MODELING BENDING TWO-LAYERED AXYSYMMMETRICAL SHELL FINITE ELEMENT

K Kurochka, e-mail: kurochka@gstu.by, I. Stefanovski

Pavel Sukhoi State Technical University of Gomel, Belarus
igorst@pisem.net

Currently, more and more applications are multi-layer elements [1]. Consider the stress-strain state of a two-layer rigidly clamped conical shell, zero (internal) layer which is reinforced with fibers of constant cross-section in the meridional direction, the first in the district. The solution to this problem will be sought by the method of finite elements.

As we consider the deflection of the required quantity of a conical shell w = w(s), which we use to simulate the axisymmetric finite element with two nodes with three degrees of freedom in each $\{g_0\}^T = \{u\ w\ \beta\}$, где $\{g_0\}$ – vector of nodal degrees of freedom finite element; β – angle in the radial direction..

Use the principle of virtual displacements [2], which in the case of a two-layer conical shell can be rewritten in the form (1),

$$\left\{\bar{g}\right\}^T\{R\} = \int_{s_i}^{s_{i+1}}\int_0^{2\pi}\int_{-\frac{h_0}{2}}^{\frac{h_0}{2}}\left\{\bar{\varepsilon}^0\right\}^T\left\{\sigma^0\right\}dzd\varphi ds + \int_{s_i}^{s_{i+1}}\int_0^{2\pi}\int_{\frac{h_0}{2}}^{\frac{h_0}{2}+h_1}\left\{\bar{\varepsilon}^1\right\}^T\left\{\sigma^1\right\}dzd\varphi ds, \qquad (1)$$

where the number in the index indicates the number of the shell layer, 0 corresponds to the inner layer,

$\{g\}^T = \{u_i \quad w_i \quad \beta_i \quad u_{i+1} \quad w_{i+1} \quad \beta_{i+1}\}$ – displacement vector;

$\{R\}^T = \{R_{si} \quad R_{\varphi i} \quad M_{\beta i} \quad R_{si+1} \quad R_{\varphi i+1} \quad M_{\beta i+1}\}$ – force vector;

$\{\varepsilon\}^T = \{\varepsilon_s \quad \varepsilon_\varphi \quad \kappa_s \quad \kappa_\varphi\}$ – strain vector;

$\{\sigma\}^T = \{\sigma_s \quad \sigma_\varphi \quad \chi_s \quad \chi_\varphi\}$ – stress vector;

s_i – coordinate of the i-th node; bar over a variable indicates the variation characteristic.

To approximate the displacement we use the following shape functions:

$$u(s) = a_1 + a_2 s\,;\ \ w(s) = a_3 + a_4 s + a_5 s^2 + a_6 s^3\,.$$

Hooke's law [3] in the case of structurally inhomogeneous anisotropic displacement in the meridional and circumferential direction of the shell layers takes the form (respectively)

$$\left\{\sigma^0\right\} = \begin{Bmatrix} B_{11} & B_{12} & 0 & 0 \\ B_{12} & B_{22} & 0 & 0 \\ 0 & 0 & -zD_{11} & 0 \\ 0 & 0 & 0 & -zD_{22} \end{Bmatrix}\left\{\varepsilon^0\right\};\ \ \left\{\sigma^1\right\} = \begin{Bmatrix} B_{22} & B_{12} & 0 & 0 \\ B_{12} & B_{11} & 0 & 0 \\ 0 & 0 & -zD_{22} & 0 \\ 0 & 0 & 0 & -zD_{11} \end{Bmatrix}\left\{\varepsilon^1\right\} \qquad (2)$$

$$B_{11} = \frac{(1-\omega_z)E_c}{1-\nu_c^2} + \omega_z E_{11};\ B_{22} = \frac{(1-\omega_z)E_c}{1-\nu_c^2} + \omega_z E_{22};\ B_{12} = \frac{(1-\omega_z)E_c}{1-\nu_c^2}\nu_c + \omega_z E_{12};$$

$$\|D_{11}\| = \|Q_{11}\|^{-1};\ \|D_{22}\| = \|Q_{22}\|^{-1};$$

$$Q_{11} = \frac{2(1-\omega_z)(1+\nu_c)}{E_c} + \omega_z \Gamma_{11};\ Q_{22} = \frac{2(1-\omega_z)(1+\nu_c)}{E_c} + \omega_z \Gamma_{22};$$

where B, D, Q – effective tangential and transverse shear stiffness and compliance reinforced layer.

Physical components (2) are defined by:

$$E_{11} = \omega E_a + (1-\omega)E_c + \frac{E_c E_a (\omega \nu_a + (1-\omega)\nu_c)^2}{\omega(1-\nu_a^2)E_c + (1-\omega)(1-\nu_c^2)E_a};$$

$$E_{12} = \frac{E_c E_a (\omega \nu_a + (1-\omega)\nu_c)}{\omega(1-\nu_a^2)E_c + (1-\omega)(1-\nu_c^2)E_a}; \; E_{22} = \frac{E_c E_a}{\omega(1-\nu_a^2)E_c + (1-\omega)(1-\nu_c^2)E_a}; \tag{3}$$

$$\Gamma_{11} = \frac{2(1+\nu_c)(1+\nu_a)}{\omega(1+\nu_c)E_a + (1-\omega)(1+\nu_a)E_c}; \; \Gamma_{22} = 2\frac{\omega(1+\nu_a)E_c + (1-\omega)(1+\nu_c)E_a}{E_a E_c};$$

where E_a, E_c, ν_a, ν_c – Young's modulus and Poisson's ratio of the reinforcing fibers and the binder. Structural reinforcement parameters - the intensity of the reinforcement in the layer plane (ω) and adjustment layer (ω_z): $\omega = \frac{d}{l}; \omega_z = \frac{\delta}{h}$; where δ, d – dimensions of the reinforcing fibers in height and width; h, l – element size in height and width. Layers of fiber-reinforced shell of constant cross section. Vector of nodal forces {R}, containing the load acting on the shell element

$$\{R\} = \int_0^{2\pi} \int_{s_i}^{s_{i+1}} \{u\}\{p\} ds d\varphi, \qquad \{p\} = \begin{Bmatrix} p_s \\ p_n \\ 0 \end{Bmatrix}, \tag{4}$$

where p_s provides a distributed load in the direction of the coordinate line s, p_n – distributed load perpendicular to the median plane of the shell.

After making the necessary changes is easy to compute $\{R\} = [k]\{g\}$, where κ – stiffness matrix:

$$[k] = \int_{-\frac{h_0}{2}}^{\frac{h_0}{2}} \int_0^{2\pi} \int_{s_i}^{s_{s+1}} \left(\varepsilon_s^0 \sigma_s^0 + \varepsilon_\varphi^0 \sigma_\varphi^0 + \kappa_s^0 \chi_s^0 + \kappa_\varphi^0 \chi_\varphi^0\right) dz ds d\varphi + \int_{\frac{h_0}{2}}^{\frac{h_0}{2}+h_1} \int_0^{2\pi} \int_{s_i}^{s_{s+1}} \left(\varepsilon_s^1 \sigma_s^1 + \varepsilon_\varphi^1 \sigma_\varphi^1 + \kappa_s^1 \chi_s^1 + \kappa_\varphi^1 \chi_\varphi^1\right) dz ds d\varphi. \tag{5}$$

After determining the displacement components may be calculated strain and stress tensors.

Cone discretized ten and thirty axisymmetric finite elements. The decision by the proposed algorithm was compared with the solution of [3]. The maximum error of solutions did not exceed 9,3 % when the number of elements equal to 10 and 5,4 % when the number of finite elements equal 30.

The advantage of the proposed mathematical model and methods of its use is the use of axisymmetric finite elements allowing for the investigated sample sheath apply fewer nodes than when using other types of elements.

References

1. Reddy, J. N. Mechanics of Laminated Composite Plates: Theory and Analysis, Second Edition – New York: CRC Press, 2004. – 856 p.
2. Zienkiewicz, O.C. Finite Element Method in Engineering Science. – London: McGraw-Hill Inc., 1972. – 432 p.
3. Andreev, A.N. Multi-layer anisotropic shell and plates: bending, resistance, fluctuations / A.N. Andreev, U.V. Nemirovsky. – Novosibirsk: Nauka, 2001. – 288 p.