent. Such a stroke is called a bergstrich (fig. 2.17, *a*).



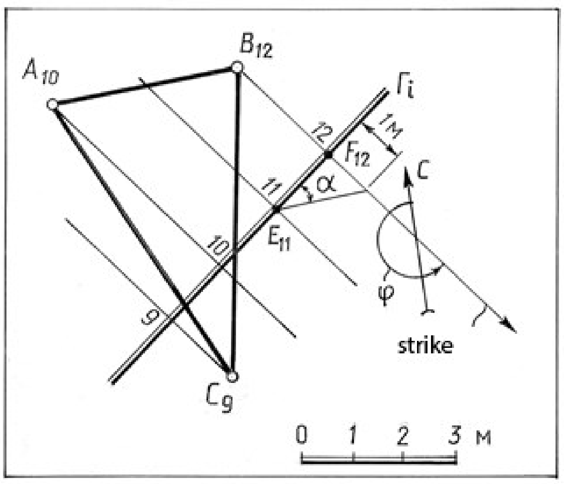
a *b*

Fig. 2.17

The plane can be specified by the angle of incidence and the direction of strike (fig. 2.17, *b*). This method of defining a plane is used in topography, geology, etc. For solving most metric and positional problems, it is convenient when the plane is specified by contours. Conducting contour lines on a plane is called plane graduation.

Example.

Determine the angles of incidence α and strike φ of the plane Γ, given by the triangle *ABC* (fig. 2.18).



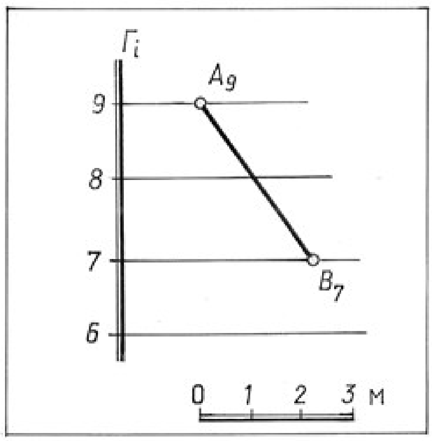


Fig. 2.18 Fig. 2.19

***Solution*:**

Having graduated the segments *AB* and *CD*, we connect the points with the same marks with straight lines. These will be the contours of the specified plane. The scale of the fall of the plane is drawn perpendicular to the contours. Using a right-angled triangle, one leg of which is segment $E_{11}F_{12}$, and another segment equal to the unit of height, we determine the angle of inclination of the line of the greatest slope of the plane Γ to Π_0. Then, having established the direction of the strike, we construct the strike angle φ. Problems on the mutual belonging of a point and a straight line of a plane in projections with numerical elevations are solved by conventional methods.

A straight line in a plane is built on two points, the elevations of which are determined at the intersection of the projection of the straight line with the horizontal lines of the plane (fig. 2.19).

A point in a plane is drawn using an arbitrary straight plane. To determine the elevation of the point, the auxiliary line is graduated.

When designing engineering structures in projections with numerical marks, it is very often necessary to solve two types of problems: drawing a straight line in the plane with a given slope *i*; carrying a plane through a straight line with a given slope *i*.

Example.

In the plane specified by the scale of the slope Γ_i, through the point A_8 draw a straight line with a slope i = 1:3 (fig. 2.20).

***Solution*:**

The interval of the straight line to be plotted is $l = 1 / i = 3$ scale units. Consequently, the point of the desired straight line, which has a mark of *7*, must lie on the horizontal plane with a mark of *7* and is removed from the point A_8 by the value of the line interval l = 3 units. Draw a circle with radius R = 3 through point A_8 and find its intersection points with the 7th horizontal of plane Γ. Point A_8 and the resulting points B_7 and C_7 define two straight lines that satisfy the condition of the problem.

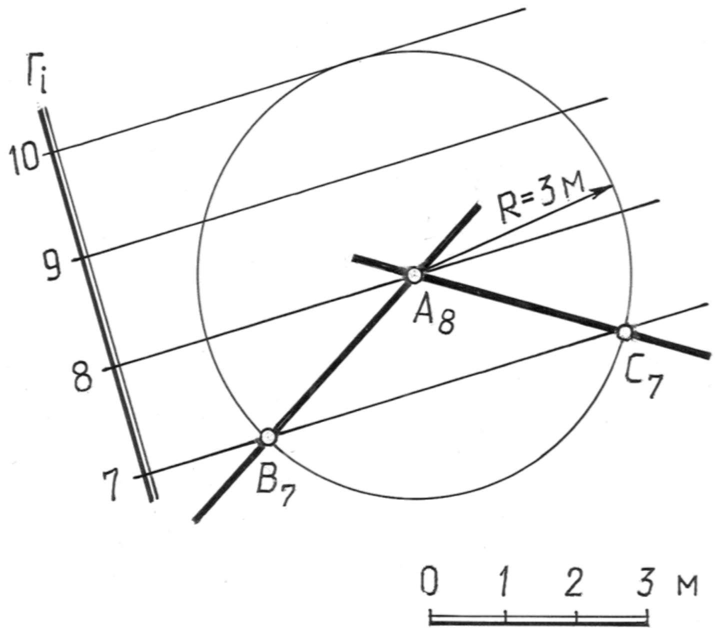


Fig. 2.20

Example.

Through the inclined line AB (A_2B_5) draw the plane Γ ($Γ_i$), the slope of which is i = 1:2 (fig. 2.21).

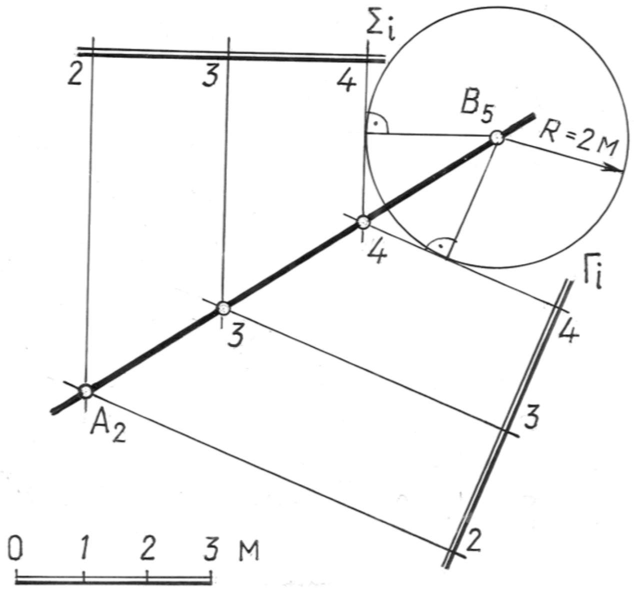


Fig. 2.21

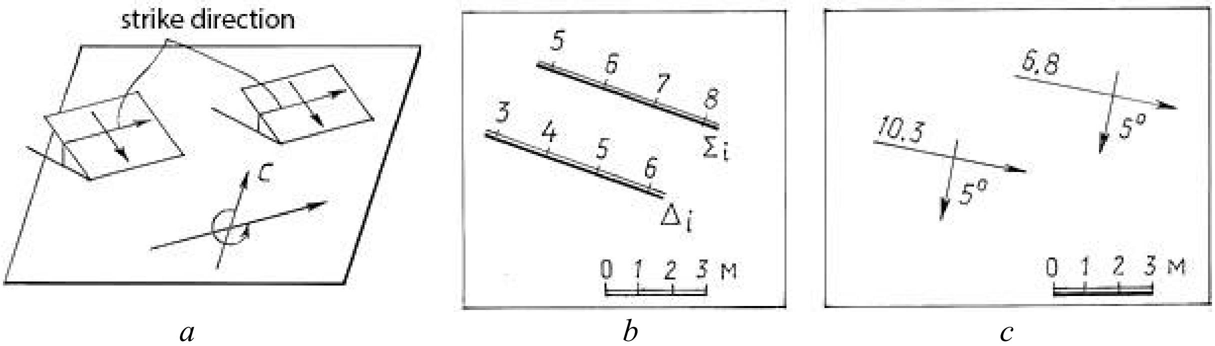
***Solution*:**

The sought plane Γ is tangent to the surface of a straight circular cone, whose generators have a slope equal to the slope of the plane. The horizontals of a cone – circle whose radii differ by the value of the plane interval. Drawings are drawn in the following order:

1) from an arbitrary point of a straight line with an integer mark (in the figure 2.21 point B (B_5) is used), a circle is drawn with a radius equal to the value of the plane interval R = 2 (the horizontal of the cone, the height of which is equal to one);

2) from the nearest point of division of the straight line C (C_4), a tangent is drawn to the constructed circle. This tangent is the horizontal line with the mark *4* of the desired plane.

Parallel planes. A necessary and sufficient condition for the parallelism of two planes is the parallelism of their lines of the greatest slope (fig. 2.22).



a *b* *c*

Fig. 2.22

In the drawing in projections with numerical marks (fig. 2.22), the scales of the slopes of the parallel planes should be parallel, have equal intervals, and the marks should increase in the same direction. A sign of the parallelism of the planes is also the equality of their strike angles and slopes (angles of incidence).

2.6. Surface projections

In projections with numerical marks, the shape of any surfaces is sufficiently fully characterized by their contours. Surface contours are the lines of intersection of this surface with horizontal planes. Thus, in projections with numerical elevations, surfaces are specified by a linear armature. Wireframe lines are surface contours with integer and fractional numeric elevations.

Polyhedrons in projections with numerical marks are depicted by the projections of the vertices, indicating their marks, or by the projection and mark of one of the faces and the slopes of other faces (fig. 2.23).

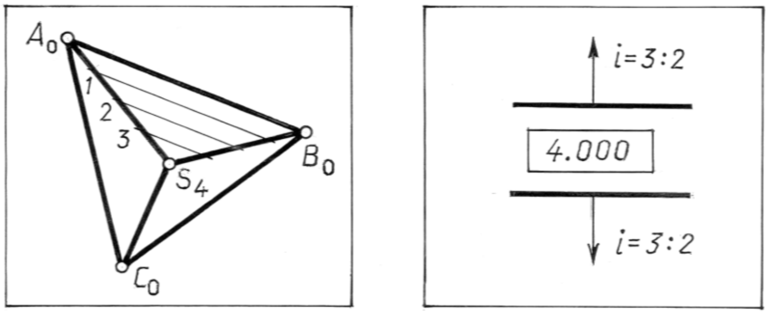


Fig. 2.23

Conical surface. A straight cone, as a surface of equal slope, is depicted by the projection of its vertex *S* with an indication of the elevation and contours (circles) (figure 2.24, *a*). The graded projection of any generatrix of such a cone is the scale of the slope of the surface and its line of the greatest slope. In fig. 2.24, *b* shows the setting by the contours of an inclined elliptical cone with circular horizontal sections.

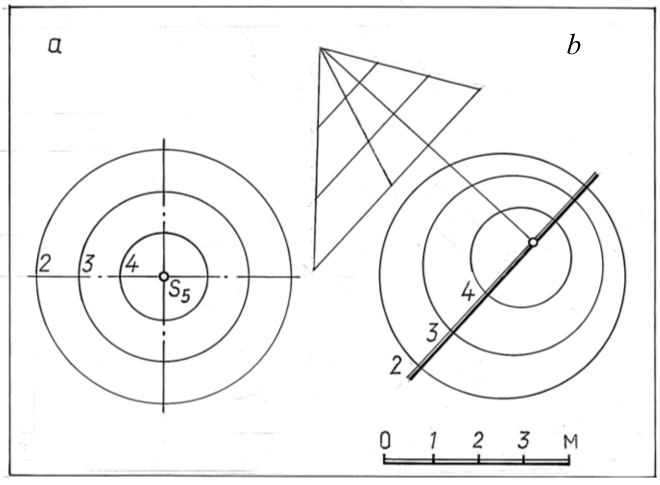


Fig. 2.24

A surface of equal slope (fig. 2.25, 2.26) is a ruled surface, all generators of which make a constant angle with the horizontal plane. Such a surface can be formed if a straight circular cone with a vertical axis and generators of a given slope is moved along a certain guide, leaving the axis of the cone vertical. The surfaces of slopes of embankments and notches on curved sections of roads are surfaces of the same slope.

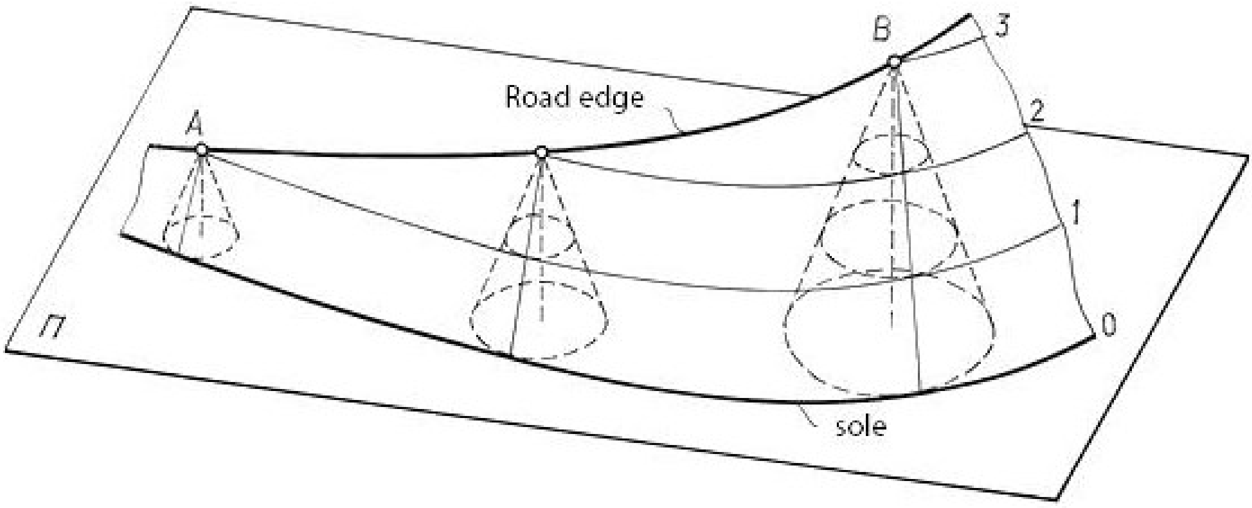


Fig. 2.25

In fig. 2.26 the construction of contour lines of the surface of equal slope is shown. Here, each surface contour is an envelope of a family of contour lines of cones. Moreover, all contours of this family have the same elevation. So, in fig. 2.26 the contour of the surface with elevation *1* goes around the contour family of the cone with the same elevation.

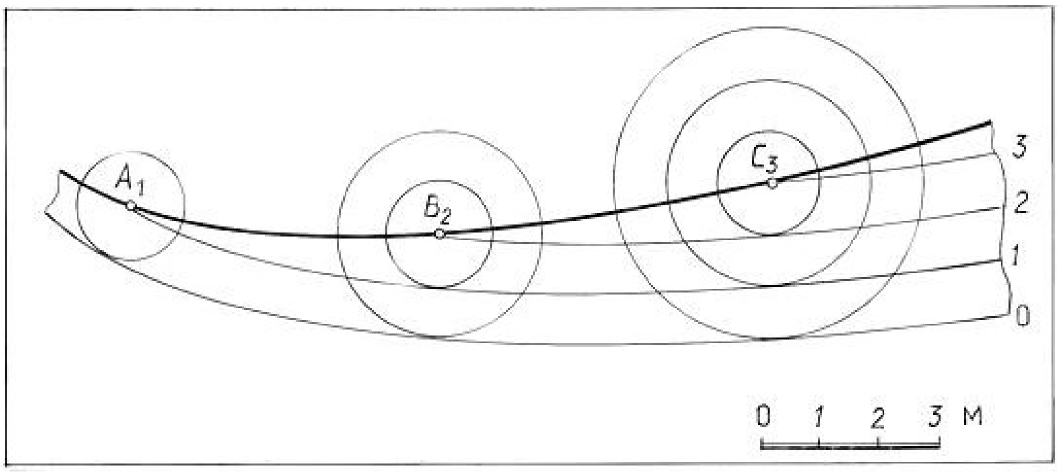


Fig. 2.26

The surface of a piece of land is an example of a so-called topographic surface, the formation of which is not subject to any geometric law. The topographic surface is set on the plan by contours, which are obtained as a result of the intersection of the surface by horizontal planes (fig. 2.27). The distances between the secant horizontal planes are selected depending on the terrain and on the scale of the drawing. They are usually multiples of one or five meters. With a poorly expressed terrain, when the horizontal lines do not sufficiently characterize the unevenness of the earth's surface, intermediate horizontal lines are drawn.

They are drawn on the plans with a dashed line. The direction of descent is indicated by a berg stroke – a short line, which is drawn perpendicular to the horizontal and directed from it in the direction of the descent.

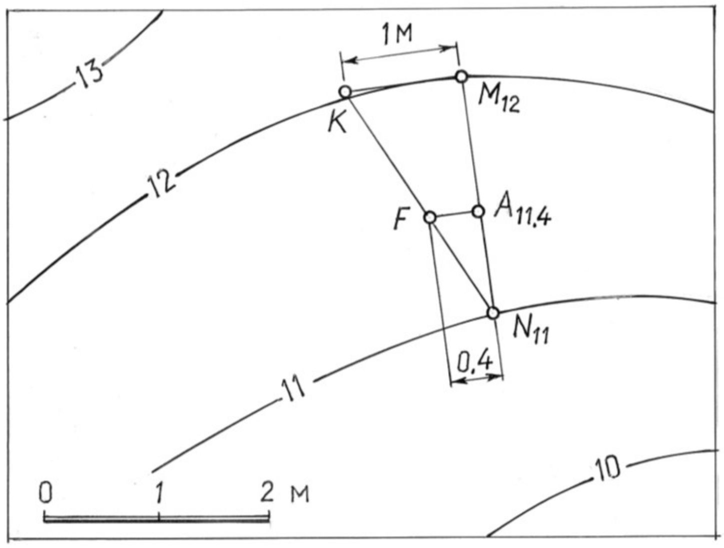


Fig. 2.27

When solving problems on a topographic surface, it is assumed that the straight line connecting two points of adjacent contours belongs to the surface.

The construction of a point on a topographic surface is reduced to finding its elevation. In fig. 2.27, the elevation of point *A*, belonging to the topographic surface and located between the contours *11* and *12*, is defined as follows: a segment *MN* is drawn through point *A*, connecting the points of two adjacent contours, then a right-angled triangle *NMK* is constructed, the leg *KM* of which is 1 m in the drawing scale. Point *A* divides the segment *MN* into two parts, proportional to the excess.

2.7. Creation of intersection of geometric shapes in projections with numerical marks

Since each of the surfaces (including the plane) is depicted using a family of contour lines, the intersection line of surfaces (planes) can be constructed as a set of intersection points with the same elevations.

Let's consider examples of constructing lines of intersection of various geometric shapes in projections with numerical marks.

Example.

Construct the line of intersection of the planes Γ and Δ, given by the scales of the slopes (fig. 2.28).

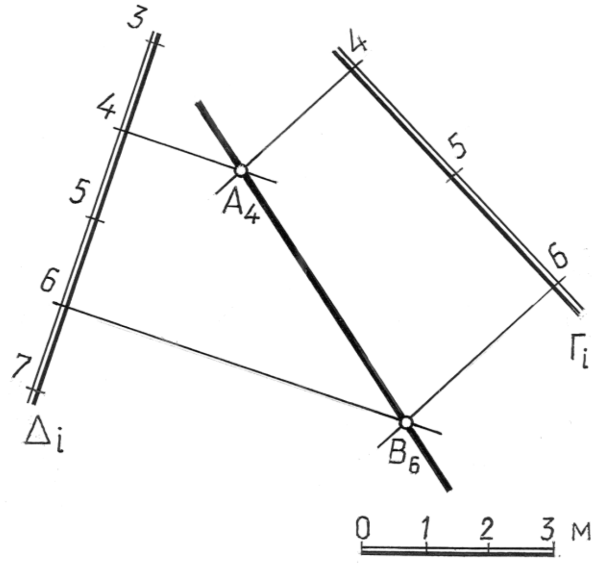


Fig. 2.28

***Solution*:**

Since the line of intersection of the planes is a straight line, to construct it, it is enough to find the intersection points of two pairs of contour lines of the same height, for example, contours *5* and *7*. Points A_5 and B_7 define line *AB*, which is the line of intersection of the given planes.

Example.

Construct the line of intersection of the planes Г and Δ, given by the scales of the slopes, provided that the horizontals of these planes are parallel (fig. 2.29).

***Solution*:**

The horizontals of the given planes are parallel, but the planes themselves are not parallel, since the intervals are not equal and, therefore, the angles of inclination to the projection plane.

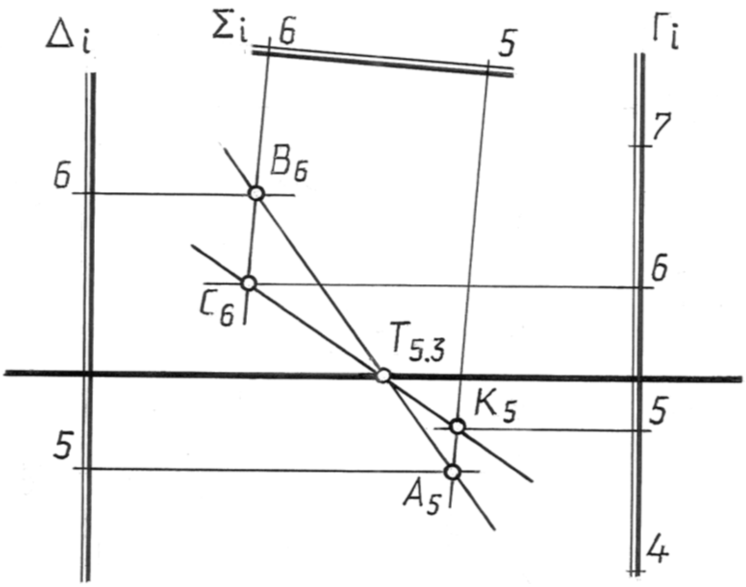


Fig. 2.29

The contours of the given planes are parallel, therefore, their horizontal traces, which are zero contour lines, are parallel. The line of intersection of these planes will be parallel to this plane.

To determine the point, through which the sought line of intersection of the given planes will pass, an auxiliary plane ∑ is drawn. This plane is specified by an arbitrary scale of the slopes Σ_i. Then the lines of intersection of the given planes with the construction plane ∑ are constructed. The horizontals of these planes intersect, so it is not difficult to construct their intersection lines:

– A_5B_6 – lines of intersection of planes Δ and ∑;

– C_6K5 – the line of intersection of the planes Γ and ∑.

The point $T_{5.3}$ of the intersection of lines A_5B_6 and C_6K_5 belongs to all three planes, and therefore, the line of intersection of the given planes.

The problem is solved in a similar way if the contours of the given planes are not parallel, but intersect outside the drawing. Since in this case the direction of the intersection line is unknown, two auxiliary planes are entered and two points are determined, belonging to the desired line of intersection of the planes.

Example.

Construct a line of intersection of the topographic surface with a horizontally projecting (vertical) plane *A* (fig. 2.30).

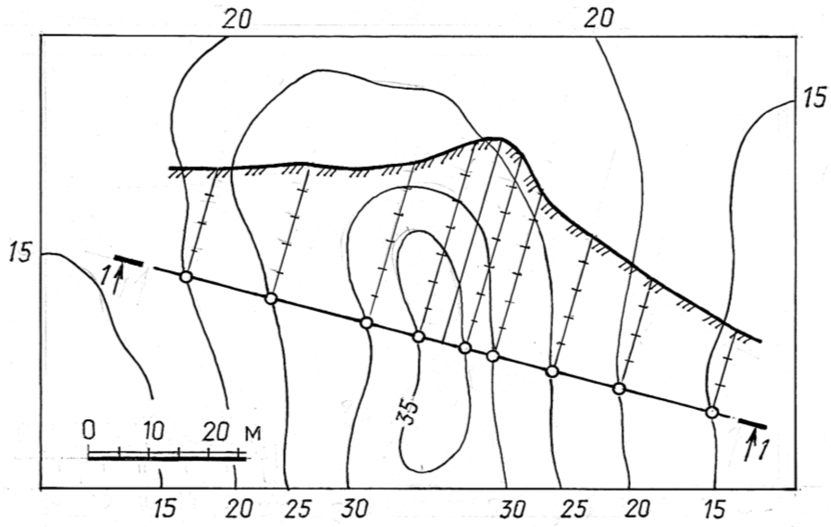


Fig. 2.30

The section of a topographic surface by a vertical plane is called a surface profile. The profile can be displayed in a free space of the drawing (taken out profile) or combined with the drawing of the topographic surface (superimposed profile).

***Solution*:**

To build the superimposed profile (fig. 2.30), the points of intersection of the projection of a given plane (lines *1–1*) with the contours of the topographic surface are determined, then perpendiculars to line *1–1* are drawn from these points, on which, in the drawing scale, the excess of the intersection points above the selected level line – the base of the profile are plotted. The smooth line connecting the constructed points is the profile of the topographic surface.

In fig. 2.31 shows the construction of the outlined profile of the same topographic surface. To build the taken out profile, a line is drawn – the base of the profile and a vertical line that sets the vertical scale. To the base of the profile from the plan (fig. 2.31), the plots are transferred, defining the points of intersection of the contour lines of the topographic surface with the given plane. From the obtained points, the perpendiculars to the base of the profile are restored to the intersection with the horizontal lines having the same numeric marks. The points obtained in this way are connected by a smooth line that forms a section profile.

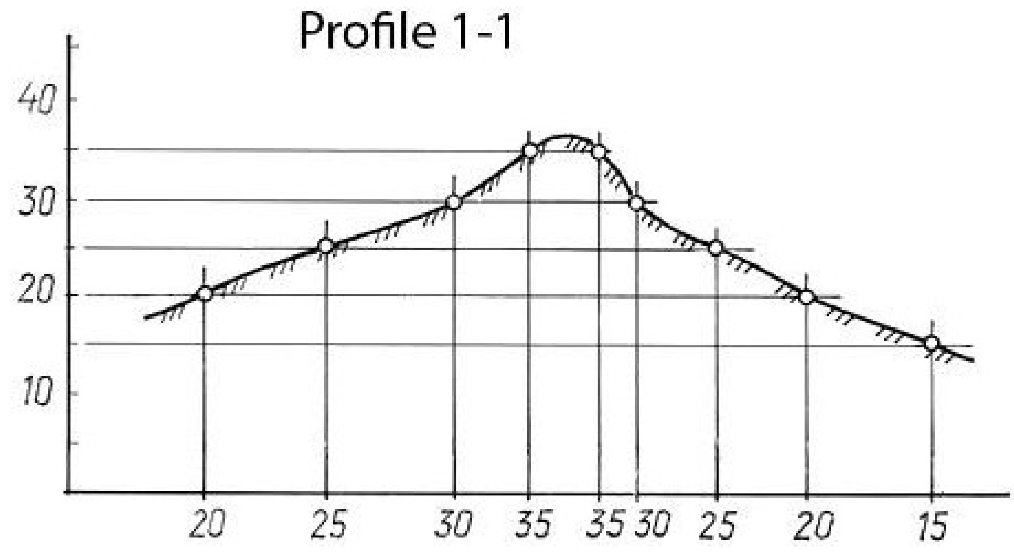


Fig. 2.31

Example.

Construct a line of intersection of the topographic surface with an inclined plane (fig. 2.32).

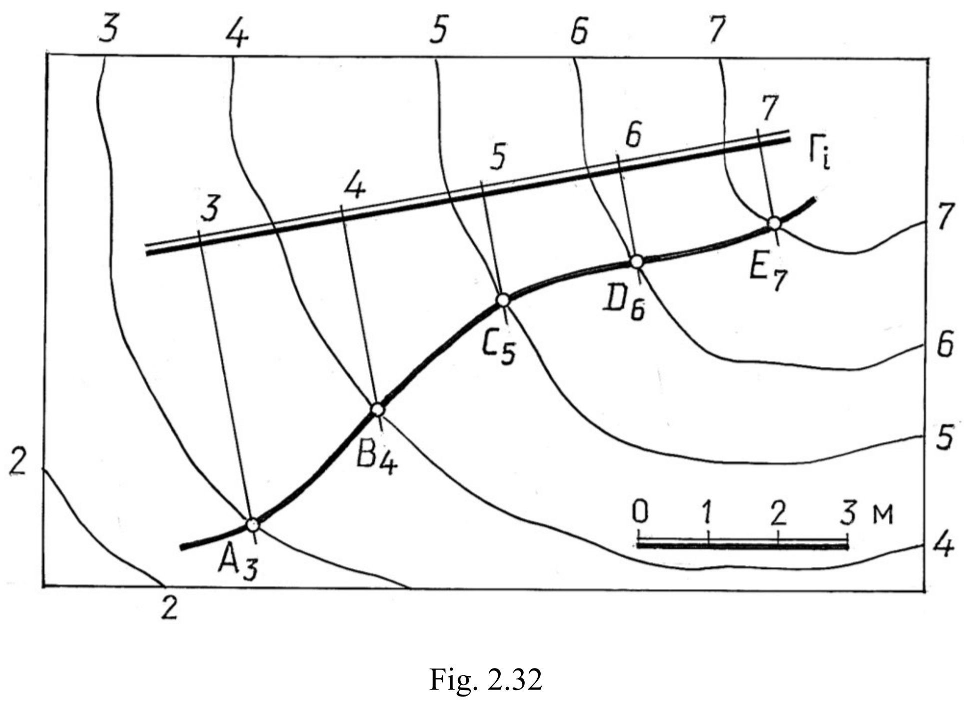


Fig. 2.32

***Solution*:**

The line of intersection of the topographic surface by the plane passes through the points of intersection of their contours with the same elevations. Connecting the constructed points with a smooth line, we get the required intersection line.

The completed constructions are clear from the drawing.

If the contours of the topographic surface and the plane within the drawing do not intersect (or do not intersect at all), you can apply the well-known method of auxiliary section planes (figure 2.29). Fig. 2.33 shows the construction of lines of intersection of the topographic surface with the plane Γ, the contours of which do not intersect. For this, two auxiliary planes Σ and Δ are drawn. Plane Σ intersects the topographic surface along line A_{24} B_{23} (the arc of the intersection line is replaced by a straight line segment for simplicity). The same plane intersects the given plane along the straight line C_{23} K_{24}. The point $T_{23.5}$ the intersection of straight lines $A_{24}B_{23}$ and $C_{23}K_{24}$ belongs to the line of intersection of the topographic surface and the plane Γ. Similarly, the point of the desired intersection line is constructed – point $M_{23.4}$. Depending on the required accuracy, you can plot any number of points belonging to the intersection line.

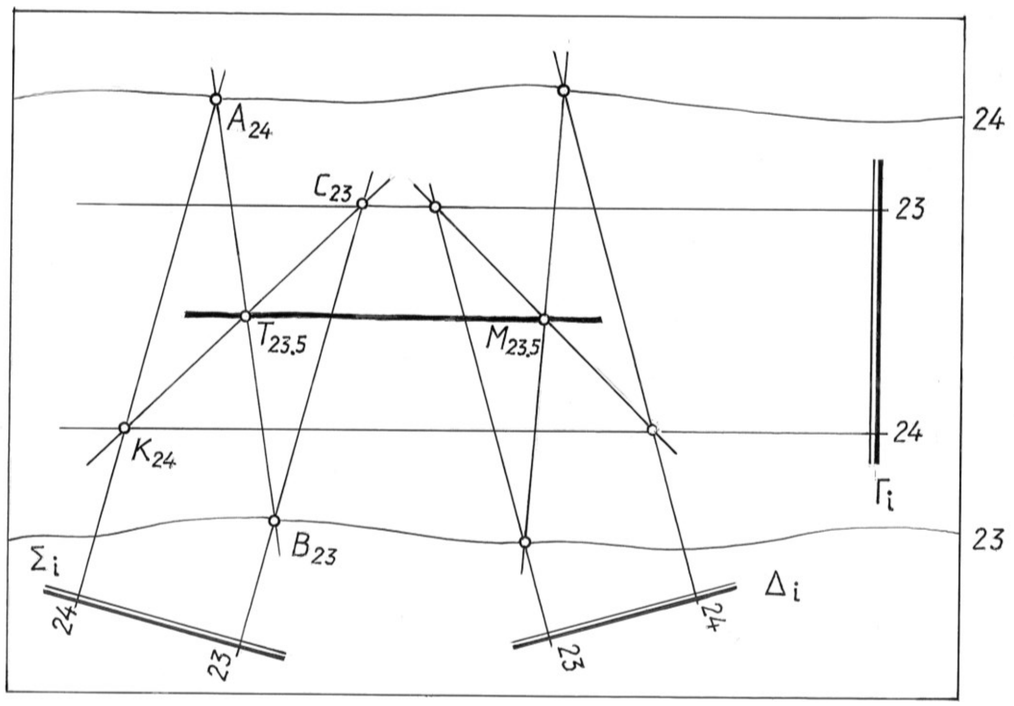


Fig. 2.33

In fig. 2.34 shows the solution to the same problem using auxiliary secant vertical planes (the method of profiles). The given topographic surface and plane Γ are intersected by two auxiliary horizontally projecting planes Σ and Δ, and section profiles are constructed by these planes.

When constructing the profile, the vertical scales are chosen arbitrarily and, to simplify, the arcs of the lines along which the auxiliary planes intersect the topographic surface are replaced by line segments. All constructions are clear from the drawing.

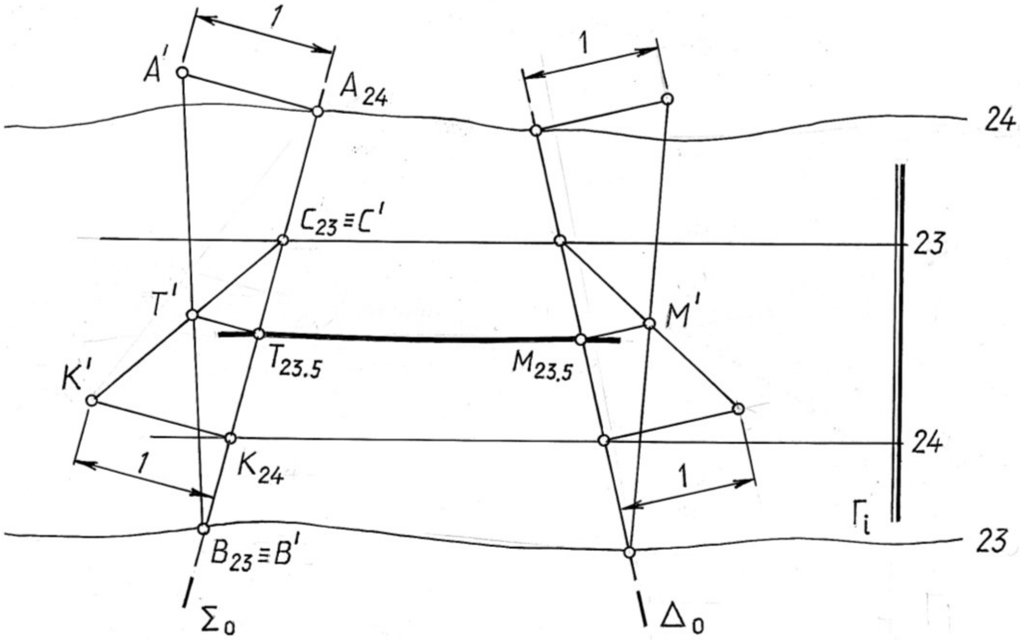


Fig. 2.34

Example.

Construct a line of intersection of the topographic and conical surfaces (fig. 2.35).

***Solution*:**

The construction of the line of intersection of surfaces is reduced to finding the points of intersection of their contours, which have the same elevation.

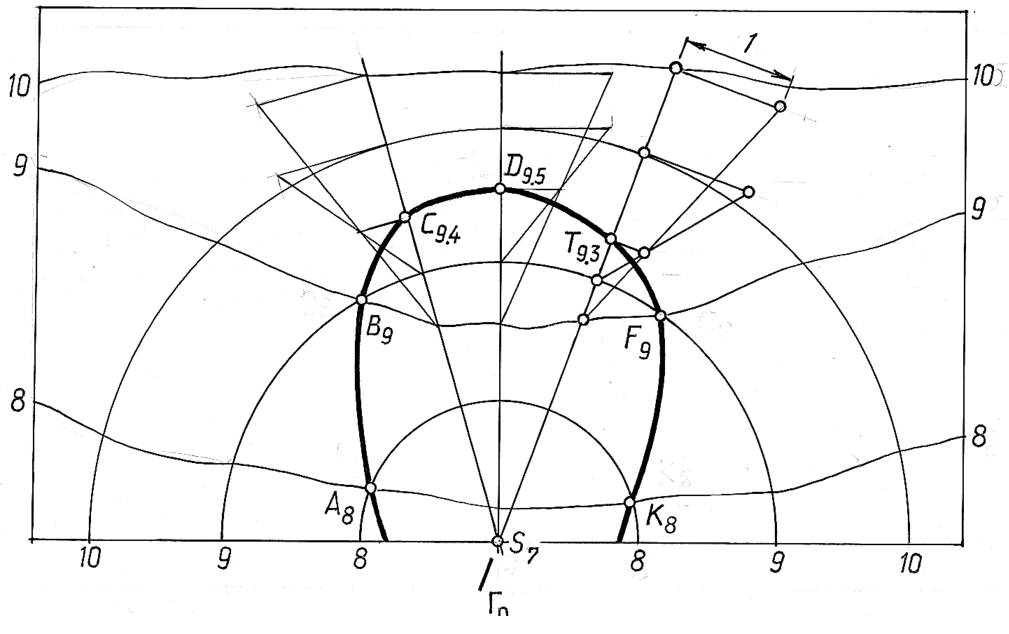


Fig. 2.35

To determine the point *T*, belonging to the intersection line and located between the horizontals *9* and *10*, an auxiliary vertical plane Г was used. The profiles of the section of the topographic and conical surfaces by the plane Г were constructed (conventionally, the curves are replaced by line segments).

The construction of points of intersection of a straight line with a plane or surface in projections with numerical elevations is similar to solving the same problem in other projection methods, namely:

1) the straight line is enclosed in an auxiliary cutting plane-mediator;

2) the line of intersection of the mediator's plane with the given plane (surface) is built;

3) the point (points) of intersection of the constructed line with the given straight line is marked.

Let's consider solving problems using examples. Note that a vertical plane (the method of profiles) or a plane of general position (the method of contours) can be used as an intermediary.

Example.

Determine the point of intersection of the straight line *AB* with the plane Γ (fig. 2.36).

***Solution*:**

To solve the problem, a horizontally projecting plane Δ is drawn through a given straight line, intersecting the given plane along a straight line *MN*. By replacing the projection planes, an additional projection of the straight line *AB* and the line of intersection of two planes are built. First, an additional projection *K*' of the desired intersection point is determined, and then a horizontal projection.

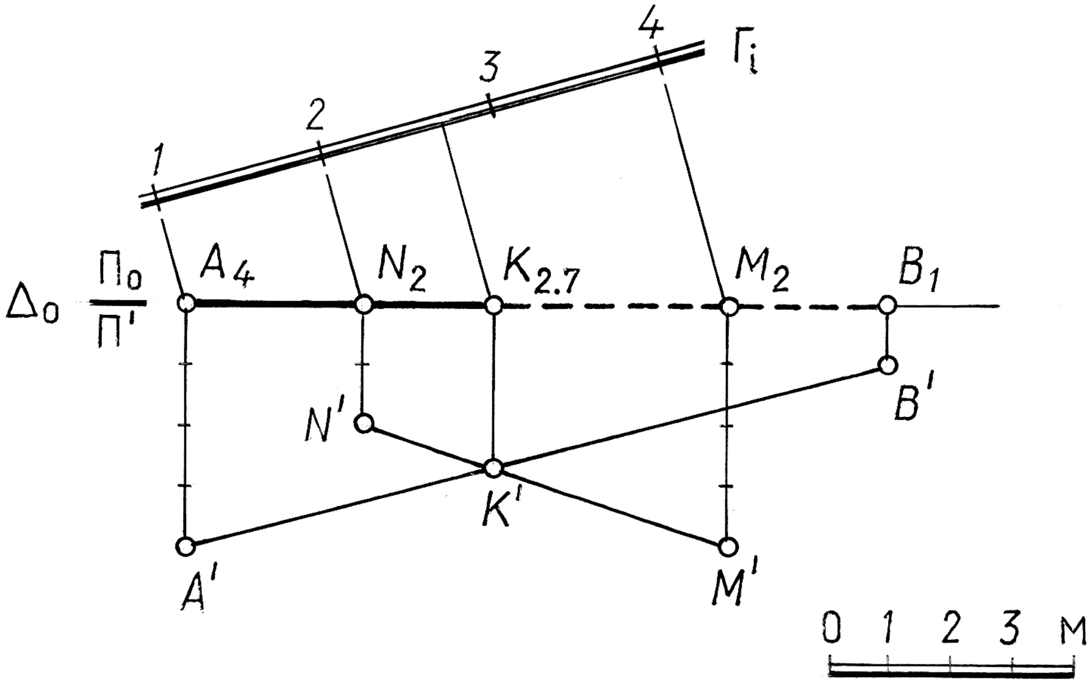


Fig. 2.36

The solution to this problem can be performed using a plane in general position.

Example.

Determine the point of intersection of the straight line *AB* with the plane Γ (fig. 2.37).

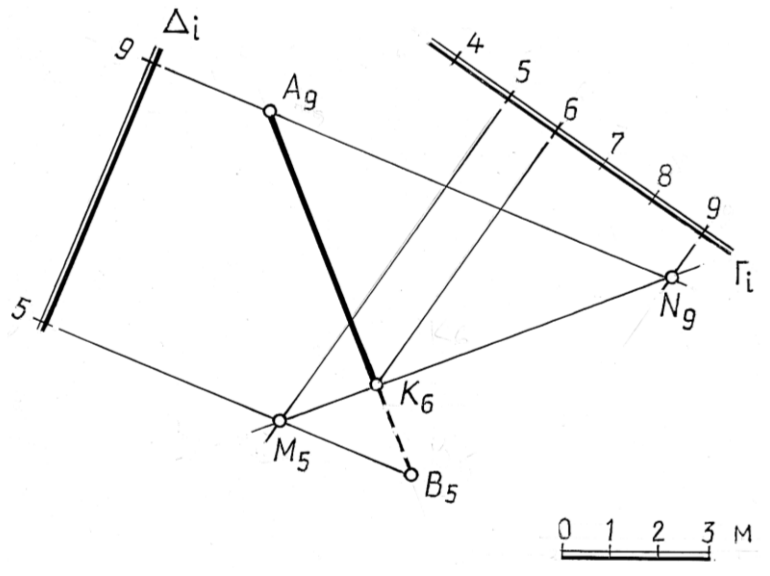


Fig. 2.37

***Solution*:**

An arbitrary auxiliary plane in general position Δ_i is drawn through line *AB*, defined by the contours (h_9, h_5).

The contour lines of the auxiliary plane are drawn through points *A* and *B* so that they intersect the contour lines within the drawing that have the same elevations of the given plane Γ_i.

Then the line of intersection of the auxiliary plane Δ_i with the plane Γ is constructed – the straight line *MN*. Point *K* – the point of intersection of the straight line *AB* and the line *MN* – is the sought point of intersection of the straight line with the plane Γ. The elevation of the point *K* is determined by the scale of the slope of the plane Γ.

Example.

Measure the distance from point *A* to plane Γ (fig. 2.38).

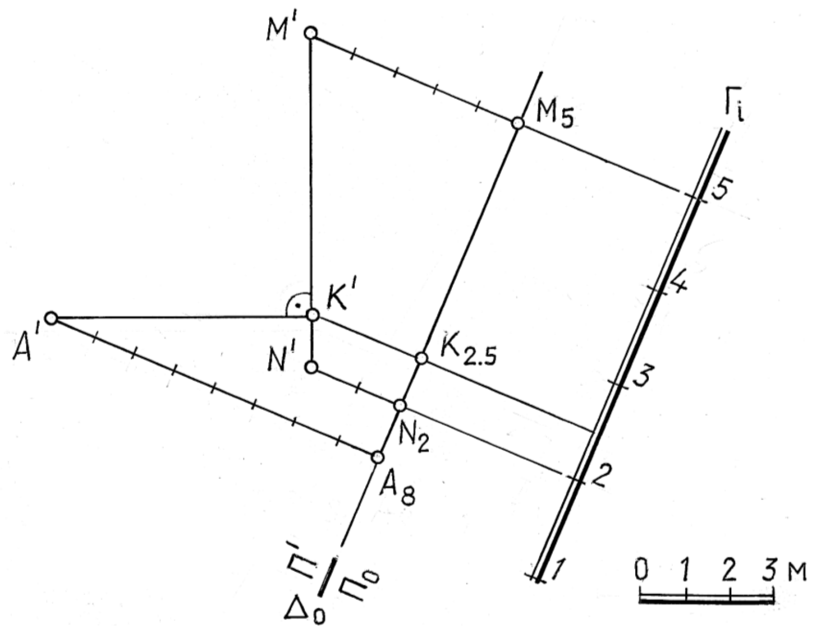


Fig. 2.38

***Solution*:**

From the point A we drop a perpendicular to the plane Γ, find the point K – the point of intersection of this perpendicular with the plane Γ, and then – the natural size of the line segment AK.

It is known earlier that the horizontal projection of the perpendicular to the plane makes a right angle with the horizontals of the same name of this plane. What is the same, the projection of the line perpendicular to the plane is parallel to the scale of incidence of this plane. Consequently, to solve the problem, a line parallel to the scale of incidence of the plane Γ_i is drawn through the point A_8. To find the point of intersection of the constructed perpendicular with the plane, an auxiliary horizontally projecting plane Δ is drawn through it, intersecting the given plane along the line MN. With the help of replacement of planes of projections the additional projection of the line of intersection of two planes MN, which is the line of the greatest slope of the plane Γ and the perpendicular to the plane AK, which is perpendicular to any line of this plane, including the line of the greatest slope, is constructed. $A'K'$ is the natural dimension of the perpendicular to AK, and the point K is the base of the perpendicular.

Example.

Construct the points of intersection of a straight line with a topographic surface (fig. 2.39, 2.40).

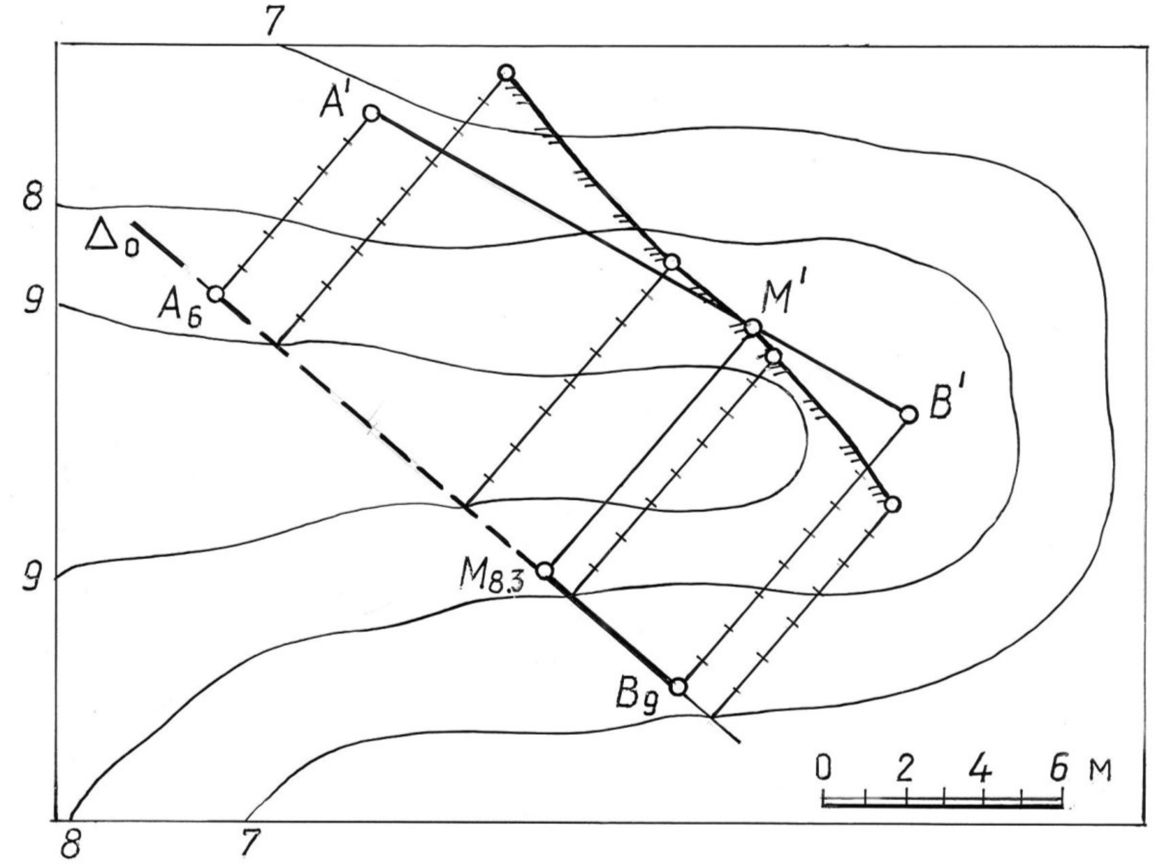


Fig. 2.39

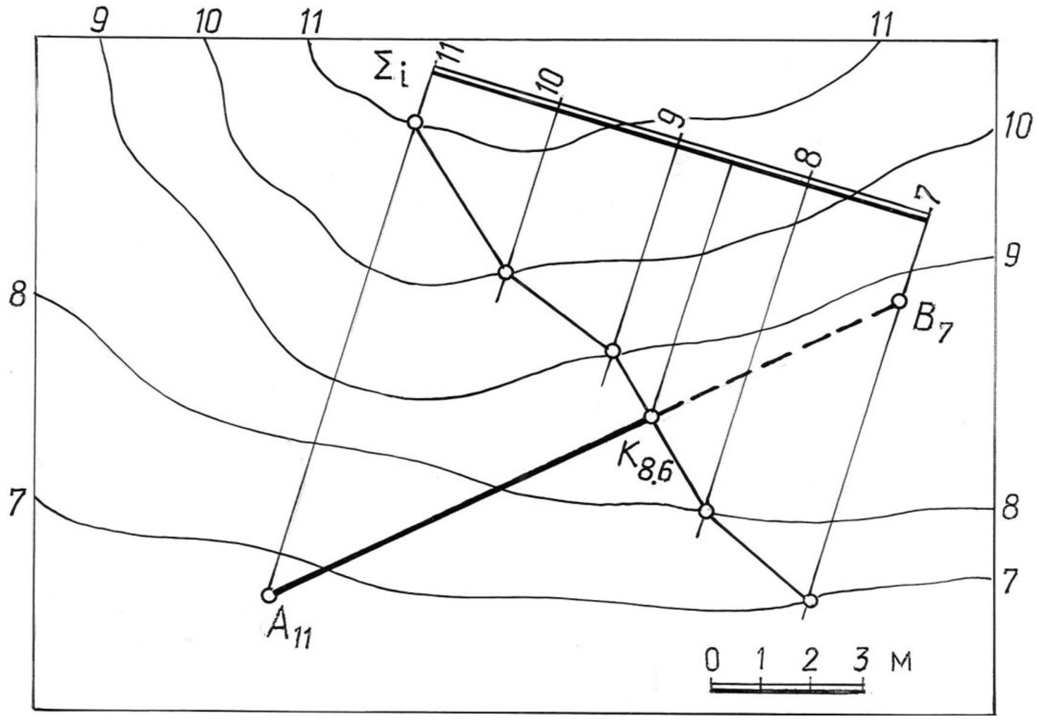


Fig. 2.40

***Solution*:**

A horizontally projecting plane Δ is drawn through the straight line *AB* (fig. 2.39) and the profile of this section of the topographic surface is built. *A*' *B*' – projection of a straight line in section. The point of intersection of the projection *A*' *B*' with the profile of the topographic surface determines the projection *M*' – the point of intersection of the given straight line with the topographic surface. Having drawn the line of the projection connection, we determine the horizontal projection of this point.

If a graduated projection of a straight line is specified in the drawing, then to solve the problem it is rational to use a general mediator plane, as shown in fig. 2.40. To determine the point of intersection of the straight line *AB* with the topographic surface, a plane of general position ∑ is drawn through the straight line *AB*. A construction plane is specified in the drawing using contours, which are drawn so that they do not intersect contours within the drawing with the same elevations of the topographic surface. Then, a section of the topographic surface was constructed by an intermediate plane and the point of intersection of the constructed line and the given straight line *AB* was marked, point *K* is the sought point of intersection of the straight line with the topographic surface. The elevation of point *K* is determined by the scale of the slope of the construction plane ∑.

2.8. Design of engineering structures in projections with numerical marks

The method of projections with numerical marks is widely used in the design of engineering earthworks. And an example of such structures are various horizontal construction sites.

A common task is to determine the boundaries of earthworks when organizing a construction site, which is expressed in determining cutting and filling lines, organizing slopes according to the task received. If the level of the site is higher than the level of the terrain surface, then the construction site is performed in the form of an embankment, if lower, then in the form of a cut. The planes and surfaces that limit the construction site on all sides and connect it to the terrain are called slopes. The slopes of the slopes are selected depending on the type of soil and are set when designing construction sites.

When designing construction sites, the following previously considered geometric problems are solved:

1) drawing planes with a given slope through line segments that limit the site in plan;

2) drawing surfaces with a given slope through the arcs of the curves that bound the site;

3) construction of lines of intersection of adjacent slopes (two planes, two surfaces or a plane with a surface);

4) construction of the intersection of surfaces or slope planes with a topographic surface – defining the boundaries of earthworks.

Let us illustrate what has been said with a specific example.

Example.

On a flat slope, design the slopes and determine the boundaries of earthworks for a horizontal construction site with an elevation of 50 m and a ramp ("ramp" – a gentle entry or descent to a horizontal site) (figure 2.41). The slopes of the embankment slopes $i_{н} = 2{:}3$, the slopes of the excavation slopes $i_{в} = 1{:}1$, the slope of the ramp $i_{а} = 1{:}2$.

***Solution*:**

1. We first draw a graph of the scale of the slopes and graphically determine the values of the intervals for the slopes of the excavation and embankment.

2. Determine the zero work points (the zero work point is the point at which the site profile intersects with the terrain profile and, therefore, no earthwork is required in this place). Slope horizontal 50 intersects the site contour with elevation 50 at points A_{50} and B_{50} along the zero works line. Above it there will be a notch, below it – an embankment.

3. We build the scale of the slopes for the slopes of the embankments and notches of the horizontal site, drawing them perpendicular to the sides of the construction site. We build the contours of the slopes.

4. We build the contours of the slopes of the ramp. From a geometric point of view, this problem is reduced to building a plane of a given slope through an inclined straight line. The ramp crosses the slope plane along the $K_{47}M_{45}$ line, which helps to determine the zero work points on the ramp (points N and L).

5. We build the lines of intersection of adjacent slopes by constructing the intersection points of the contour lines of the slopes with the same elevations.

6. We build the boundaries of earthworks by determining the intersection points of the contour lines of the slopes and slopes with the same elevations.

7. For the more visual expression of the direction of the slope at the top of the edges of the slopes, strokes are applied perpendicular to the horizontal lines (GOST 21.108-78). The distance between long strokes is 3–4 mm; between short and long – 1.5–2 mm. The strokes are drawn with the same thickness equal to 0.1–0.15 mm.

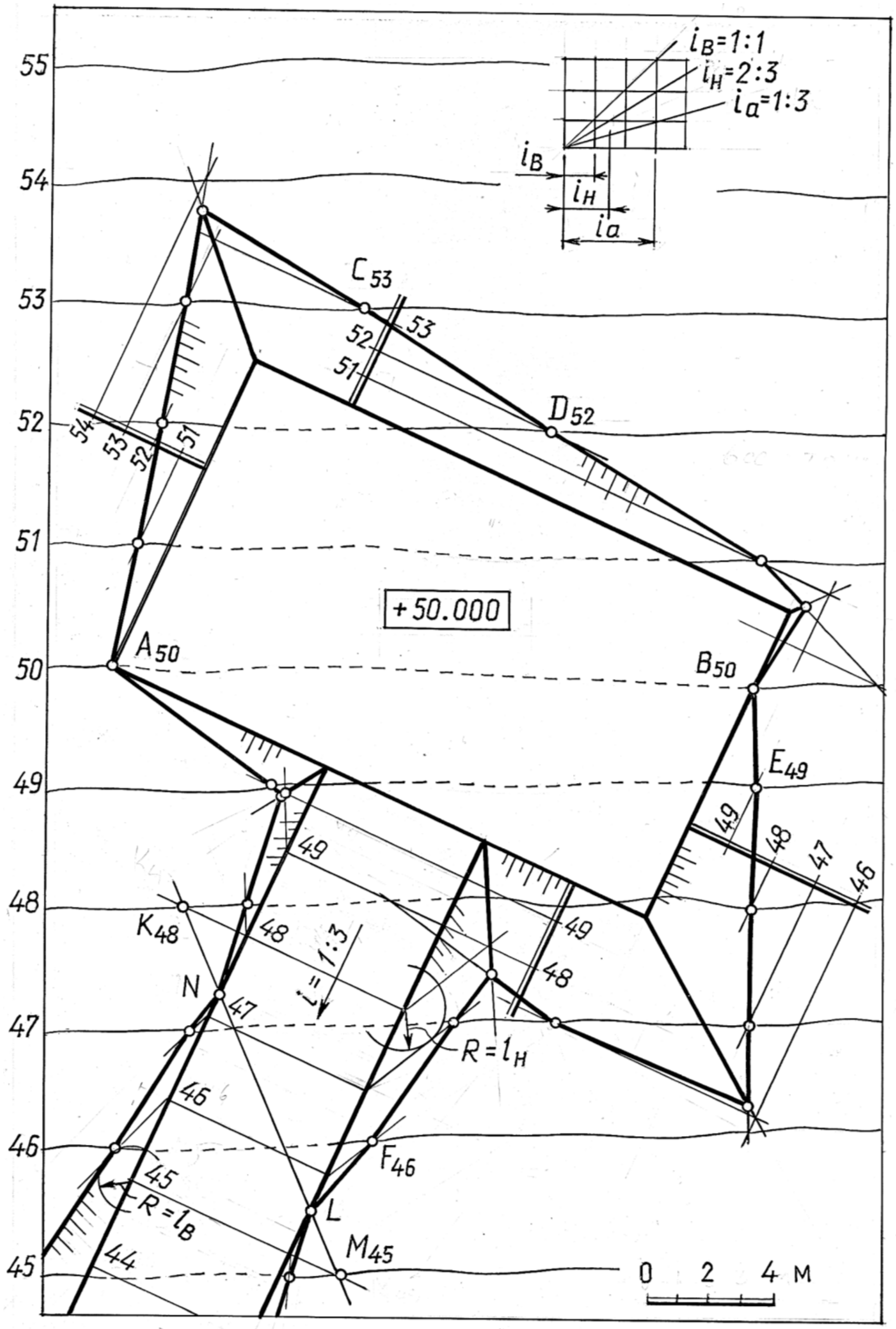


Fig. 2.41

3. BASICS OF SHADOW GEOMETRY

3.1. Drawing shadows on an orthogonal drawing

In descriptive geometry, when constructing shadows, they study graphical methods for determining the boundaries of shadows and do not consider the physical basis of shadows (the intensity of the light source, light glare, etc.).

When constructing shadows, it is assumed that light propagates in a straight line. If shadows are constructed in sunlight, then the light rays are considered parallel, since the light source is almost removed to infinity.

The main geometric task of constructing shadows is to determine the boundaries (contours) of proper and falling shadows.

The unlit part of the surface is called its own shadow. The line that separates the illuminated part of the body's surface from its own shadow is called the contour of its own shadow.

The shadow from one object to another, or from one part of the surface to another, is called a falling shadow, and the line bounding it is called the outline of the falling shadow. The contour of the falling shadow is built from the contour of its own.

In an orthogonal drawing, the direction of the light beam is assumed to be parallel to the diagonal of the cube whose faces coincide with the projection planes. In this case, the projections of the light beam are inclined to the *O*x axis at an angle of 45° (fig. 3.1).

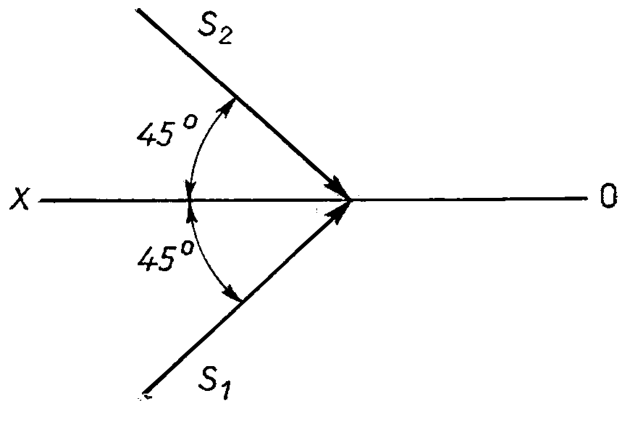


Fig. 3.1

3.2. Point shadows

The shadow from point *A* to any surface Γ is the point of intersection of the light ray *S* passing through this point *A* with the surface Γ.

The shadow from point *A* to the projection plane is the trace on this plane of the light ray *S*, passing through point *A*.

Figure 3.2 shows the construction of the shadow from point *A* on the plane of projections Π_2 and Π_1. To construct the shadow of this point, projections of the light beam are made through its projections and its traces are plotted on the frontal plane of the projections (A_2^T) and on the horizontal plane of the projections (A_1^T).

Of these two shadows, the first A_2^T is real, and the second A_1^T is imaginary. The shadow of a point on the frontal plane of the projections is real because the ray in this example intersects the frontal plane of the projections earlier than the horizontal plane of the projections.

The shadow from point A to an arbitrary flat shape is the point A^T where the light beam passing through point A intersects with the flat shape.

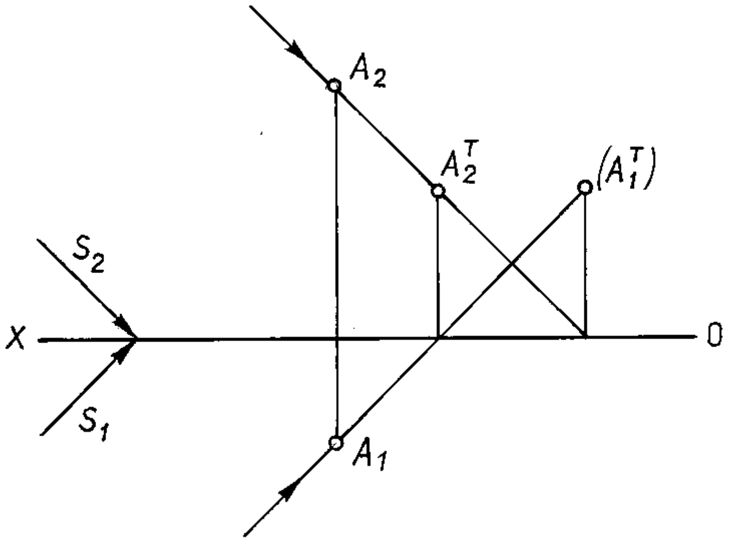


Fig. 3.2

Fig. 3.3 shows the construction of the shadow A^T from point A on the plane of the triangle.

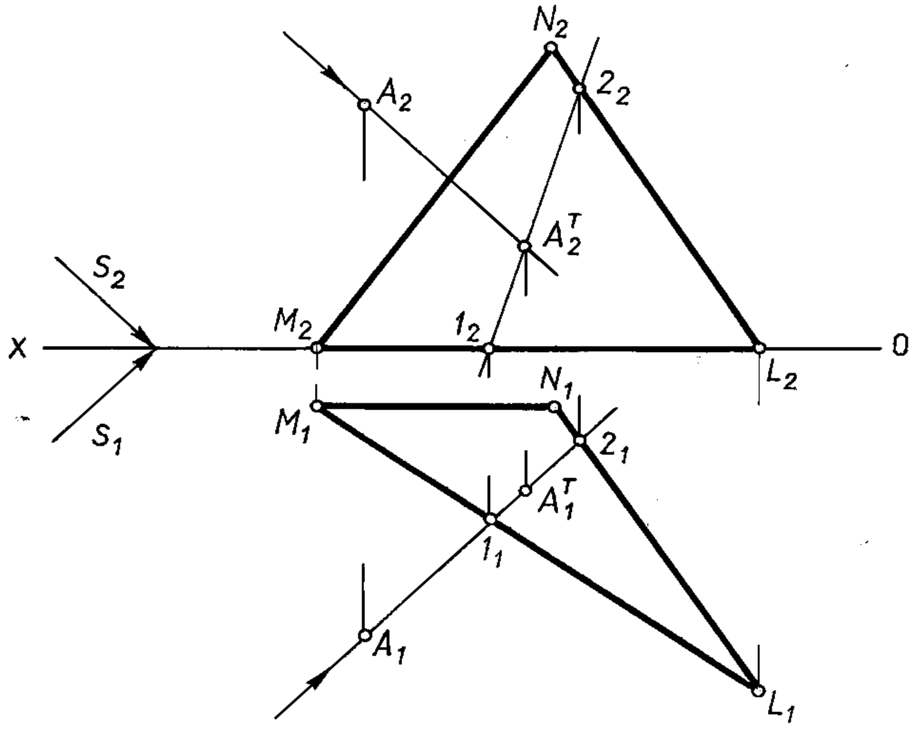


Fig. 3.3

The constructions are performed in the following order:

1. Projections of the light beam are drawn through the projections of point *A*.

2. The point of intersection of the light beam with the specified plane is determined (in fig. 3.3, the projection plane is used to determine the point of intersection of the light beam with the plane).

3.3. Shadows from a straight line

The shadow from a straight line to a surface is the line of intersection of the ray plane with this surface. A ray plane is a plane that passes through a given straight line parallel to the light beam.

The shadow of a straight line can be a point straight, polyline or curve.

The process of constructing the shadow of a straight line segment on two projection planes is recommended to be carried out in this sequence (fig. 3.4).

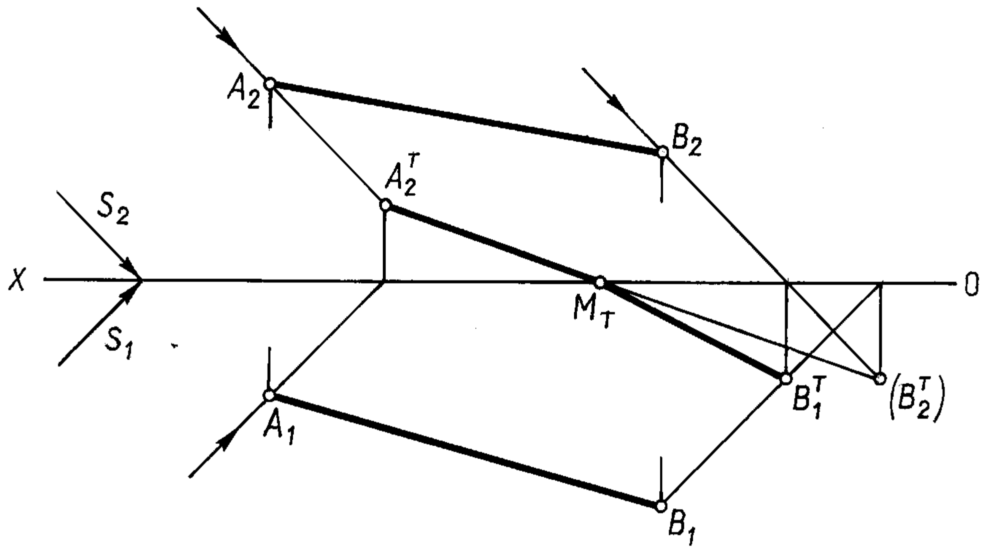


Fig. 3.4

1. Build the shadow of the segment on one of the planes, assuming that the second does not exist. So, in the example in figure 3.4, the shadow of the segment on the front plane of the projections is first constructed.

2. If the constructed shadow intersects the projection axis, then at this point, the shadow will be refracted and move from one projection plane to another.

The refracted shadow of the line segment will be directed to this point. In fig. 3.4, such a point is the real shadow of point *B* on the horizontal plane of the projections.

Let us note the regularities of the arrangement of the shadows of the segments of the straight lines of the particular position.

If the line segment is parallel to the plane, then the shadow from it on this plane is parallel to the segment and equal to it in magnitude.

If the line is perpendicular to the plane, then the shadow falling from it on this plane coincides with the projection of the light beam on this plane.

If the line is parallel to the direction of the light beam, then the shadow of it is a dot. Parallel lines have parallel shadows.

The shadow properties of these particular position lines are used to construct the shadows of more complex geometric shapes.

3.4. Shadows from a flat shape

The incident shadow of a flat shape on the projection plane can be constructed as a set of shadows from the vertices and shadows from its sides. Thus, the construction of the shadow of a flat figure on the plane of projections can be reduced to the well-known definition of shadows from points and straight line segments.

The construction of the shadow of a flat figure on two planes of projections must be carried out in the same sequence, what was recommended for building the shadow of a straight line (fig. 3.3).

So in fig. 3.5, first of all, the incident shadow of a triangle on the frontal plane of projections is constructed under the assumption that there is no horizontal plane of projections.

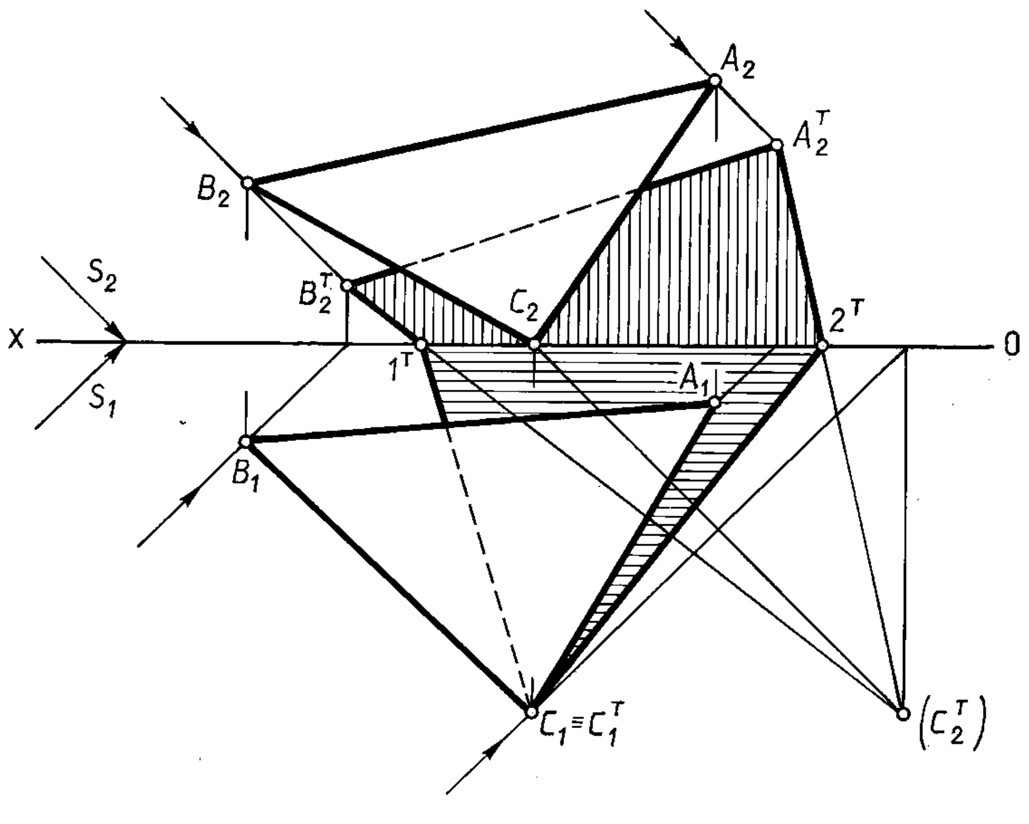


Fig. 3.5

The real part of the shadow will be the part that is located above the projection axis. After connecting the points of refraction of the shadow with the actual shadow of point *C* on the horizontal plane of the projections, the construction is completed.

Depending on the position of the flat figure in relation to the direction of the light rays, and in relation to the projection planes, the illuminated or shadow side of the flat figure can be projected onto one or another projection plane. To determine which side of a flat shape, i.e. illuminated or shaded, is projected onto the given projection plane, you need to compare the order of the projections of individual points on the contour of the flat shape with the order of their shadows.

So in fig. 3.5, the triangle is projected on both projection planes with the illuminated side, since the sequence of the point designations on the contour of both the frontal and horizontal projections of the triangle, on the contour of its shadow when reading them, for example, "clockwise", is the same.

One of the invariant properties of parallel projection states that the projection of a plane figure parallel to the plane of projections is equal to this figure. Since the shadows on the projection plane are parallel oblique projections, the shadow on the projection plane from any flat figure parallel to this projection plane is equal to the flat figure itself (figure 3.6). Therefore, the shadow from a circle, whose plane is parallel to the projection plane, is a circle of the same radius.

Therefore, to draw the contour of the incident shadow of a circle on the plane of projections parallel to it, it is enough to build a shadow from the center of the circle and build a circle of the same radius as the circle bounding the circle.

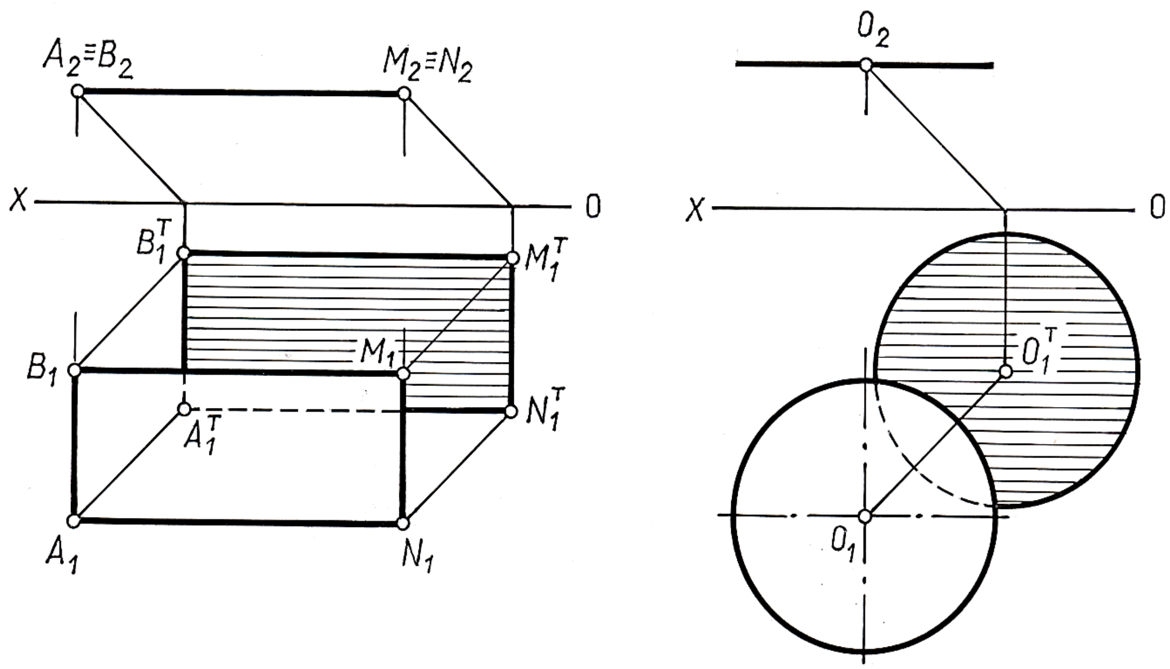


Fig. 3.6

3.5. Shadow of a three-dimensional figure

Any surface that borders a spatial body is divided by the contour of its own shadow into two parts: illuminated and in shadow. The incident shadow from this surface is limited by the shadow from the contour of the surface's own shadow. Therefore, you can build a shadow from a three-dimensional shape:

1) build a falling shadow, then use the contour of the falling shadow to determine the contour of the surface's own shadow;

2) define the contour of its own shadow and then build the falling shadow from the contour of its own shadow.

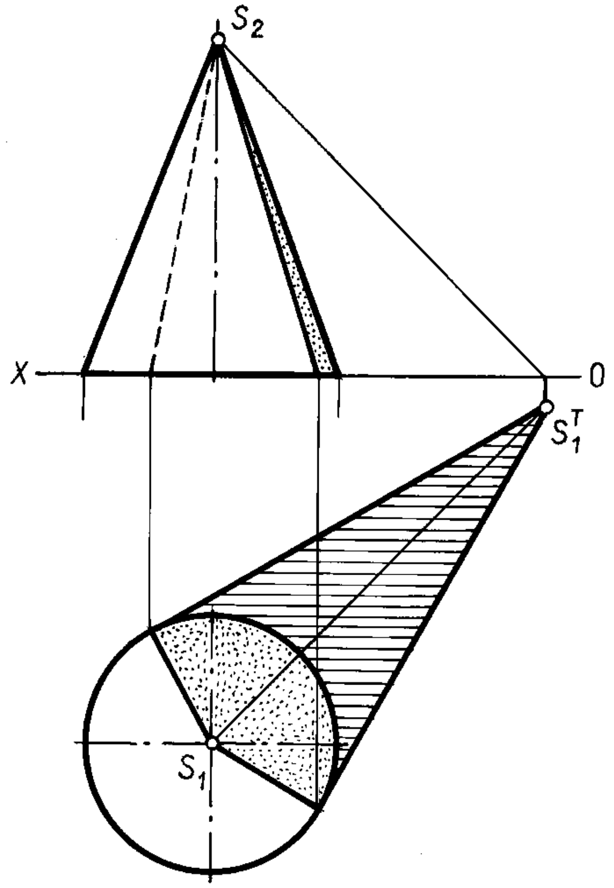


Fig. 3.7

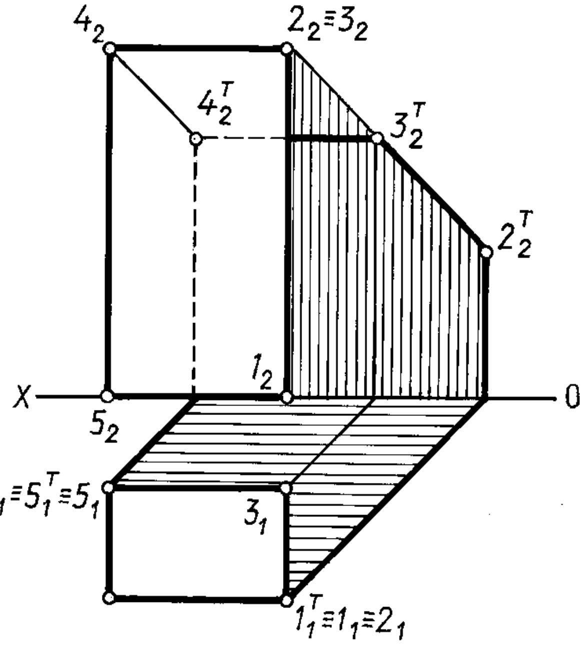


Fig. 3.8

Fig. 3.7 shows the construction of the proper and incident shadows of a straight circular cone, the base of which is located in the horizontal plane of the projections.

First, the shadow on the horizontal plane of the projections from the vertex of the cone is determined. Then two straight lines tangent to the circle of the base of the cone are drawn from the constructed point. The points of contact of these lines to the base circle determined the position of the cone generators, which are the contour of the cone's own shadow.

Fig. 3.8 shows the construction of the shadow of a straight prism. First, the faces that are in their own shadow are defined, and the contour of their own shadow is indicated. Then the contour of the falling shadow is constructed from the contour of its own shadow.

Fig. 3.9 shows the definition of the contour of the falling shadow from the pipe on the slope of the roof of the building.

The problem is reduced to determining the shadows from points and straight lines on an arbitrary plane (roof slope).

The constructions are made using the method of auxiliary secant ray planes Q and Σ.

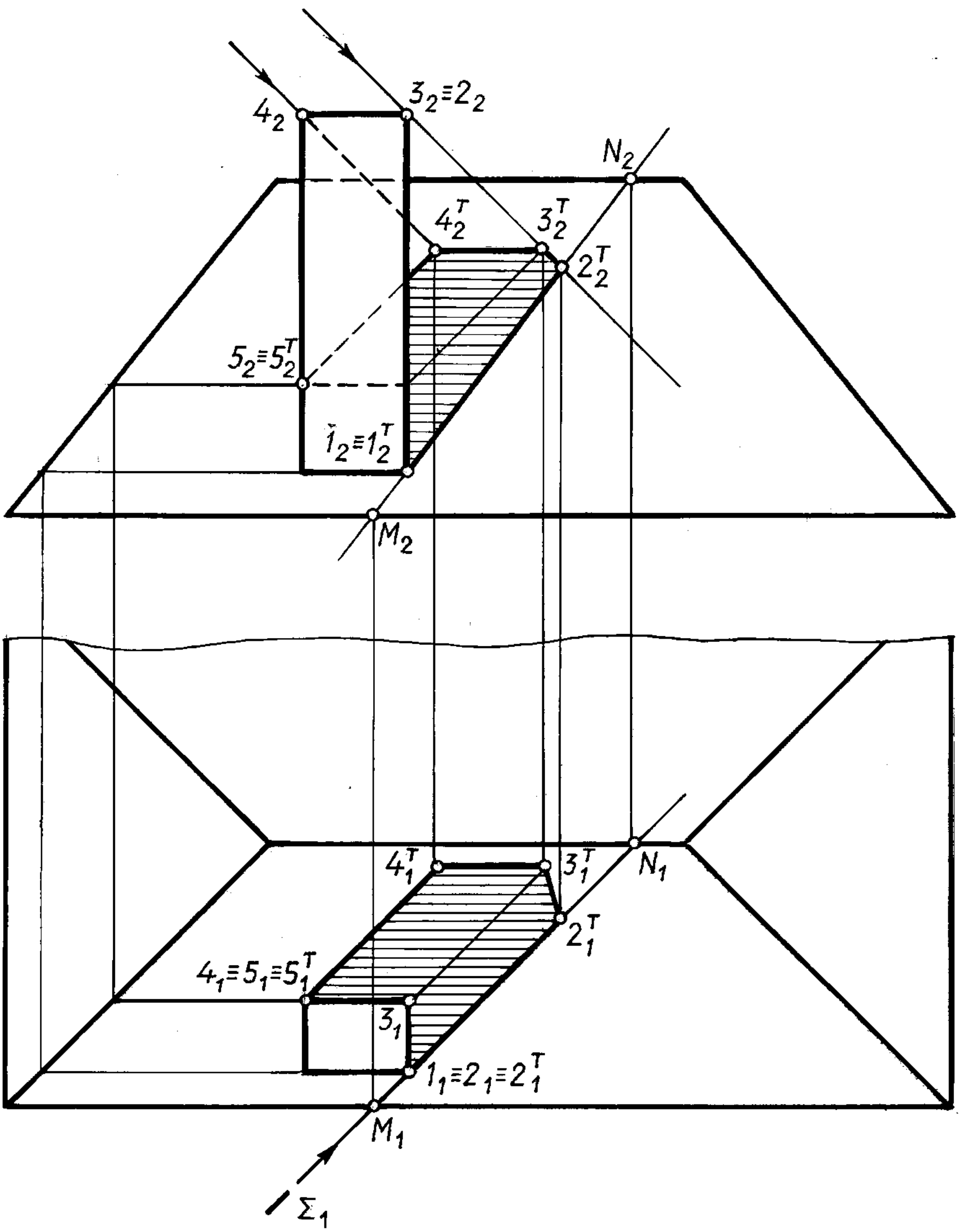


Fig. 3.9

3.6. Drawing shadows in axonometry

The basic rules for constructing shadows, set out in relation to orthogonal projections, remain in force when constructing shadows in axonometric projections. Features – only in setting the direction of the light beam.

In axonometric projections the direction of the light rays can be taken by any one, but it is necessary to observe the condition of plausibility of illumination, and also remember that the shadow is a means of revealing the shape and giving the drawing the greatest expressiveness. The rays should not be too flat or too steep; the best angle of inclination of the light beam to the horizon can be considered 30°–40°. The direction of the light beam is set by its axonometric S' and secondary S_1' projections (fig. 3.10).

To construct the shadow of the point B' (fig. 3.10), the axonometric projection B' is used to carry out the axonometry of the beam parallel to the given direction S', and through the secondary projection B_1' we draw a straight line parallel to the secondary projection of the beam S_1'. The point of intersection of the ray with its secondary projection is the shadow of point B_1^T.

Depending on the location of the point in space, the shadow can fall on the horizontal plane. Thus, from the construction of the shadow of point A (fig. 3.10), it can be seen that the shadow from point A lay on the horizontal plane outside the vertical profile plane.

To determine the shadow of point A_3^T of point A on the profile plane, it is necessary from the intersection point of the secondary projection of the light beam with use the y-axis to draw a vertical line until it intersects with the axonometry of the ray passing through point A'.

To construct the shadow of the segment AB in axonometry in fig. 3.10, the imaginary shadow (A_1^T) of the point A is used.

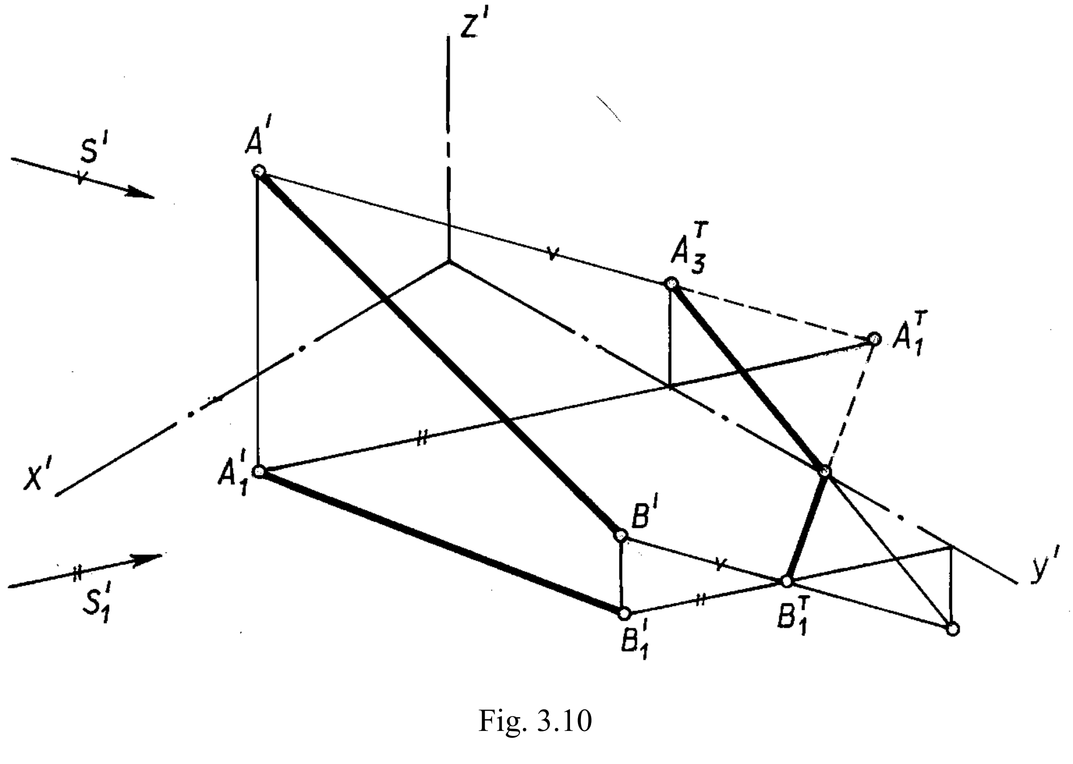


Fig. 3.10

Figure 3.11 shows the shadows from the stairs and the shadows from the pedestal, which has the shape of a truncated parallelepiped.

Note that in axonometry, the shadow of a vertical segment on a horizontal plane coincides with the direction of the secondary projection of the light beam, and the shadow of any straight line on a plane parallel to it is parallel to the straight line itself.

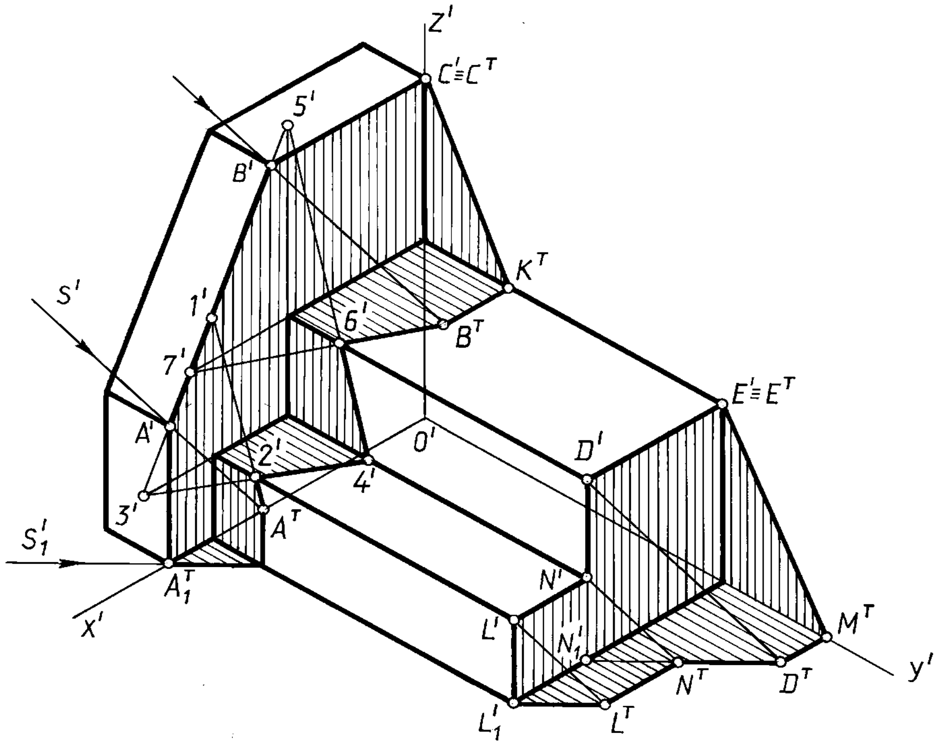


Fig. 3.11

The construction of the shadow of the stairs is clear from the drawing.

In fig. 3.11 the shadows on the stairs from the pedestal are constructed in the following order:

1. The shadow from the vertical segment $A'A'_1$ is determined.

2. The point of intersection of the inclined line *A'B'* with the vertical plane of the first stage is determined – point *1'*. From point A^T the shadow moves along the vertical plane to point *1'* and ends at point *2'*.

3. The inclined line *A'B'* is continued until it intersects with the horizontal plane of the step and the point *3'* is obtained. Connecting the points *2'* and *3'*, we obtained the shadow of the segment *A'B'* on the horizontal plane of the first step, which ends at point *4'*.

On the second step, the shadow of the segment *A'B'* is constructed similarly. The completed constructions are clear from the drawing.

4. The shadow from the horizontal segment *B'C'* on the horizontal plane is parallel to the segment itself, and on the facade plane goes to the point C^T(point *C'* is a shadow itself).

3.7. Drawing shadows in perspective

The construction of shadows in perspective is fundamentally no different from the construction of shadows in axonometric projections, except that in perspective, in the general case, the projections of light rays are directed to the corresponding vanishing points.

In perspective projections, the image of the sun or the direction of the light beam is set, if it is parallel to the picture. Since the sun is considered to be removed to infinity, its secondary projection must be on the horizon line. If the sun is in front of the viewer, then its image is above the horizon line, if it is behind the viewer, then its image in the picture is below the horizon line.

Most often the direction of light rays in the perspective is taken parallel to the plane of the picture. Then the secondary projections of the light rays should be parallel to the line of the earth.

In fig. 3.12 the incident shadows from the vertical segment *AB* are plotted at different relative positions of the light source:

3.12, *a* – the sun is located in front of the segment, to the left of it; the point *S*' of the vanishing point of the light rays is below the horizon line.

3.12, *b* – the sun is behind the segment and to the left of it, the point *S*' is located above the horizon line.

3.12, *c* – in some cases, to simplify the construction of shadows in perspective, the direction of light rays is taken parallel to the plane of the picture (fig. 3.14).

The projections of the light rays in figure 3.12 are directed from left to right. Obviously, when the light source is positioned to the right, the projections of the light rays will be directed from right to left.

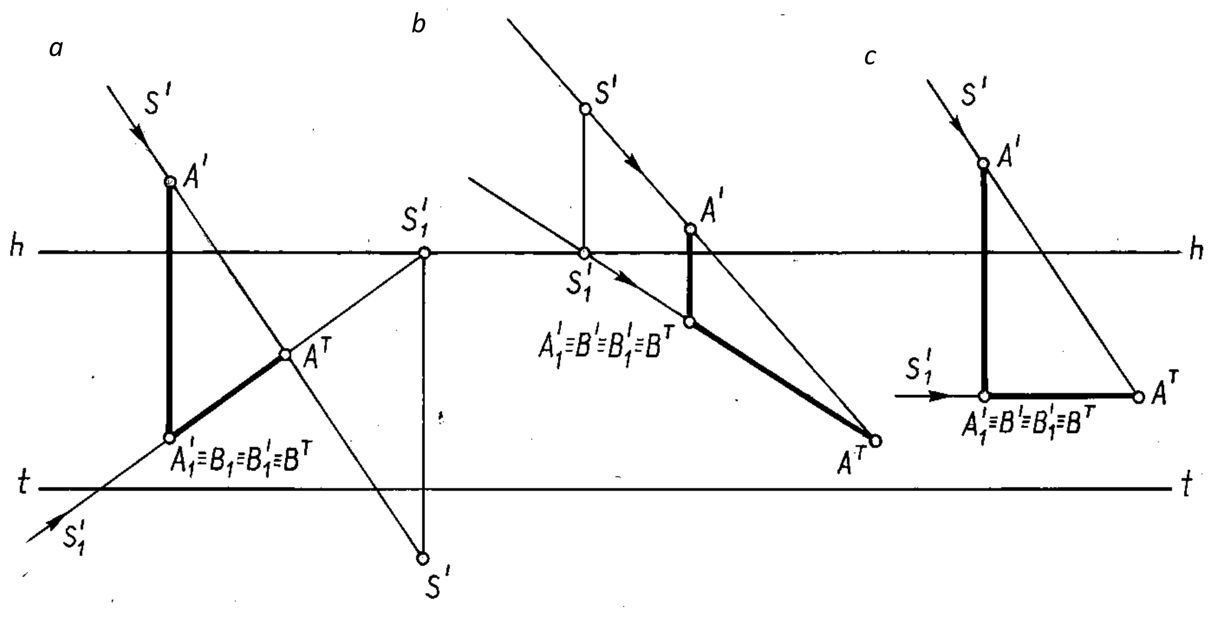


Fig. 3.12

Fig. 3.13 shows the construction of the shadow of the segment *AB*, when the sun is behind the viewer.

To find the shadow A^T point *A* in perspective, you need to draw the perspective of the light beam through *A*' and *S*', and through the secondary projection A_1' – the secondary projection of the beam (straight $A_1'S_1'$). The intersection of the ray's perspective with its secondary projection will determine the shadow of point *A* on the subject plane.

The shadow of point *B* is constructed similarly.

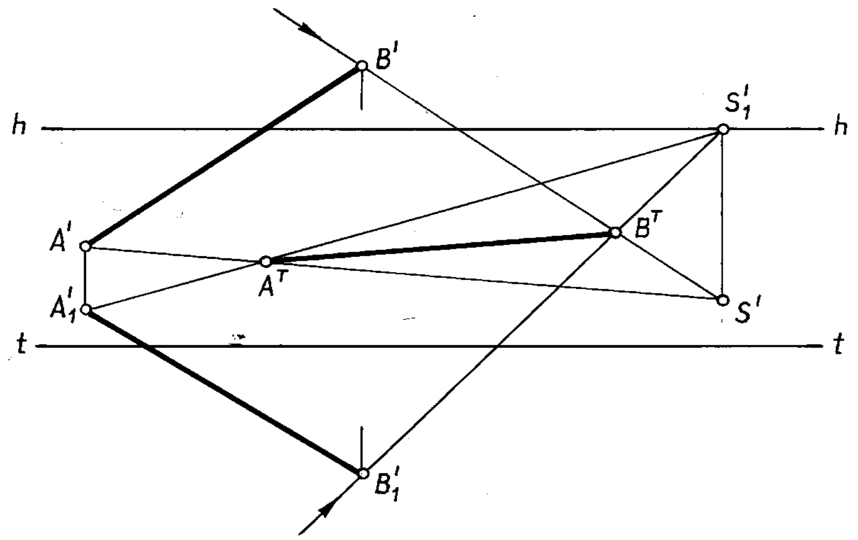


Fig. 3.13

The obvious convenience in constructing shadows in perspective is inherent in the case when the light rays are taken parallel to the plane of the picture. It should be borne in mind that in this case, not only the secondary projections are parallel to each other, but also the light rays themselves.

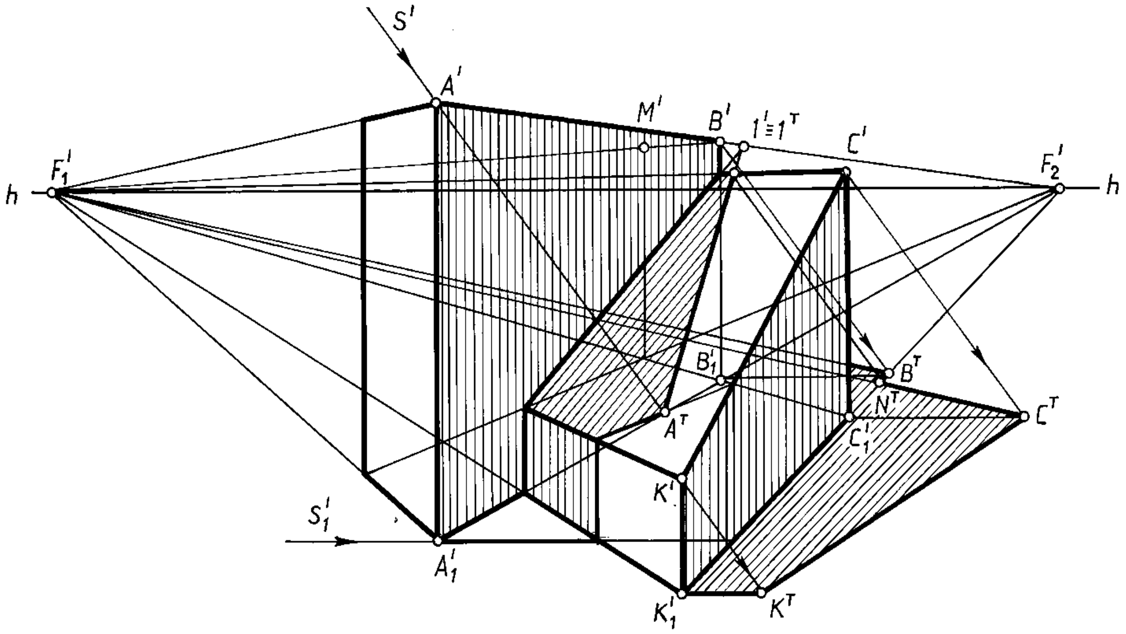


Fig. 3.14

Fig. 3.14 shows the proper and incident shadows of two prisms when the light rays are parallel to the picture.

Determining the contours of falling shadows in perspective, when the light rays are parallel to the plane of the picture, is essentially no different from determining the contours of falling shadows in axonometry. The completed constructions are clear from the drawing.

REFERENCES

1. Tarasov, V. V. Descriptive geometry. Short course for the practical classes: student book, part 2 / V. V. Tarasov, L. S. Koritko, Y. I. Sadovski, I. M. Shubert; cor. V. V. Tarasov. – Minsk : BNTU, 2011. – 117 p.
2. Bogolyubov, S. K. Engineering Drawing / S. K. Bogolyubov, A. Voinov; transl. from the Russian by Leonid Levant. – M. : Mir, 1968. – 352 p.
3. Bradley, Thomas. Practical geometry, linear perspective, and projection; including isometrical perspective, projections of the sphere, and the projection of shadows, with descriptions of the principal instruments used in geometrical drawing / T. Bradley. – London : Baldwin and Cradock, 2008. – 290 p.
4. Gordon, F. Engineering graphics and descriptive geometry in 3-d / F. Gordon. – Canada : Macmillan Company of Canada, 1977. – 368 p.
5. Krylov, N. Descriptive geometry / N. Krylov, P. Lobandievsky, S. Men; transl. from the Russian by G. Yankovsky. – M. : Mir, 1974. – 383 p.
6. Bhatt, N. D. Engineering Drawing / N. D. Bhatt – 50th edition. – Mumbai : Charotar publishing house, 2011. – 738 p.
7. Walsh, C. J. Engineering drawing and descriptive geometry / C. J. Walsh. – Cambridge: Harvard Univ. Press, 2014. – 247 p.
8. Watts, E. F., Rule J. T. Descriptive geometry / E. F. Watts, J. T. Rule – NY: Prentice-Hall, Inc, 1946. – 321 p.
9. Shah, M. B., Rana, B. C. Engineering Drawing / M. B. Shah, B. C. Rana – Delhi : Pearson Education, 2007. – 484 p.
10. K. Venkata, R. Textbook of Engineering drawing / K. Venkata Reddy – Hyderabad : BS Publication, 2008. – 377 p.
11. Hawk, M. C. Theory and problems of descriptive geometry / M. C. Hawk. – NY : McGraw-Hill Company, 1962. – 220 p.

Учебное издание

САДОВСКИЙ Юрий Игоревич
ШУБЕРТ Ирина Михайловна
САДОВСКАЯ Елена Александровна
ТЕЛЕШ Евгений Александрович

СПЕЦИАЛИЗИРОВАННЫЕ РАЗДЕЛЫ НАЧЕРТАТЕЛЬНОЙ ГЕОМЕТРИИ ДЛЯ СТРОИТЕЛЕЙ И АРХИТЕКТОРОВ

Учебно-методическое пособие
для студентов специальностей
7-07-0732-01 «Строительство зданий и сооружений»,
7-07-0732-02 «Инженерные сети, оборудование зданий и сооружений»,
6-05-0718-01 «Инженерная экономика»,
6-05-0732 «Экспертиза и управление недвижимостью»,
6-05-0719-01 «Инженерно-педагогическая деятельность»

На английском языке

Редактор *К. С. Мельникова*
Компьютерная верстка *А. В. Степанкиной*

Подписано в печать 19.12.2024. Формат 60×84 $^{1}/_{8}$. Бумага офсетная. Ризография.
Усл. печ. л. 6,86. Уч.-изд. л. 4,55. Тираж 100. Заказ 721.

Издатель и полиграфическое исполнение: Белорусский национальный технический университет.
Свидетельство о государственной регистрации издателя, изготовителя, распространителя
печатных изданий № 1/173 от 12.02.2014. Пр. Независимости, 65. 220013, г. Минск.