ON THE PROPERTIES OF SOLUTIONS OF A NONLINEAR FILTRATION PROBLEM WITH A SOURCE AND MULTIPLE NONLINEARITIES

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In this paper, we studied the conditions for global solvability and unsolvability of a nonlinear filtration equation

$$\rho(x)\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left(\left|\frac{\partial u^m}{\partial x}\right|^{p-2}\frac{\partial u^m}{\partial x}\right)+\rho(x)u^{\beta},\qquad (x,t)\in R_+\times(0,+\infty) \tag{1}$$

with nonlinear boundary condition

$$-\left|\frac{\partial u^m}{\partial x}\right|^{p-2}\frac{\partial u^m}{\partial x}(0,t)=u^q(0,t),\qquad t>0 \tag{2}$$

and initial value condition

$$u(x,0)=u_0(x)\geq 0,\qquad x\in R_+ \tag{3}$$

where $p>1+1/m,\ \beta,q>0,\ \rho(x)=x^{-n},\ n\in R,\ u_0(x)$ – bounded, continuous, non-negative and non-trivial initial data.

Equation (1) occurs in various areas of natural science [1, 3–5]. For example, equation (1) is considered in mathematical modeling of the thermal conductivity of nanofluids, in the study of problems of fluid flow through porous media, in problems of the dynamics of biological populations, polytropic filtration, structure formation in synergetics and nanotechnologies, and in a number of other areas [1–4].

Equation (1) is called a parabolic equation with variable density [1] and in case $m(p-1)-1>0$ corresponds to the equation of slow filtration [2-3]. Problem (1)–(3) has been intensively studied by many authors (see [2, 6–17] and references therein) for various values of numerical parameters.

In [17], the authors, considering problem (1)-(3) in the case $m=1,\ \rho(x)=1$, proved that for $0<\beta\leq 1$ and $q\leq(p-1)(2-n)/(p-n)$ any non-trivial solution of problem (1)-(3) is global. If $\beta<1$ and $q>(p-1)(2-n)/(p-n)$, then each solution of problem (1)-(3) is unbounded in a finite time.

In work [5], the condition of global unsolvability in time of the solution of the Cauchy problem for equation (1) at was obtained that $m=1,\ \rho(x)=1$ and the critical exponent of the Fujita type $\beta = 2p-1$ was established.

Some properties of solutions to problem (1)–(3) at $\rho(x)=1,\ m=1$ were studied in [9]. They obtained the critical exponent of the global existence of the solution and the critical exponent of the Fujita type by constructing the lower and upper solutions.

In [7], the unboundedness of the solution of the following reaction-filtration model with a nonlinear boundary condition was studied

$$u_t = \Delta u^m + u^\beta, \quad (x,t) \in \Omega \times (0,T),$$

$$\frac{\partial u}{\partial \eta} = u^q, \quad (x,t) \in \partial\Omega \times (0,T),$$

$$u(x,0) = u_0(x), \quad x \in \Omega,$$

where $\Omega \in R^N$ is the bounded area. The authors showed that all positive solutions exist globally in the case $m>1$ if and only if $\beta, q \le 1$, and in the case $m \le 1$ when $\beta \le 1,\ q \le 2m/(m+1)$.

As is known, degenerate equations may not have classical solutions. Therefore, its solution is understood in a generalized sense.

Definition 1. A function is called a weak solution to problem (1)-(3) at $\Omega = \{R_+ \times (0,T)\}$, if $0 \le u(x,t) \in C(\Omega),\ \left|\frac{\partial u^m}{\partial x}\right|^{p-2} \frac{\partial u^m}{\partial x} \in C(\Omega)$, and if it satisfies (1)–(3) in a generalized sense at Ω, where $T>0$ is the maximum lifetime.

1. Main results

Below, we will determine the condition of solvability and unsolvability in general in terms of time for solving problem (1)–(3) in the case of slow filtration. It is assumed that $p > 1 + 1/m$.

Theorem 1. *If* $q \le \frac{m(1-n)+1}{p-n}(p-1)$ *and* $0<\beta\le 1$ *, then any solution to problem (1)–(3) is global.*

Remark 1. *Theorem 1 shows that the critical exponent of the global existence of a solution to problem (1)–(3) is equal to*

$$\left\{\beta=1,\ 0<q\le\frac{(2-n)(p-1)}{p-n}\right\}\cup\left\{q=\frac{(2-n)(p-1)}{p-n},\ \beta\le 1\right\}$$

Theorem 2. *If* $\beta<1$ *and* $q\ge\frac{(m(1-n)+1)(p-1)}{(p-n)}$*, then the solution of problem (1)–(3) is unbounded in a finite time.*

Theorem 3. *If* $\beta>2p-1$ *and* $q<(p-1)(m-mn+1)/(p-n)$*, then the solution of problem (1)–(3) is unbounded in a finite time.*

Theorems 1-3 are proved in the same way as in [13, 16].

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МОДЕЛЬ ФОРМИРОВАНИЯ ОПТИМАЛЬНОЙ ИНВЕСТИЦИОННОЙ ПРОГРАММЫ ПРИ ЗАДАННЫХ БЮДЖЕТЕ И ПРОГРАММЕ ПРОИЗВОДСТВА

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При вложении инвестиций в реальную экономику банкам и другим инвесторам целесообразно учитывать не только инвестиционную программу, но и финансовую, производственно-хозяйственную и социально-экономическую деятельность предприятию. Поэтому лицо принимающих решение (ЛПР) и его команде интересно исследовать