

## Wye-connected current transformers simplified model validation in MATLAB-Simulink

**Abstract.** This paper presents the wye-connected current transformers (CTs) simplified mathematical model with the initial data based on nominal parameters (nameplate data) of the simulated CT. Designed user-friendly graphical interface allows to specify the CT data (nominal parameters, remanence and burdens) in convenient form. Results of the carried out experiments confirmed the CT model validity.

**Streszczenie.** Niniejszy artykuł przedstawia uproszczony model matematyczny układu przekładników prądowych połączonych w gwiazdę, w którym dane źródłowe oparte zostały o parametry znamionowe (tabliczkę znamionową) symulowanych przekładników. Przyjazny dla użytkownika interfejs graficzny pozwala na określenie danych przekładników prądowych (parametry znamionowe, magnetyzm szczątkowy, obciążenia) w wygodny sposób. Wyniki przeprowadzonych eksperymentów potwierdziły poprawność modelu przekładnika prądowego (**Weryfikacja eksperymentalna uproszczonego modelu połączenia gwiazdowego przekładników prądowych w programie MATLAB-Simulink**).

**Keywords:** current transformer model, relay protection, magnetization curve, saturation, Simulink, Matlab, SimPowerSystems.

**Słowa kluczowe:** model przekładnika prądowego, przekaźniki zabezpieczeniowe, krzywa magnesowania, nasycenie, Simulink, Matlab, SimPowerSystems.

### Introduction

Information sources about the currents of the protected object are current transformers (CTs), which secondary currents are transmitted to the digital relay protection analog inputs. Because of CT core saturation in emergency mode secondary current waveform may substantially differ from the primary current waveform. To evaluate the digital relay behavior nowadays are widely used test sets which allow to deliver current signals (that are equivalent in form and capacity of the CT secondary currents) to the digital relay analog inputs. One way to obtain such data signals is the mathematical modeling method. It allows with the help of computer simulation using protected object and measuring CT mathematical models to obtain CTs instantaneous primary and secondary currents waveforms in the different operational modes.

Paper [1] describes the wye-connected current transformers mathematical model implementation technique in MATLAB-Simulink environment using SimPowerSystems component library. This library contains many electrical power components but it lacks a CT model [2]. This circumstance greatly restricts the Simulink-SimPowerSystems environment usage for digital protection and automation devices modeling for which the CT is the major information source about the protected object operational mode.

The purpose of this paper is to validate the adequacy of the wye-connected current transformers mathematical model with the initial data based on nominal parameters (nameplate data) of the simulated CT. Obtained during computer simulation CT secondary currents can further be transmitted to the digital relay protection analog inputs with the help of test set (e.g. Omicron) or used as input signal for digital relay protection models developed in Matlab-Simulink environment or another dynamic simulation system.

### The wye-connected current transformers mathematical model

The major issue during CT modeling is the absence in the publicly available sources a CT core dimension: core area and mean magnetic path length. More often than not are known the following CT parameters: accuracy class, rated accuracy limit factor (the ratio between the nominal overcurrent and the primary rated current), rated resistive secondary burden and the CT ratio. Another two important characteristics – the secondary winding resistance and the excitation curve are also often unknown.

Electrical steel magnetization curve does not depend on CT core dimension and CT nominal parameters. It's represents only the steel property, whereas CT excitation curve depends on these parameters. Magnetization curves of the electrical steels using for CT cores production in general case are similar. Therefore, for a simplified CT model it can be assumed that all the CT cores are made of the same steel grade and have the same magnetization curve. As such characteristic assumed the typical average magnetization curve provided in [3] (fragment of this curve is shown in Fig.1).

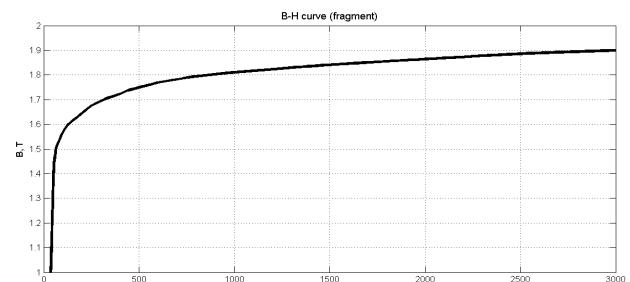


Fig.1. Typical magnetization curve [3]

If assume that the load connected to the secondary windings and neutral conductor of the wye-connected CTs (Fig.2) consists only of a digital protection device and copper cables then the cumulative inductance of these loads ( $L_{b(A,B,C)}$ ,  $L_0$ ) because of their little value can be disregarded. Assuming that the secondary winding is fully distributed along the core perimeter it can also be disregarded secondary winding inductance ( $L_{ct}$ ).

The wye-connected CTs mathematical model is based on the single current transformer mathematical model [4]. The wye-connected CTs mathematical model contains three single CT equations and constructed in accordance with the Fig.2.

$$(1) \quad \begin{cases} \frac{dB_j}{dt} = \frac{1}{w_2 s_c} [(R_{ct} + R_{bj}) \cdot i_{2j} + R_0 \cdot i_0], & j = A, B, C; \\ i_{2j} = \frac{w_1}{w_2} \cdot i_{1j} - \frac{l_c}{w_2} \cdot H_j; \\ B_{mj} = f(H_j); \\ i_0 = i_{2A} + i_{2B} + i_{2C}. \end{cases}$$

where:  $R_{ct}$  – secondary winding resistance,  $R_b$  – actual resistive secondary burden,  $s_c$  – core area,  $R_0$  – neutral conductor active resistance,  $l_c$  – mean magnetic path length,  $w_1$  – primary turns,  $w_2$  – secondary turns,  $B_m = f(H)$  – relation between magnetic induction  $B$  and magnetic field strength  $H$  (at nominal frequency 50 Hz),  $i_1$  – instantaneous primary current,  $i_2$  – instantaneous secondary current,  $i_0$  – instantaneous current flowing in neutral conductor.

Wye-connected current transformers mathematical model implementation in Simulink-SimPowerSystems environment with known core dimension presented in [1]. The equation system (1) includes next unknown constants:  $w_1$ ,  $w_2$ ,  $s_c$ ,  $l_c$ ,  $R_{ct}$ , but it should, however, be noted that  $w_1$  typically equal 1 or 2.

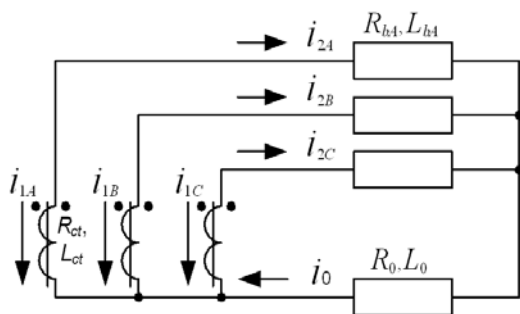


Fig.2. Wye-connected current transformers

After expressing unknown constants in terms of CT nominal parameters (nameplate data) and adopted magnetization curve the equation system which describes the wye-connected current transformers simplified model appears as follows (for the CT with a secondary rated current 5 A, winding accuracy class – 10P):

$$(2) \begin{cases} \frac{dB_j}{dt} = \frac{8334 \cdot [0.0004 I_{1n} + R_{bj}] \cdot i_{2j} + R_0 \cdot i_0}{ALF \cdot (0.0004 I_{1n} + R_{bj})}, & j = A, B, C, \\ i_{2j} = \frac{5 \cdot i_{1j}}{I_{1n}} - 2.25 \cdot 10^{-4} \cdot ALF \cdot H_j; \\ B_{mj} = f(H_j); \\ i_0 = i_{2A} + i_{2B} + i_{2C}. \end{cases}$$

where:  $I_{1n}$  – CT primary rated current,  $ALF$  – rated accuracy limit factor,  $R_n$  – rated resistive secondary burden.

### Model implementation in Matlab-Simulink

Based on (2) in MATLAB-Simulink environment the wye-connected CTs simplified mathematical model was implemented (CT block in Fig.4). Fig.3 depicts model parameters settings.

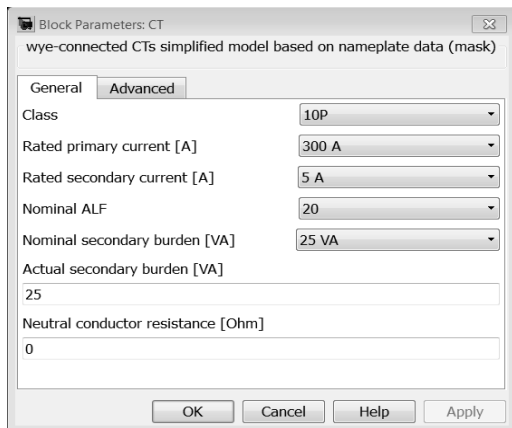


Fig.3. Model graphical interface

A structural model for wye-connected current transformers simplified model investigation was created in MATLAB-Simulink environment using SimPowerSystems component library (Fig. 4).

The structural model contains the following elements: Eq. system – three-phase equivalent supply system; CB – three-phase circuit breaker; CT – investigated wye-connected CTs simplified model (discussed in this paper); I2a, I2b, I2c, In – virtual oscilloscopes for fixing the secondary current waveform; Three-Phase-Fault – three-phase short circuitor.

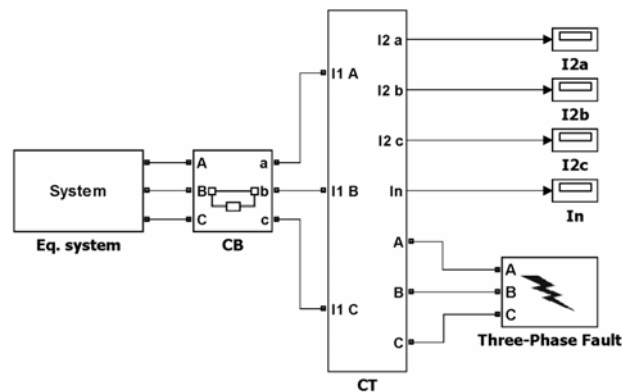


Fig.4. A structural model in MATLAB-Simulink

Consider next the model operation in the modes that are critical in terms of the protection relay functioning. All shown below waveforms are referred to a CT located in phase A. In all cases a three-phase fault was considered. The neutral conductor resistance is set to zero (see Fig. 3).

### Computational experiments

Consider the CTs operation in case of primary circuit current 13 times exceeds the CTs primary rated current. It should be noted that hereinafter, the CTs actual resistive secondary burden taken equal to the rated resistive secondary burden and relative short-circuit current (in relation to CTs primary rated current) provided for steady-state short-circuit currents. The CTs nominal parameters are the following: accuracy class – 10P;  $ALF = 10$ ;  $R_n = 0.6 \Omega$  (= 15 VA); CT ratio – 600/5 A. The obtained waveform is shown in Fig.5. As expected, distorted secondary current waveform is obtained, the error is 35%.

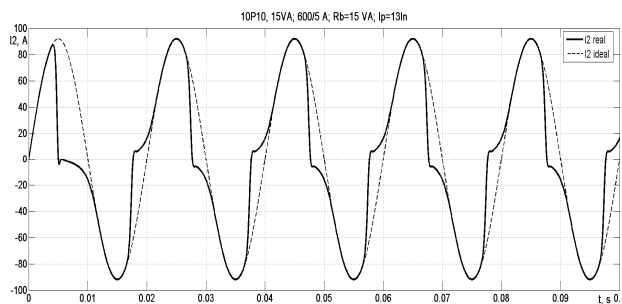


Fig.5. CTs secondary current waveform

Consider the CTs operation in case of presence a DC component in the flowing short-circuit current. CTs parameters are equal to the previous case, apart from the CT ratio, in this example it is accepted 1000/5 A. The primary time constant ( $T_a$ ) is equal 0.05 s. Primary short-circuit current 10 times exceeds the CTs rated primary current (in other words it is equal to  $ALF$ ). Fig.6 shows the secondary current waveform (solid line) and the CTs error in % (dashed line).

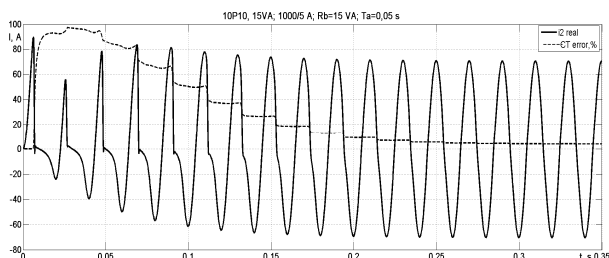


Fig.6. CTs secondary current waveform in case of a DC component presence in the short-circuit current

Due to presence of a DC component in the short-circuit current the secondary current waveform is significantly distorted in the first few periods after the fault inception and the establishment of the error margin (10%) takes about 0.23 s. Fig.7 shows the CT core magnetic induction waveform for the described above case. As can be seen from this figure, presence of a DC component in the short-circuit current results in the single-sided CT core magnetization during the first few periods. After a DC component decays to zero (0.23 s) the magnetic induction waveform becomes sinusoidal.

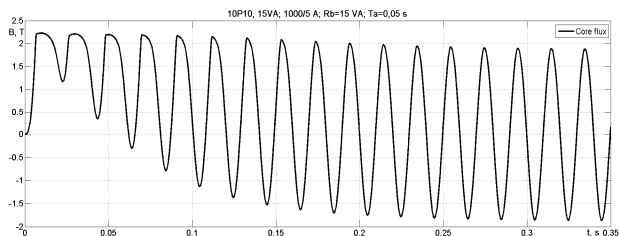


Fig.7. CT core magnetic induction waveform

CT core remanent magnetic induction strongly affects the CT operation during its energizing: depending on the sign increasing or decreasing the current transformation error. Fig. 8 shows the CT secondary current waveform in cases with the different remanent magnetic induction: 0 T and 1 T. In the first case (Fig. 8 a) the CT core is completely demagnetized and its saturation occurs at 0.027 s; in the second case (Fig. 8 b) the CT core saturation occurs at 0.0075 s.

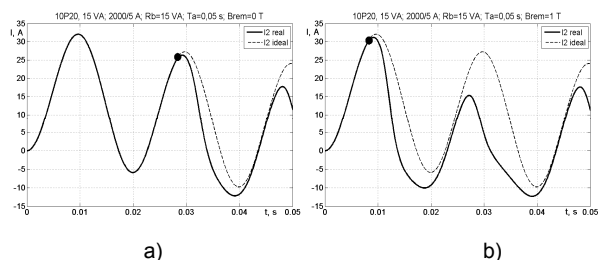


Fig.8. CTs secondary current waveform in case of different remanent magnetic induction: 0 T (a) and 1 T (b)

For a more sophisticated treatment of the proposed simplified model it was compared with a wye-connected current transformers model which considers the core dimension. Such CTs model is included in the comprehensive mathematical model of the transmission line (Fortran-based software package) [4, 5], which comprises mathematical description of the next components: supply system, power transformer, transmission line, current and voltage transformers, as well as the additional electrotechnical devices which have a significant quantitative and qualitative impact on the emergency and post-emergency modes. Fig.9 depicts the graphical interface of the discussed software package.

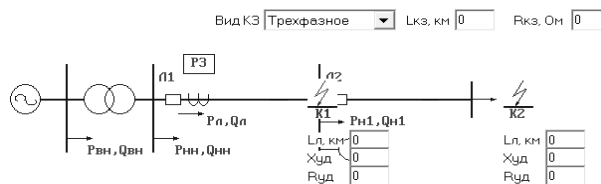


Fig.9. Software package graphical interface

The software package is designed to generate the input current signals for the 6-10 (35) kV transmission line overcurrent protection. Due to accounting a nonlinear magnetization characteristic of the power and current transformers software package allows to obtain closest to the real secondary current waveforms during fault condition.

In MATLAB-Simulink environment using SimPowerSystems component library was created a power network fragment which is in structure similar to Fig. 9 (software package). Power network fragment is shown in Fig. 10.

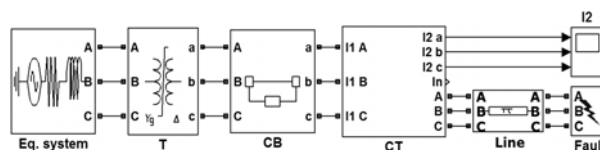


Fig.10. Power network fragment in SimPowerSystems

Network elements (equivalent system, power and current transformers, transmission line) parameters in the software package and in the Simulink-SimPowerSystems environment were set identical. The only difference is in the CTs model. Three-phase transformer parameters calculation for the Simulink-Simpowersystems network fragment was carried out in accordance with [6].

For investigation was chosen the ТПЛ-10 type CT with the following nominal parameters and core dimension [3]: CT ratio – 400/5 A, accuracy class – 10P, ALF = 13;  $R_n = 0.6 \Omega$  (= 15 VA),  $R_{ct} = 0.28 \Omega$ ,  $s_c = 0.00112 \text{ m}^2$ ,  $l_c = 0.48 \text{ m}$ ,  $w_1 = 2$ ,  $w_2 = 159$ ,  $X_{ct} = 0.1 \Omega$  (secondary winding reactance). These parameters have been set for the wye-connected CTs model in the software package. The simplified model in MATLAB-Simulink environment requires only the following parameters: CT ratio – 400/5 A, accuracy class – 10P, ALF = 13 and  $R_n = 0.6 \Omega$  (= 15 VA). For both models the actual resistive secondary burden was set to  $0.2 \Omega$  (= 5 VA) and the neutral conductor resistance was set to  $0.2 \Omega$ . The three-phase fault at the end of the line was simulated. Since the length of the transmission line in both models was assumed equal to 0, then simulated fault can be considered as short-circuit fault on the CTs secondary terminals. Obtained during simulation CTs secondary waveforms are presented in Fig.11 (solid line – MATLAB-Simulink model, dashed line – software package).

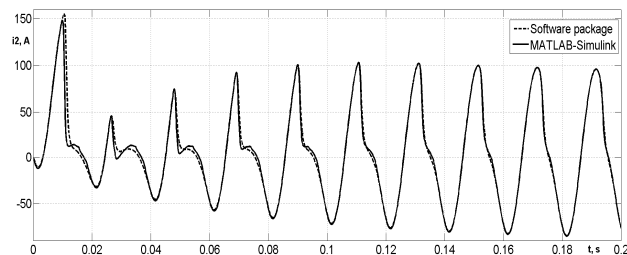


Fig.11. Compared models secondary current waveforms

As can be seen from the Fig.11 the close agreement between obtained waveforms is observed. This fact confirms the wye-connected CTs simplified model validity.

### Conclusion

1. Wye-connected CTs simplified mathematical model with the initial data based on nominal parameters (nameplate data) of the simulated CT is presented.
2. In MATLAB-Simulink environment the wye-connected CTs simplified model was implemented.
3. Results of the carried out experiments confirmed the wye-connected CTs simplified model validity.

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