$$i(t) = 10(1 - e^{-1000t}) + 2e^{-1000t} = (10 - 8e^{-1000t})\text{А}.$$

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УДК 519.6

GOLDEN SECTION IN MATHEMATICS AND ITS APPLICATIONS IN ENGINEERING

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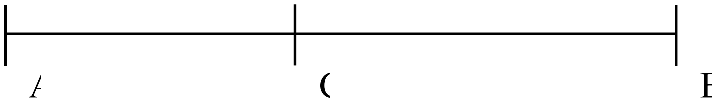
One of the most important and fascinating base concepts of Mathematics can be considered as the Golden Section (also known as golden ratio, golden mean or divine proportion) which is an irrational number holding the value $\frac{1+\sqrt{5}}{2} \approx 1.6180339887$and denoted by the Greek letter Φ.Because of thedistinctive and confounding properties of the Golden Section, not only researchersand mathematicians have studied about it, but also renaissance architects, designers and artists worked on it and incorporated the Golden section proportions indistinguished works of artifacts, sculptures, construction and paintings. The Golden Ratio is contemplated as the mostenchantingto humans' visual perception and not limited to aesthetic beauty but also be found its existence in natural world through the bodyproportions of living beings, the growth patterns of many plants, insects, mathematical series, geometrical patterns and much more.

The earliest documented reference to the Golden Section is found in the book "Elements" written around 300BCE by the prominent Greek mathematician Euclid to solve a geometrical problem.This was called the problem of division of a line segment in extreme and mean ratio. The gist of the problem is the following. A line segment AB must be divided with a point C into two parts such that the ratio between the longer segment CB and the shorter segment AC is equal to the ratio between the whole line segment AB and the longer part CB;



$$\frac{AB}{CB} = \frac{CB}{AC} \quad \text{...(1.0)}$$

Considering the relationship $AC + CB = AB$, the equation 1.0 can be rewritten as

$$x = \frac{CB}{AC} = \frac{AB}{CB} = \frac{AC+CB}{CB} = 1 + \frac{AC}{CB} = 1 + \frac{1}{x} \quad \text{........................(1.1)}$$

Hence equation to calculate the ratio x is given by:

$$x^2 - x - 1 = 0 \quad \text{...(1.2)}$$

The positive root of the equation (1.2) is the solution for the problem of division of a line segment in extreme and mean ratio and it is the Golden ratio :

$$\Phi = \frac{1+\sqrt{5}}{2} \approx 1.618 \quad \text{..(1.3)}$$

The Fibonacci sequence comply to the divine proportion as a ratio of the two preceding elements of the sequence itself. The Fibonacci sequence was introduced in 1202 with the publication of famous rabbit puzzle by Leonardo of Pisa who was later nicknamed as Fibonacci.The Fibonacci sequence is a series containing numbers in which each number is the sum of the two that precede it except the first two elements. Starting at $F_0 = 0$ and $F_1 = F_2 = 1$, the Fibonacci numbers are distinctivelycalculated from their recursion relation;

$$F_{n+1} = F_n + F_{n-1} \quad \text{..(1.4)}$$

The Binet's Formula which is a comprehensive formula for the Fibonacci numbers can be calculated using the correlation between Golden Section and Fibonacci sequence.

In order to express (1.4) for Fibonacci numbers, the equation (1.5) is used;

$$x_{n+1} = x_n + x_{n-1} \quad \text{..(1.5)}$$

This equation is a second-order, linear, homogeneous difference equation with constant coefficients, and its method of explication follows that of the analogous differential equation. The idea is to hypothesize the general form of a solution in order to find two solutions and then is multiplied by these solutions by unknown constants and add them. This leads to a general solution to (1.5), and then solved (1.4) by satisfying the specified initial values. Starting with the postulated solution to (1.5) as:

$$x_n = \lambda^n \quad \text{...(1.6)}$$

where λ is an unknown constant. Substitution of this estimate into (1.4) outputs

$\lambda^{n+1} = \lambda^{n} + \lambda^{n-1}$……………………………………………………(1.7)

Division by λ^{n+1} and rearranging gives;

$\lambda^{2} - \lambda - 1 = 0$……………………………………………………………..(1.8)

The quadratic formula used to receive two roots;

$\lambda_1 = \frac{1+\sqrt{5}}{2} = \Phi\ , \lambda_2 = \frac{1-\sqrt{5}}{2} = -\phi$……………………………………(1.9)

where Φ is the Golden Section and ϕ is the conjugate of Golden section.

Multiplication of solutions by constants, brings about

$F_n = c_1\Phi^{n} + c_2(-\phi)^{n}$…………………………………………………(2.0)

which must satisfy the elementary values $F_1 = F_2 = 1$. The unknown constants can be found simply by using the value$F_0 = F_2 - F_1 = 0$.

Application of these values results;

$c_1 + c_2 = 0$……………………………………………………………(2.1)

$c_1\Phi - c_2\phi = 1$……………………………………………………………..(2.2)

Using the equation (2.1), $c_2 = -c_1$ and substitution brings;

$c_1(\Phi + \phi) = 1$…………………………………………………………...(2.3)

Since $\Phi + \phi = \sqrt{5}$, obtained solution for $c_1$and c_2;

$c_1 = 1/\sqrt{5},\ c_2 = -1/\sqrt{5}$ ………………………………………………(2.4)

Manipulating (2.4) and (2.0), derives the Binet's formula.

$F_n = \frac{\Phi^{n} - (-\phi)^{-n}}{\sqrt{5}}$……………………………………………………………(2.5)

The Golden Section is a determining parameter in the stress analysis of beams which is a crucial application in engineering. Considering the simple beam in figure 1, the stress analysis of the beam results in the plane stress condition as illustrated in figure 2.

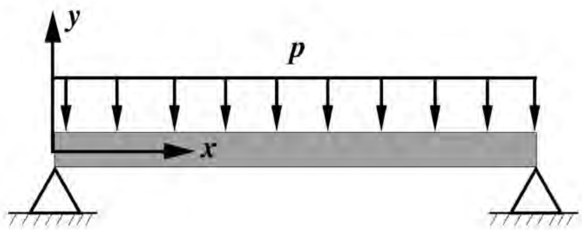
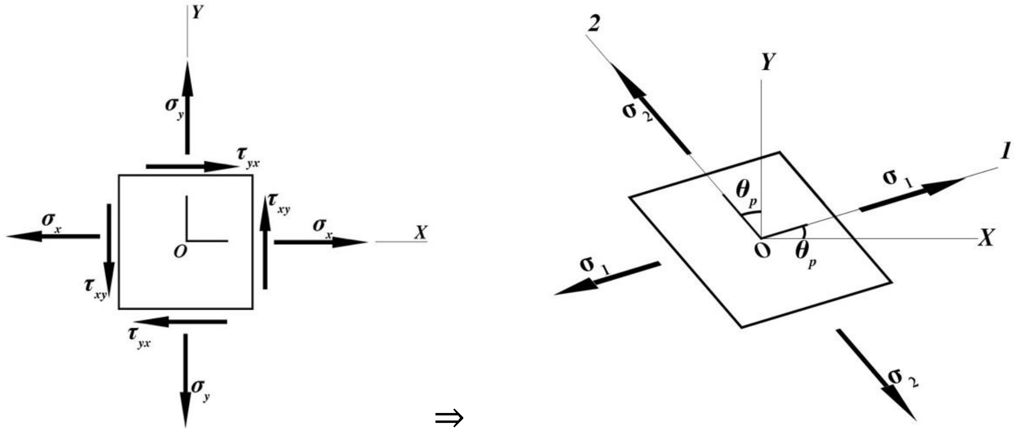


Fig.1



Stresses in given coordinate

Principal Stresses

Fig.2

The normal stresses (σ_x and σ_y) and shear stresses (τ_{xy} and τ_{yx}) are acting on the simple beam as illustrated in the figure for a given x-y coordinate system. The principal stresses σ_1 and σ_2(critical in stress analysis) are also illustrated in figure (2).

The principal stresses σ_1 and σ_2 can be expresses as;

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} \text{...(2.6)}$$

Shear stress can be expressed as;

$$\tau_{max/min} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} \text{..(2.7)}$$

In general, the beam behavior implies flexural and shearing actions in which σ_y isnegligible when compared to the other stresses acting on the beam. That is $\sigma_y = 0$ for all practical purposes. Furthermore, using the specific condition in which$\sigma_x = \tau_{xy}$, (and as mentioned earlier $\sigma_y = 0$) the followingrelationships for the principal stresses are obtained;

$$\sigma_{1,2} = \sigma_x\left(1 \pm \sqrt{5}\right) \text{...(2.8)}$$

and

$$\sigma_1 = \sigma_x(1.61803) = \sigma_x(\Phi) \text{ tension.................................(2.9)}$$

$$\sigma_2 = \sigma_x(-\,0.61803) = \sigma_x\left(-\frac{1}{\Phi}\right)\text{compression}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.0)$$

The maximum shear stress can be expressed as;

$$\tau_{max} = \sigma_x\left(\frac{\sqrt{5}}{2}\right), \text{ where } \sqrt{5} = \frac{1+\Phi^2}{\Phi}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.1)$$

Therefore, it is evident that the golden ratio is a crucial determining parameter in stress analysis of beams.

Over the past few years, with the extensive old city renovations, the construction of infrastructure and the reconstruction and extension of mine factories, blasting engineering has become a pivotal application. Due to these implications,the degree of difficulty and the scope of blasting projects have also escalatedrequiring the technology used and quality to besuperior. Nevertheless, due to the complexity of having to deal with large numbers of equations, some blasting technicians easily tend to get confused during the designing stage.Observing the development of blasting technology in China, the forerunners have derived some essential equations based on the engineering applications and blasting mechanisms, providing a concrete base for technicians to implement blasting design and construction.Furthermore, the effective results differ from the ones estimated from some individual equations, which doesn't give a clear guidance to the design and construction process, making it even more complicated to ensure the safety of blasting projects.In order to conquer many of these complications and to provide a comprehensible guidance in construction and design, golden section method is used in blasting engineering.

For engineering practices done over a span of 20 years, it has proved that an optimized blasting effect can be obtained when the charge length is 0.618 (this value is closely linked to the golden ratio) times the blast hole depth. Also,itwas evident that administrating golden section in defining the length of charge can greatly diminish the toe rate and block rate, and improve the blasting effect which was concluded after comparing the corresponding block rate and toe rate of blasting by varying different charging heights in numerous deep hole blasting tests in various scaled explosion regions in diverse environments.

Surface blasting which deals with mine stripping, hydropower engineering and cutting excavations, administrates the golden section method in designing process and to analyze blasting parameters in order to obtain an optimized effective result.

Moreover, to facilitate the efficiency of mining equipments and to reduce the time taken for excavation procedures, this golden section method is employed in the determination of the rational charge construction of the trenching zone and the optimal location for the first blasting region. In addition, finding the rational length and reserved length of blasting notch for demolition construction, golden section method is applied which proved satisfying results after a number of high-rise tubular building blasting.

Engineering is also a profession which incorporates many other fields including the designing aspect belonging to architecture and aesthetics. For better perception of a building, a product or even a logo which is introduced, having used the golden section brings about an unprecedented level of perception among the receivers. The key fact for this is that the golden ratio is acknowledged for its capability in providing a sense of aesthetic appeal in beauty, balance and harmony of design. Nowadays, it is even used to style and design of everyday consumer products. Both engineers and architects make blueprints incorporating the golden ratio, in order to get an eye catching and an optimized design. Graphic designers use the golden section in logo designing and photo creations. The web developers use this concept in building astounding websites. The design of violin, guitars and high-quality speaker wires also uses the golden section as a base. One of the main tech giants of the present day, Apple uses golden ratio in their product designs. Furthermore, Golden Ratio is used in designing product logoswhichcomprisesof an image that should leave a memorable impact on the subconscious and conscious minds of consumers.

In contrast, the golden section is one of the most useful constants in mathematics used from ancient times to the present. Many people including engineers, architects and artists explored this concept in order to bring about a revolutionary change in each field. Due to that fact, even to this day the usage of golden section is immensely evolving. In this article the application of golden section in engineering is utilized.

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УДК 519.6

ЗАДАНИЕ ДВИЖЕНИЯ МАНИПУЛЯТОРА ПРИ ПОМОЩИ ВЕКТОРОВ

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Аннотация: в данной работе рассматриваются общие сведения о роботах типа манипулятор, а также способ задания движения манипулятора методом векторов.

Ключевые слова: манипулятор, движение манипулятора, глобальная система координат, …

Введение

В наши дни существует множество различных конструкций роботов, и в этой работе мы разберем робот конструкции “манипулятор”.

Но как же определяется положение манипулятора в пространстве, как задать его движение и как подвести захват к нужной точке? В этой работе мы разберем все это, и приведем пример математических вычислений.

В качестве образца будем использовать робот манипулятор HIWIN Ra-605. Соответственно, далее мы будем ссылаться именно на эту модель.

Рабочее пространство манипулятора.

Рассматриваемый манипулятор, ввиду свой конструкции, имеет весьма специфическое рабочее пространство (рисунок 1):