# Dynamic Features of Spectra of Single and Quasi-Periodic Measuring Signals

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*Received 28.01.2022
Accepted for publication 30.05.2022*

**Abstract**

Solving the problems of spectral processing of single and quasi-periodic signals in measurement and diagnostic systems is directly related to their isolation against the background of external interference or noise. The purpose of this work was to study single and quasi-periodic signals, i. e. signals limited in time, presented as separate components; development of a mathematical apparatus that connects the individual components of the original, time-limited signal, with the spectral characteristics of the periodic signal, which is obtained from the original by its periodization.

The paper analyzes the spectrum of a quasi-periodic signal, which is presented from spectral density regions separated by spectral components with zero amplitude. The process of signal periodization is considered on the example of unipolar rectangular pulses. The representation of the analyzed complex single signal in the form of a linear combination of given functions, limited in time by the duration of the considered signal, was chosen, and it was determined that it is most logical and efficient to use radio-frequency pulses. The spectral density of the signal under consideration is presented as the sum of the spectral densities of radio-frequency pulses of the same width with a varying carrier frequency. The original signal is presented as the sum of the constituent components (radio-frequency pulses), which form a time-limited frequency spectrum – a quastr. As a result, the correlation of the considered quasi-periodic signal with the parameters of the periodic signal (amplitude, period, and initial phase) is shown.

A format for representing time-limited signals in the form of components related to the spectral characteristics of a periodic signal, obtained from the original signal by periodization, has been developed. The formed mathematical apparatus allows simplifying the algorithmic support of measuring systems by eliminating the correlation signal processing.

**Keywords:** quasi-periodic measuring signals, spectrum harmonics, spectral density, decomposition basis.

**DOI:** 10.21122/2220-9506-2022-13-2-128-138

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***Для цитирования:***
*U.V. Suchodolov, A.V. Isaev, A.A. Sheinikau.*
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Приборы и методы измерений.
2022. – Т. 13, № 2. – С. 128–138.
**DOI:** 10.21122/2220-9506-2022-13-2-128-138

***For citation:***
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2022, vol. 13, no. 2, pp. 128–138.
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# Спектральное представление измерительных одиночных и квазипериодических сигналов

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*Поступила 28.01.2022*
*Принята к печати 30.05.2022*

Решение задач по спектральной обработке одиночных и квазипериодических сигналов в системах измерения и диагностики непосредственно связанно с их выделением на фоне внешних помех или шумов. Целью работы являлось исследование одиночных и квазипериодических сигналов, т. е. сигналов, ограниченных во времени, представленных в виде отдельных компонент; разработка математического аппарата, связывающего отдельные компоненты исходного, ограниченного во времени сигнала, со спектральными характеристиками периодического, который получен из исходного путём его периодизации.

В работе проведён анализ спектра квазипериодического сигнала, который представлен из участков спектральной плотности, разделённых спектральными составляющими с нулевой амплитудой. Рассмотрен процесс периодизации сигнала на примере однополярных прямоугольных импульсов. Выбрано представление анализируемого сложного одиночного сигнала в виде линейной комбинации заданных функций, ограниченных по времени длительностью рассматриваемого сигнала. Определено, что наиболее логично и эффективно в качестве линейной комбинации заданных функций использовать радиоимпульсы. Представлена спектральная плотность исследуемого сигнала в виде суммы спектральных плотностей радиоимпульсов той же длительности с изменяющейся несущей частотой. Исходный сигнал представлен как сумма составляющих компонент (радиоимпульсов), которые формируют ограниченный во времени частотный спектр – квастр. В результате показана корреляция рассматриваемого квазипериодического сигнала с параметрами периодического сигнала (амплитудой, периодом и начальной фазой).

Разработан формат представления ограниченных во времени сигналов в виде компонент, связанных со спектральными характеристиками периодического сигнала, который получен из исходного путём его периодизации. Сформированный математический аппарат позволяет упростить алгоритмическое обеспечение измерительных систем за счёт исключения корреляционной обработки сигнала.

**Ключевые слова:** квазипериодический измерительный сигнал, гармонические составляющие спектра, спектральная плотность, базис разложения.

**DOI:** 10.21122/2220-9506-2022-13-2-128-138

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## Introduction

Measuring signals are the main physical quantities parameters information carriers in measuring systems. In case of this signal being random, the aggregate of individual instantaneous values of its main parameter is considered to be information. Pulse measuring systems most often use the frequency-domain parameters of the signal. However, the spectral method is used to analyze the measuring signals most often in both cases.

According to the Fourier transform theory, the representation of any signal in the frequency domain is valid only when it meets the Dirichlet conditions, i. e. the signal must start at infinity and end at infinity, and, therefore, it must be stable over the entire time interval in which its spectral characteristic will not be a function of time [1]. This condition is not feasible for the analysis of real quasi-periodic processes and signals. Usually in this case window Fourier transforms [2] or wavelet transforms [3, 4] are used. However, the wavelet transform theory also contains a number of simplifications that use orthogonal basis systems, which, in turn, are mathematical abstractions.

Distortion in the operation of individual elements of various measuring and diagnostic systems leads to the formation of additional non-normalized quasi-periodic signal structures [5]. Therefore, when developing algorithms to automatically remove interference from a useful signal, the main condition is to establish an accurate dependence between local signal disturbances and changes in the values of individual components of its spectrum. It was shown in [6] that changes in the values of signal parameters lead to a significant violation of the initial distribution of spectral components, some of the latter undergoing more significant changes than others. Therefore, to increase measurement accuracy one can replace the low-sensitivity registration of the signal temporal parameters changes with the registration of the characteristic spectral components parameters changes, the latter being more sensitive to deviations of the measured parameter and less sensitive to deviations that are due to the instability of the elements in the measuring systems.

Having taken recent development of modern computing base into account, using a hardware-mathematical approach to study of quasi-periodic signals is most efficient, since by controlling individual parameters of the signal under study (amplitude, pulse-repetition interval, pulse duration, and other time parameters), it is possible to indirectly control the parameters of its spectrum. One also can distinguish useful signal from interference using the numbers of the necessary spectral components. In this case to select the numbers of spectral components with minimum measuring signal parameters instability sensitivity one must study the dynamic features of measuring signal spectrum.

The purpose of this work was to develop a mathematical apparatus that allows one to represent single and quasi-periodic time-limited signals as a set of spectral components, the basis of which are impulses, parameters of said impulses being determined by conditional periodization, to provide a unified approach to describing the spectra of these signals over finite time intervals.

## Main part

Quasi-periodic signals, which occupy an intermediate position between periodic and non-periodic signals, are among the most common signals used in measurement and diagnostic systems, said signals reflect the process of transforming spectral density into a discrete spectrum. Their main feature is that they are time-limited [7, 8]. Therefore, given the different approach to describing the spectra of periodic and non-periodic signals, this creates difficulties when considering quasi-periodic signals. At the same time, modeling the discrete spectrum formation process from the spectral density provides a general description for both types of spectra. This problem can be solved in two ways. The first one considers the process of a discrete spectrum formation from the spectral density during signal periodization. However, a spectrum-limited signal is a signal that is infinite in time. Consequently, when sampling such a signal, an infinite number of samples will be obtained. Then, to restore the original signal (including obtaining its discrete spectrum), it is necessary to take into account all the readings, which is impossible due to its unlimited duration [7].

The second method involves obtaining a periodic sequence of signals, which is the sum of individual signals delayed relative to each other in time [1]. However, such treatment of a real signal as a periodic one leads to an error due to the finite duration of the measurement process. And in this case, it is necessary to determine the influence of the discrete spectrum formation error that depends on the number of repetitions in the periodization process. Thus, with an increase in the number of pulses more and more zeros appear in the spectral density of the signal during periodization [1]. Taking into account that each zero of the spectral density is at a strictly defined frequency and has a strictly defined and measurable zero amplitude at the same time, it can be assumed that it is the spectral component of the discrete spectrum, which was formed as a result of full periodization, and it is a part of a continuous spectrum at the same time. This representation is the main connecting component. Therefore, the spectrum of the quasi-periodic signal itself can be considered combined, i. e. consisting of spectral density regions separated by spectral components with zero amplitude. For example, consider the process of periodization of a signal represented as unipolar rectangular pulses (Figure 1).

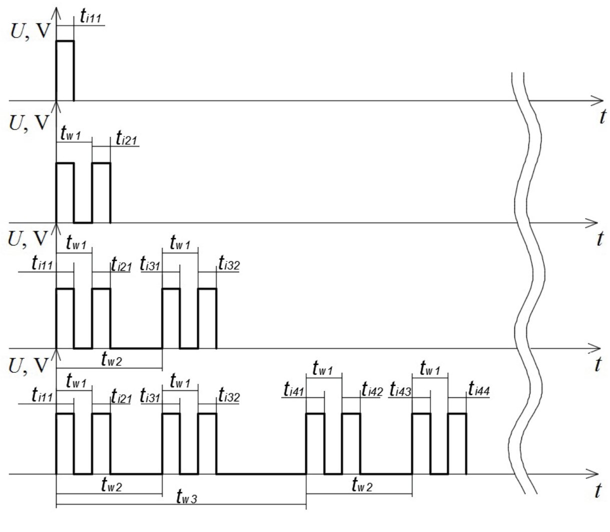



**Figure 1** – Periodization of rectangular pulses

Let's set the following conditions:

$t_{i11} = t_{i12} = t_{i21} = t_{i22} = t_{ixx} = t_i$.

Then, according to [9], the complex spectral density of such a sequence is defined as the spectral densities sum of the pulses, that are presented in the sequence. The equation for the first pulse in the sequence can be written as:

$$S_{11}(\omega)=\frac{E}{\pi j}(1-e^{-j\omega t_i}), \quad (1)$$

where $E$ is pulse amplitude; $t_i$ is pulse duration; ω is current frequency of the rectangular pulse sequence.

Accordingly, the equation for the second pulse is:

$$S_{12}(\omega)=\frac{E}{\pi j}(1-e^{-j\omega t_i})e^{-j\omega t_{w1}}, \quad (2)$$

where $t_{w1}$ is delay between two pulses.

The expression for the sequence of these two pulses is:

$$S_{p1}(\omega)=S_{11}(\omega)+S_{12}(\omega)=\frac{E}{\pi j}(1-e^{-j\omega t_i})+\frac{E}{\pi j}(1-e^{-j\omega t_i})e^{-j\omega t_{w1}}=$$
$$=\frac{E}{\pi j}(1-e^{-j\omega t_i})(1+e^{-j\omega t_{w1}}). \quad (3)$$

According to the displacement theorem [9], for the second same pair of pulses with numbers 3 and 4, the equation will look like:

$$S_{p2}(\omega)=S_{21}(\omega)+S_{22(\omega)}=\frac{E}{\pi j}(1-e^{-j\omega t_i})e^{-j\omega t_{w2}}+$$
$$+\frac{E}{\pi j}(1-e^{-j\omega t_i})e^{-j\omega t_{w2}}e^{-jn\omega t_{w1}}= \quad (4)$$
$$=\frac{E}{\pi j}(1-e^{-j\omega t_i})(1+e^{-j\omega t_{w1}})e^{-j\omega t_{w2}},$$

where $t_{w2}$ is delay between double pulses.

Then the complex spectral density of the sum of two identical double pulses has the form:

$$S_{pp1}(\omega)=S_{p1}(\omega)+S_{p2}(\omega)=\frac{E}{\pi j}(1-e^{-j\omega t_i})(1+e^{-j\omega t_{w1}})+$$
$$+\frac{E}{\pi j}(1-e^{-j\omega t_i})(1+e^{-j\omega t_{w1}})e^{-j\omega t_{w2}}= \quad (5)$$
$$=\frac{E}{\pi j}(1-e^{-j\omega t_i})(1+e^{-j\omega t_{z1}})(1+e^{-j\omega t_{z2}}).$$

Transforming this expression, we will have:

$$S_{pp1}(\omega)=$$
$$=\frac{8E}{\pi j}(\sin\frac{\omega t_i}{2})(\cos\frac{\omega t_{z1}}{2})(\cos\frac{\omega t_{z2}}{2})e^{-j\omega\frac{t_i}{2}}e^{-j\omega\frac{t_{z1}}{2}}e^{-j\omega\frac{t_{z2}}{2}}. \quad (6)$$

Repeating the resulting impulse combination after a delay time $t_{w3}$ and taking into account the displacement theorem, we obtain an expression for its complex amplitude:

$$S_{pp2}(\omega)=\frac{E}{\pi j}(1-e^{-j\omega t_i})(1+e^{-j\omega t_{w1}})(1+e^{-j\omega t_{w2}})e^{-j\omega t_{w3}}. \quad (7)$$

Then the general expression for the complex spectral density of the resulting pulse train will be as follows:

$$S_{ppp1}(\omega) = S_{pp1}(\omega) + S_{pp2}(\omega) = \frac{E}{\pi nj}(1 - e^{-j\omega t_i})(1 + e^{-j\omega t_{w1}})(1 + e^{-j\omega t_{w2}}) + \frac{E}{\pi j}(1 - e^{-j\omega t_i})(1 + e^{-j\omega t_{w1}})(1 + e^{-j\omega t_{w2}})e^{-j\omega t_{w3}}. \quad (8)$$

Transforming this expression, we will have:

$$S_{ppp1}(\omega) = \frac{16E}{\pi j}(\sin\frac{\omega t_i}{2})(\cos\frac{\omega t_{w1}}{2})(\cos\frac{\omega t_{w2}}{2})(\cos\frac{\omega t_{w3}}{2})e^{-j\omega\frac{\omega t_i}{2}} e^{-j\omega\frac{\omega t_{w1}}{2}} e^{-j\omega\frac{\omega t_{w2}}{2}} e^{-j\omega\frac{\omega t_{w3}}{2}}. \quad (9)$$

Therefore, the general expression for the spectral densities modulus of the constructed impulse arrays can be written as:

$$|S(\omega)| = \frac{2EK}{\pi}|\sin\frac{\omega t_i}{2}| \times \prod_{L=1}^{K}|\cos\frac{\omega t_{zL}}{2}|, \quad (10)$$

where $E$ is pulse amplitude; $t_{zL}$ is delay between sets of impulse combinations; $t_i$ is pulse duration; ω is current spectral density frequency; $K$ is number of consecutive rectangular pulses combinations.

Accordingly, the spectral density modulus envelope zeros are determined from the expressions:

$$\sin\frac{\omega t_i}{2} = 0 \text{ and } \cos\frac{\omega t_{zL}}{2} = 0. \quad (11)$$

Therefore,

$$\omega_{0t_i} = N \cdot 2\pi / t_i, \quad \omega_{0w} = N \cdot \pi / Kt_w, \quad (12)$$

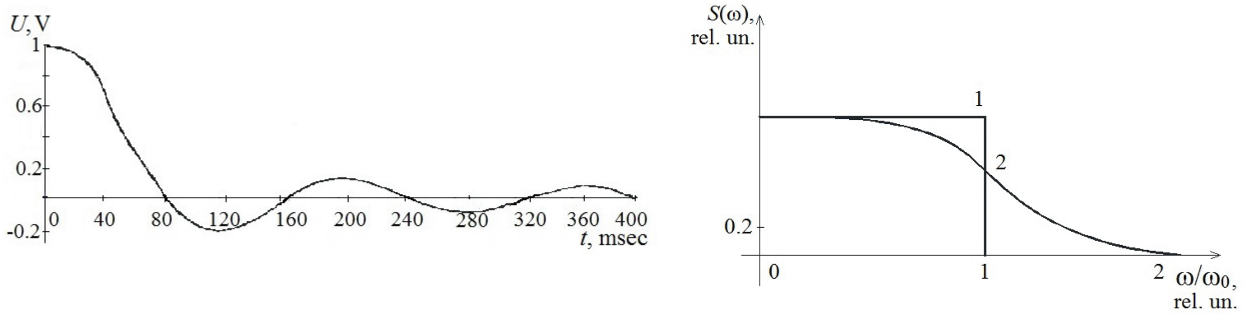
where $N$ = 1, 2, 3... etc. is zero number.

From expressions (10), it can be concluded that with an increase in the number of zeros the distance between them decreases and the spectral density increases in the region of frequencies that are multiples of the periodization frequency, and, therefore, the components of the expected discrete spectrum are formed. This suggests that with such a periodization, due to an increase in the number of zeros, the process of transformation of the spectral density into a discrete spectrum occurs. However, with such a signal analysis, it is impossible to determine a discrete spectrum formation end time, since with each new periodization step and regardless of the number of steps, the amplitudes of the spectral components change to indicate that in order to obtain high accuracy, it is necessary to obtain an amplitude-frequency spectrum corresponding to the maximum number of repetitions, and this leads to an increase in the analysis time. An increase in the analysis time in turn leads to the additional error appearance, the latter associated with the signal under study repetition parameters instability.

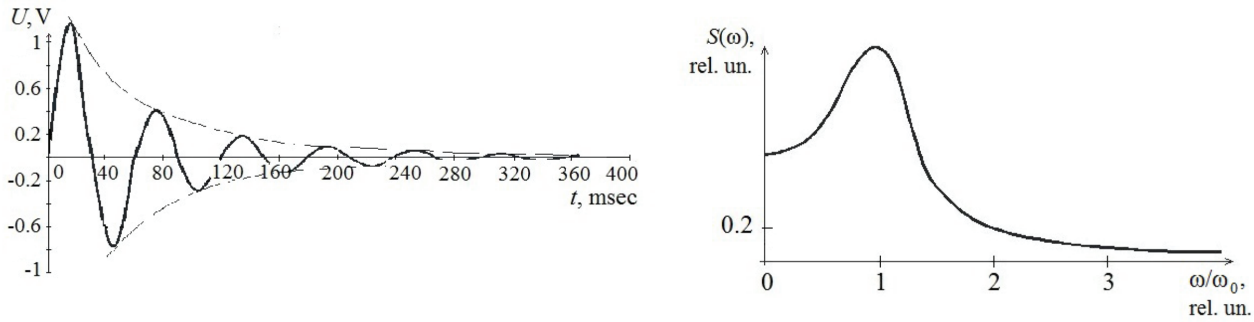
To analyze a single signal, let's represent it as a linear combination of given functions, which are limited in time by the duration of the consi-dered signal. According to the signal Fourier series expansion theory, only signals of infinite duration have physical meaning, which does not reflect real processes [2]. Thus such a method cannot be used to expand a time-limited quasi-periodic signal. Therefore, when choosing the expansion basis, it is most expedient to represent it as a set of harmonic signals, taking into account the limitations imposed by practice. A solution that satisfies the requirements is the use of various time-limited and often encountered in real life signals as a basis. Various types of sequences can serve as such signals, the most optimal of them being various damped oscillations with fast Fourier series convergence. For example, sinusoidal pulses with exponentially decaying amplitude, or a limited spectrum signal of the form $\sin(\omega t)/\omega t$, the general form and spectral density of which are shown in Figure 2.

However, the construction and circuit implementation of complex pulse sequences are associated with the introduction of additional errors associated with the limited stability of such circuits. In addition, on the signal duration limiting the amplitude-frequency spectrum for a signal in the form $\sin(\omega t)/\omega t$, ideally having a finite spectrum, is strongly distorted, up to the loss of the advantage in limiting the spectrum. Therefore, when analyzing quasi-periodic sequences, it is most optimal to use radio pulses, as presented in [10, 11], but it has to be taken into account that generally the initial phase is not equal to zero inside each radio pulse.

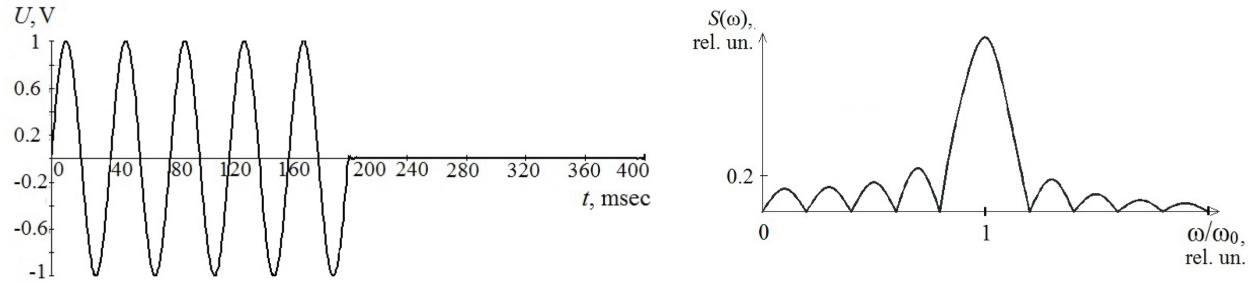
Thus, the signal will be decomposed into radio pulses with different multiple carrier frequencies [12], the initial phases and amplitudes being obtained as a result of the Fourier expansion of the generated periodic signal, and the carrier frequency of an individual radio pulse will then correspond to the frequency of a separate spectral component of the periodized signal. Consequently, the analyzed signal spectral density will consist of the sum of its constituent radio pulses spectral densities. Figure 3 shows the decomposition of a sequence of bipolar rectangular pulses into time-limited spectrum components – radio pulses.




*a*




*b*




*c*

**Figure 2** – General view and spectral density of signals limited in time: *a* – sinusoidal signal with exponentially decaying amplitude; *b* – signal of the form sin(ω*t*)/ω*t*; *c* – radio pulse

Therefore, a time-limited quasi-periodic signal $S(t_1,t_2)$ (i. e., a signal existing in the time interval $[t_1,t_2]$) can be represented as the sum of time-limited spectral densities $\varphi_k(t_1,t_2)$, which are a set of non-periodic signals (radio pulses) described in the frequency domain by the spectral density [13]:

$$S(t_1,t_2) = \sum_{k=0}^{K} C_k \phi_k(t_1,t_2), \tag{13}$$

where $C_k$ is expansion coefficients that determine the spectrum of a quasi-periodic signal.

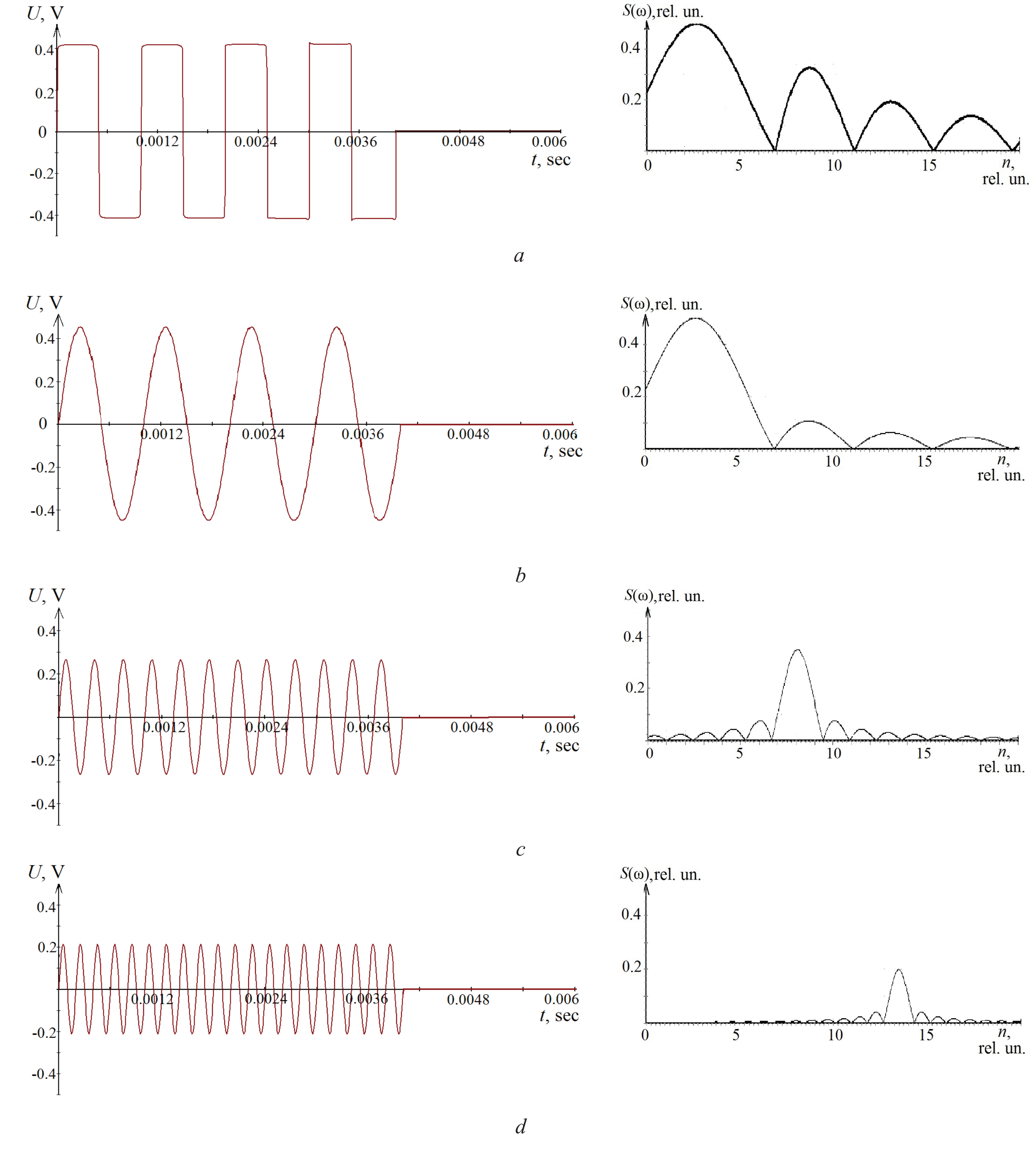



**Figure 3** – An example of the spectral density of a quasi-periodic signal and its components: *a* – a limited pulse sequence and its spectral density; *b*, *c*, *d* – constituent radio pulses and their spectral densities

The result of such an expansion is the total spectral density of the studied pulse sequence shown in Figure 4. For convenience, it was proposed to name this kind of expansion a quastr.

At the same time, it should be noted that a separate component of the quastr has a physical meaning, since it is essentially a radio pulse with quite simply measurable parameters. This distinguishes it from the representation of the investigated pulse only in the form of spectral density, the latter having frequency components with infinitely small amplitudes and continuous frequency components. At the same time, the quastr reflects the real process and corresponds to practice [8, 12]. The components of the quastr (radio pulses) being not periodic functions, they are described in the frequency domain by the spectral density, which links the quasi-periodic signal to its representation in the form of a spectral density. In this case, the spectral density of each component of the quastr, as well as the spectral density of the other components, is distributed over the entire frequency range, and the value of the spectral signal density sum of all radio pulses at a certain frequency can be calculated. Thus, the gain in the transition from the spectrum to the quastr is associated with the emerging opportunity to calculate and to measure the components of a non-periodic signal, and hence the opportunity to localize errors due to measuring systems individual elements functional peculiarities. This will make it possible to establish an unambiguous relationship between local signal variations and changes in its quastr components, which will allow the development of algorithms for automatic noise suppression in measuring systems, express diagnostics of malfunctions in electrical machines and other electrical equipment using one or more signals, and so on.

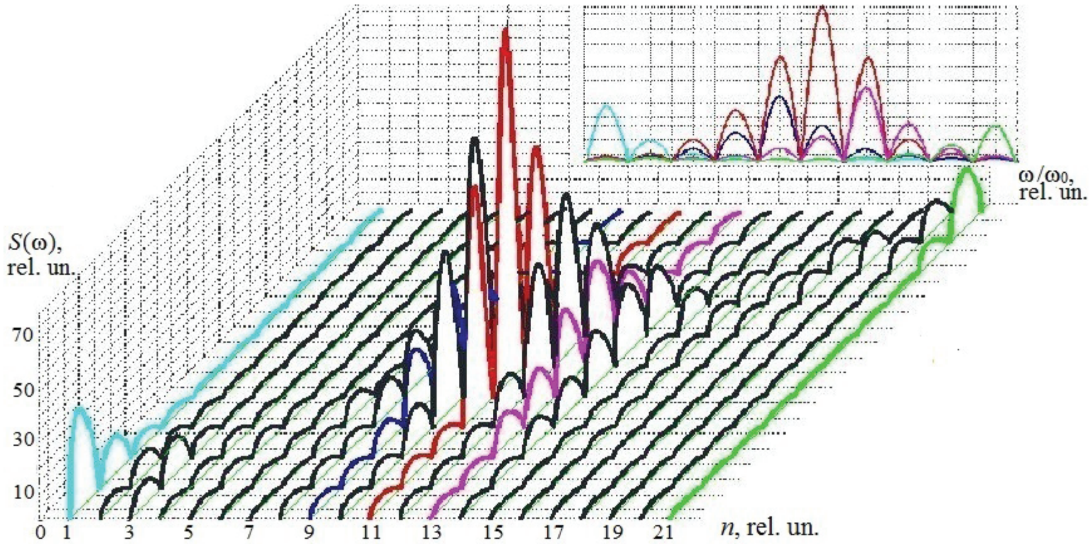



**Figure 4** – Volumetric representation of a quasi-periodic signal in the form of a quastr

Let us obtain an expression for the spectral density, presented as the sum of the spectral densities of its constituent radio pulses. The use of coherent radio pulses as frequency components, as it is shown in [8, 12], does not allow one to accurately and fully relate the processes and parameters of the quastr and spectral density components, i. e. the absence of phase shift in radio pulses is a special case, and the general view of a radio pulses sequence with an arbitrary phase delay is shown in Figure 5.

Let's represent the signal under consideration as a sum of four separate pulse components:

1. θ is incomplete pulse period associated with the phase shift θ;
2. 1, 2, ..., $m$ is full periods of the harmonic signal delayed in time by $T_c$ relative to each other together with the delay time $t_\theta$;
3. $P_+$ is completed half-cycle of the last pulse;
4. $P_-$ is incomplete part of the half-cycle of the last pulse.

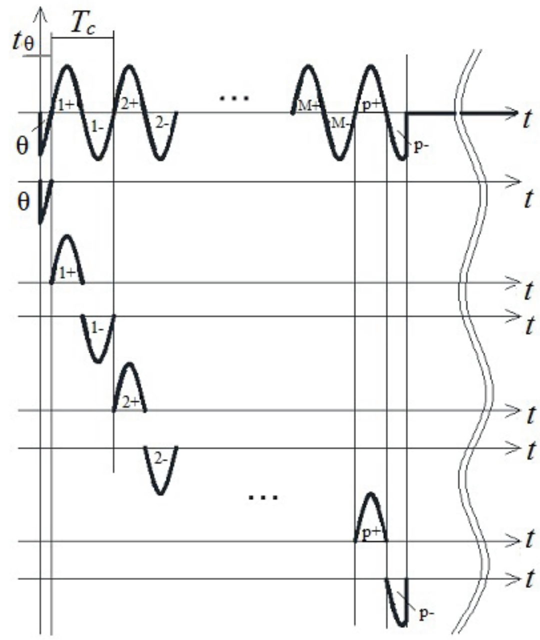



**Figure 5** – Representation of a phase-delayed signal of a sequence of radio pulses

Thus, taking into account the phase shift by time $t_\Theta$, the presented harmonic signal spectral density will be determined by the expression:

$$S_c(\omega) = S_{i\theta}(\omega) + \sum_{k=1}^{m} S_{i_k}(\omega) + S_{ip+}(\omega) + S_{ip-}(\omega), \quad (14)$$

where $S_\theta(\omega)$ is spectral density of a part of the sig is number of complete periods of the harmonic signal; $S_{ip+}(\omega)$ is spectral density of the completed part of the last signal period; $S_{ip-}(\omega)$ is the spectral density of the last incomplete pulse.

According to [1], the spectral density of a sinusoidal pulse is determined by the formula:

$$S_i(\omega) = \int_{t_1}^{t_2} (-1)\sin\left(\frac{2\pi}{T_c}t\right) e^{-j\omega t} dt, \quad (15)$$

where $t_1$ and $t_2$ is start and finish times of the impulse under consideration; $T_c$ is full period of the signal under study; ω is current frequency.

Taking into account expression (15), expression (14) will be of the form:

$$S_c(\omega) = \int_0^{t_\theta} E_m \sin\left(\frac{2\pi}{T_c}t\right) e^{-j\omega t} dt + \int_{t_\theta}^{t_\theta + mT_c} E_m \sin\left(\frac{2\pi}{T_c}t\right) e^{-j\omega t} dt + \int_{t_\theta + mT_c}^{t_\theta + mT_c + T_c/2} E_m \sin\left(\frac{2\pi}{T_c}t\right) e^{-j\omega t} dt + \int_{t_\theta + mT_c + T_c/2}^{(m+1)T_c} E_m \sin\left(\frac{2\pi}{T_c}t\right) e^{-j\omega t} dt, \quad (16)$$

where $E_m$ is amplitude of impulse under consideration.

Or, using the displacement theorem [9]:

$$S_c(\omega) = \int_0^{t_\theta} E_m \sin\left(\frac{2\pi}{T_c}t\right) e^{-j\omega t} dt + \int_0^{mT_c} E_m \sin\left(\frac{2\pi}{T_c}t\right) e^{-j\omega t} e^{-j\omega t_\theta} dt + \int_0^{T_c/2} E_m \sin\left(\frac{2\pi}{T_c}t\right) e^{-j\omega t} e^{-j\omega(t_\theta + mT_c)} dt +$$
$$+ \int_0^{T_c/2 - t_\theta} E_m \sin\left(\frac{2\pi}{T_c}t\right) e^{-j\omega t} e^{-j\omega(t_\theta + mT_c + T_c/2)} dt. \quad (17)$$

Let us form an expression for each part of the expression (17).

1. The solution of equation for $S_\theta(\omega)$ will look like:

$$S_\theta(\omega) = \int_0^{t_\theta} E_m \sin\left(\frac{2\pi}{T_c}t\right) e^{-j\omega t} dt = E_m \left( \frac{(-j\omega)\sin\left(\frac{2\pi}{T_c}t\right) - \left(\frac{2\pi}{T_c}\right)\cos\left(\frac{2\pi}{T_c}t\right)}{(-j\omega)^2 + \left(\frac{2\pi}{T_c}\right)^2} e^{-j\omega t} \right)\Bigg|_0^{t_\theta}. \quad (18)$$

After a series of transformations and calculations, equation (18) has the following form:

$$S_\theta(\omega) = E_m \frac{\frac{2\pi}{T_c}}{\omega^2 + \left(\frac{2\pi}{T_c}\right)^2} \left( \left( j\omega \frac{T_c}{2\pi} \sin\left(\frac{2\pi}{T_c}t_\theta\right) + \cos\left(\frac{2\pi}{T_c}t_\theta\right) \right) e^{-j\omega t_\theta} + 1 \right). \quad (19)$$

2. Let us write the spectral density of the full period of the impulse under consideration as the sum of two identical pulses, opposite in sign and shifted relative to each other by a time $t_{w1} = T_c/2$ , i. e.:

$S_i(\omega) = S_{i+}(\omega) + S_{i-}(\omega)$.

Then the spectral density of the first positive pulse will have the form:

$$S_{i_1+}(\omega) = \int_0^{T_c/2} (-1)\sin\left(\frac{2\pi}{T_c}t\right) e^{-j\omega t} e^{-j\omega t_\theta} dt. \quad (20)$$

After integration, expression (20) has the form:

$$S_{i_1+}(\omega) = $$
$$= \left( E_m \frac{(-j\omega)\sin\left(\frac{2\pi}{T_c}t\right) - \left(\frac{2\pi}{T_c}\right)\cos\left(\frac{2\pi}{T_c}t\right)}{(-j\omega)^2 + \left(\frac{2\pi}{T_c}\right)^2} e^{-j\omega t_\theta} e^{-j\omega t} \right)\Bigg|_0^{T_c/2}$$

or

$$S_{i_1+}(\omega) = E_m \frac{\frac{2\pi}{T_c}}{\omega^2 + \left(\frac{2\pi}{T_c}\right)^2} e^{-j\omega t_\theta} \left(1 - e^{-j\omega \frac{T_c}{2}}\right). \quad (21)$$

Then, according to displacement theorem, the spectral density of the negative half-wave of the first pulse, delayed by time $t_{w1} = T_c/2$, is determined by the expression:

$$S_{i_1-}(\omega) = E_m \frac{\frac{2\pi}{T_c}}{\omega^2 + \left(\frac{2\pi}{T_c}\right)^2} e^{-j\omega t_\theta} e^{-j\omega T_c/2} \left(1 - e^{-j\omega\frac{T_c}{2}}\right). \quad (22)$$

Then the expression for the spectral density of a single period of the radio pulse will have the form:

$$S_i(\omega) = S_{i_1+}(\omega) + S_{i_1-}(\omega) =$$

$$= E_m \frac{\frac{2\pi}{T_c}}{\omega^2 + \left(\frac{2\pi}{T_c}\right)^2} \left(1 - e^{-j\omega\frac{T_c}{2}}\right)\left(1 + e^{-j\omega\frac{T_c}{2}}\right) e^{-j\omega t_\theta}. \quad (23)$$

The total spectral density of the full periods of the considered harmonic signal has the form:

$$S_i(\omega) = \sum_{k=1}^{m} S_{i_k}(\omega) e^{-j\omega k T_c} = E_m \frac{\frac{2\pi}{T_c}}{\omega^2 + \left(\frac{2\pi}{T_c}\right)^2} e^{-j\omega t_\theta} \times$$
$$\times \left(1 - e^{-j\omega\frac{T_c}{2}}\right)\left(1 + e^{-j\omega\frac{T_c}{2}}\right)\left(\frac{\sin\frac{\omega m T_c}{2}}{\sin\frac{\omega T_c}{2}}\right) e^{-j\omega\frac{(m-1)T_c}{2}}. \quad (24)$$

3. Similarly to the previous calculations, let us define the spectral densities of the considered signal remaining components. The expression for the spectral density of the completed half-cycle of the last pulse has the form:

$$S_{P+}(\omega) = E_m \frac{\frac{2\pi}{T_c}}{\omega^2 + \left(\frac{2\pi}{T_c}\right)^2} \left(1 - e^{-j\omega\frac{T_c}{2}}\right) e^{-j\omega t_\theta} e^{-j\omega m T_c}, \quad (25)$$

and for the incomplete one:

$$S_{p-}(\omega) = E_m \frac{\frac{2\pi}{T_c}}{\omega^2 + \left(\frac{2\pi}{T_c}\right)^2} \left(\left(j\omega\frac{T_c}{2\pi}\sin\left(\frac{2\pi}{T_c}t_\theta\right) + \cos\left(\frac{2\pi}{T_c}t_\theta\right)\right) e^{-j\omega\left(\frac{T_c}{2} - t_\theta\right)} + 1\right) e^{-j\omega(t_\theta + mT_c + T_c/2)}. \quad (26)$$

Then the general expression for the spectral density of the considered signal can be represented as:

$$S(\omega) = E_m \frac{\frac{2\pi}{T_c}}{\omega^2 + \left(\frac{2\pi}{T_c}\right)^2} (\left(1 - e^{-j\omega\frac{T_c}{2}}\right) e^{-j\omega t_\theta} \left(\left(1 + e^{-j\omega\frac{T_c}{2}}\right)\left(\frac{\sin\frac{\omega m T_c}{2}}{\sin\frac{\omega T_c}{2}}\right) e^{-j\omega\frac{(m-1)T_c}{2}} + e^{-j\omega m T_c}\right) +$$
$$+ \left(\left(j\omega\frac{T_c}{2\pi}\sin\left(\frac{2\pi}{T_c}t_\theta\right) + \cos\left(\frac{2\pi}{T_c}t_\theta\right)\right) e^{-j\omega t_\theta} + 1\right) + \left(\left(j\omega\frac{T_c}{2\pi}\sin\left(\frac{2\pi}{T_c}t_\theta\right) + \cos\left(\frac{2\pi}{T_c}t_\theta\right)\right) e^{-j\omega\left(\frac{T_c}{2} - t_\theta\right)} + 1\right) e^{-j\omega(t_\theta + mT_c + T_c/2)}). \quad (27)$$

It follows from expression (26) that the signal spectral density is directly related to the periodic signal through its main parameters – amplitude ($E_m$), period ($T_c$) and initial phase ($t_\theta$), and, therefore, to study single and quasi-periodic signals, instead of the spectral density, one can use the quastr, which represents the time-limited amplitude-frequency spectrum of the signal. Such a representation will simplify the algorithmic support of measuring systems by eliminating the correlation signal processing, which requires large computational resources [13].

## Conclusion

A format has been developed to represent single and quasi-periodic, time-limited signals in the form of components that are associated with the spectral characteristics of a periodic signal obtained by periodization and the development of a mathematical apparatus for performing these procedures.

The presented approach to the analysis of quasi-periodic signals makes it possible to simplify the algorithmic support of measuring systems by eliminating the correlation processing of the signal, which

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requires large computational resources. The practical implementation of the proposed technique will reduce the measurement time and design parameters of the measuring systems elements.

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