ON INTEGRATION OF THE MATHEMATICAL PROGRAMMING, APPLIED STATISTICS AND DECISION MAKING METHODS

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Introduction

The problem of emergency prevention and liquidation is as much practically important as theoretically complicated. By the example of this problem we intend to show how different mathematical and applied theories ([0]) could be used together in order to cope with this problem (and the problems of the same kind).

The problem statement and solution approach

The problem under consideration concerns resources allocation between the emergency liquidation departments (for instance, those dealing with fire prevention). The entire problem consists of the two interrelated subproblems (P_A and P_B).

The first one deals with the resource demands prognosis and uses as inputs the next data:

- the dates when the emergencies occurred (*A*);
- the amounts of the resource(s) used for emergency liquidation (the resource type may be anti-fire equipment, cars, chemical staff and so on) (*B*);
- the probability *P*, estimating that the allocated amount of the resource will be enough to liquidate the next emergency of that type (*C*).

The second subproblem deals with the optimal distribution of the entire amount *R* of the resource between the emergency liquidation departments on the basis of the solutions of the first subproblem and some additional information reflecting emergency complexity and frequencies.

To proceed let us introduce a random variable *x* standing for the resource amount used in emergency liquidation. One can then write

$$P(x \le w) = \int_0^w f(z)dz,$$

for continuous variable *x*, and

$$P(x \le w) = \sum_{i=0}^{i \le w} P(x = i)$$

for a discrete variable. Here *w* denotes the amount of the resource we are interested in; $P(x \le w)$ means the probability value for which *x* won't exceed *w*, that is, the resource amount allocated to emergency department will be enough to liquidate the next emergency consequences by that department alone; *f*(*z*) stands for a density function of the unknown probabilistic distribution we need to define. In order to find *f*(*z*) we need to build an empirical probabilistic distribution function on the basis of the data (*A*), (*B*). The sequence of the pairs $S = <(t_1, r_1), (t_2, r_2), ..., (t_n, r_n)>$ is a priori known, where each pair in *S* contains the time t_k elapsed since the previous emergency case (*k* -1)*th* till the emergency case *k*; and r_k means the resource amount required to liquidate the *k-th* emergency consequences. Introduce a new variable *q = r/t* showing the «rate» of the resource consumption. Then we shall deal with an integral

$$P(q \le q^{max}) = \int_0^{q^{max}} h(z)dz$$

with *h*(*z*) reprezenting empirical density of the «rates» *q* distribution. One can use a polynomial approximation of the empirical distribution function in the form

$H(z) = a_0 + a_1 z + a_2 z^2 + ... + a_m z^m$ with the power value m providing statistically adequate solution (to verify statistical adequacy one can use, for instance, the χ^2-criterion). To find the resulting value of $q_i^{\max}$ (with i standing for a resource index) from the equation above one can use some approximation-based method like the Monte-Carlo one, for instance. Given value $q_i^{\max}$, one can then directly find the amount r of the resource needed to cope with the emergences which are expected to occur within the time period T as $\omega = T \cdot q_i^{\max}$. It should be noted here that one should discriminate between restorable and not restorable type of the resource under consideration. Thus, the above estimations are given for the type of not restorable resource and can be quite easy adopted to the restorable resource type.

Now let us address the second subproblem P_B dealing with resource (re)distribution among emergency liquidation departments. It is supposed that the total amount of the resourse to be allocated may be either known or unknown. Introduce the following goal function

$$L = M \cdot \varepsilon + \sum_{j=1,n} \delta_{kj} / G_j \to \min .$$

Here, $G_j = \lambda_j \cdot \varphi_j$ estimates the tensity of the emergency liquidation department j functioning; λ_j stands for the frequencies of the emergency cases the *j-th* department has been engaged in; φ_j denotes the mean complexity estimation of the emergency case liquidated by the *j-th* department (this estimation requires a multicriteria approach, e.g. based on the Saati's hierarchy analysis). For the *k*th resource and the *j*th department the next inequality should be satisfied: $r_{kj} + \delta_{kj} \geq \omega_{kj}$, where ω_{kj} stands for the amount of the *k*th resource found in the subproblem P_A; r_{kj} denotes the amount of the *k*th resource to be assigned to the *j-th* department (the value of r_{kj} is to be found by solving the optimization problem); δ_{kj} represent extra variable to provide the subproblem P_B consistency. Informally, δ_{kj} means the additional amount of the resource allocated to the *j-th* department if the total amount of the resource k is insufficient. We also introduce the next constraint $\sum_{j=1,n} r_{kj} = R_k + \varepsilon$ which means that the sum of the resource of the type k assigned to all departments cannot exceed the available total amount R_k. It is clear that in general case one needs to use some extra variable ε to provide the above constraint. Also it should be clear now that this extra amount ε must be as small as possible, so we use a big constant M as a penalty for usage of additional resource amount ε. Finally, we introduce the next constraints

$$R_k \cdot \frac{G_j}{\sum_{k=1,n} G_k} - \delta_{kj} \leq r_{kj} \leq R_k \cdot \frac{G_j}{\sum_{k=1,n} G_k} + \delta_{kj},$$

demanding to provide deviation from the values r_{kj} as low as possible ($k = \overline{1,K}; j = \overline{1,n}.$). By this we obtained a linear optimization problem with non negative variables.

Conclusion

The described approach was used in a real program for the needs of practical utilization.

References

O.V. German, V.B. Taranchuk, L.V. Shkolnikov. On the formulation and methods for solving optimization problem of resource requirements for emercom departments // Science Online Journal Technosphere Technology Security: 2014, № 3 (55). – 8 p. / Mode of access: http://ipb.mos.ru/ttb/2014-3/2014-3.html. − Date of access: 8.10.2014.