Low-Frequency Admittance of Capacitor with Working Substance "Insulator–Partially Disordered Semiconductor– Insulator"

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Abstract

The study of the electrophysical characteristics of crystalline semiconductors with structural defects is of practical interest in the development of radiation-resistant varactors. The capacitance-voltage characteristics of a disordered semiconductor can be used to determine the concentration of point defects in its crystal matrix. The purpose of this work is to calculate the low-frequency admittance of a capacitor with the working substance "insulator–crystalline semiconductor with point *t*-defects in charge states (-1), (0) and (+1)–insulator".

A layer of a partially disordered semiconductor with a thickness of 150 μ m is separated from the metal plates of the capacitor by insulating layers of polyimide with a thickness of 3 μ m. The partially disordered semiconductor of the working substance of the capacitor can be, for example, a highly defective crystalline silicon containing point *t*-defects randomly (Poissonian) distributed over the crystal in charge states (-1), (0), and (+1), between which single electrons migrate in a hopping manner. It is assumed that the electron hops occur only from *t*-defects in the charge state (-1) to *t*-defects in the charge state (0) and from *t*-defects in the charge state (+1).

In this work, for the first time, the averaging of the hopping diffusion coefficients over all probable electron hopping lengths via *t*-defects in the charge states (-1), (0) and (0), (+1) in the covalent crystal matrix was carried out. For such an element, the low-frequency admittance and phase shift angle between current and voltage as the functions on the voltage applied to the capacitor electrodes were calculated at the *t*-defect concentration of $3 \cdot 10^{19}$ cm⁻³ for temperatures of 250, 300, and 350 K and at temperature of 300 K for the *t*-defect concentrations of $1 \cdot 10^{19}$, $3 \cdot 10^{19}$, and $1 \cdot 10^{20}$ cm⁻³.

Keywords: partially disordered semiconductor, low-frequency admittance of capacitor, triple-charged intrinsic point defects.

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Низкочастотный адмиттанс конденсатора с рабочим веществом «изолятор – частично разупорядоченный полупроводник – изолятор»

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Исследование электрофизических характеристик кристаллических полупроводников с дефектами структуры представляет практический интерес при создании радиационно-стойких варакторов. По вольт-фарадным характеристикам разупорядоченного полупроводника можно определять концентрацию точечных дефектов в его кристаллической матрице. Цель работы – рассчитать низкочастотный адмиттанс конденсатора с рабочим веществом «изолятор – кристаллический полупроводник с точечными *t*-дефектами в зарядовых состояниях (-1), (0) и (+1) – изолятор».

Слой частично разупорядоченного полупроводника толщиной 150 мкм отделен от металлических обкладок конденсатора диэлектрическими прослойками из полиимида толщиной 3 мкм. Частично разупорядоченный полупроводник рабочего вещества конденсатора представляет собой, например, сильнодефектный кристаллический кремний, содержащий точечные *t*-дефекты, случайно (пуассоновски) распределенные по кристаллу, в зарядовых состояниях (-1), (0) и (+1) между которыми прыжковым образом мигрируют одиночные электроны. Считается, что прыжки электронов происходят только с *t*-дефектов в зарядовом состоянии (-1) на *t*-дефекты в зарядовом состоянии (0) и с *t*-дефектов в зарядовом состоянии (0) на *t*-дефекты в зарядовом состоянии (+1).

В работе впервые проведено усреднение коэффициентов прыжковой диффузии по всем вероятным длинам прыжка электрона между *t*-дефектами в зарядовых состояниях (-1), (0) и (0), (+1) в ковалентной кристаллической матрице. Для такого элемента рассчитаны низкочастотный адмиттанс и угол сдвига фаз между током и напряжением в зависимости от приложенного на электроды конденсатора напряжения при концентрации *t*-дефектов $3 \cdot 10^{19}$ см⁻³ для температур 250, 300 и 350 К и при температуре 300 К для концентраций *t*-дефектов $1 \cdot 10^{19}$, $3 \cdot 10^{19}$ и $1 \cdot 10^{20}$ см⁻³.

Ключевые слова: частично разупорядоченный полупроводник, низкочастотный адмиттанс конденсатора, трехзарядные собственные точечные дефекты.

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Introduction

In the works [1, 2], for the first time, a variant of controlling the hopping electrical conductivity via hydrogen-like donors along a semiconductor film using an external electrostatic field $E(x) = -d\phi/dx$ perpendicular to the film surface, which does not lead to the appearance of a current and does not violate the electrical neutrality of the film as a whole, was theoretically considered. However, the hopping electrical conductivity longitudinal to the direction of the controlling external electric field was not considered in [1, 2]. The field effect was studied and the quasifrequency (low-frequency) capacitance and conductivity of silicon crystals with hopping electron migration via point two-level defects with positive and negative correlation energies in three charge states (-1), (0), and (+1) were calculated [3, 4]. However, the electrical capacity and conductivity of the "insulator-partially disordered semiconductor-insulator" structure were not investigated in [3, 4]. For the first time, the static capacitance-voltage characteristics of a ζ -diode made of crystalline silicon, in which current was carried only by electron hopping via tdefects, were calculated [5]. However, in the diode model constructed in [5], there was no averaging of diffusion coefficients over all probable electron hopping lengths via *t*-defects in three charge states (-1), (0), and (+1). Taking into account electron hopping via point defects, the temperature and frequency dependences of the dielectric permittivity of silicon irradiated with a large dose of neutrons were studied [6]. The low-frequency electrical capacitance as well as the electric field and potential distribution for the "metal-insulator-intrinsic semiconductor-insulatormetal" structure were calculated [7–9]. However, the capacitance-voltage characteristics for the structure with a disordered semiconductor layer were not calculated in [7–9]. The results of an experiment on measuring the capacitance of a thin-film capacitor (structure Al-Al₂O₃-Al) were interpreted [10] taking into account quantum effects. A method was described [11, 12] for determining, from the temperature dependences of capacitance and conductivity, the ionization energy and concentration of deep centers in an overcompensated semiconductor placed between insulator plates (40-100 µm thick polyethylene terephthalate), to which a sinusoidal voltage was applied through copper contacts. However, in [11, 12] the experimental data on the conductivity and capacitance of the studied structure were not compared with theory.

The purpose of this work is to calculate the lowfrequency admittance of a capacitor with the working substance "insulator–crystalline semiconductor with point *t*-defects in charge states (-1), (0) and (+1) with hopping migration of electrons between them–insulator".

Model of capacitor with working substance "insulator-partially disordered semiconductor-insulator"

Let a wafer of highly defective crystalline silicon (*hd*-Si) with a thickness of d_s and a surface area A be in the middle between the metal plates of a flat capacitor and separated from them by the layers of insulator (e.g., polyimide) with a thickness of d_i (Figure 1*a*). The capacitor is connected to a constant electrical voltage source. The *x* coordinate axis is perpendicular to the surface of the semiconductor wafer occupying space $-d_s/2 < x < d_s/2$, the *y* and *z* coordinate axes are parallel to the wafer surface.



Figure 1 – Cross-section of capacitor with a wafer of highly defective crystalline silicon (hd-Si) of thickness d_s separated from the metal capacitor plates by the insulator layers of thickness d_i . Across the semiconductor wafer an electric potential difference is created by two metal electrodes parallel to the plane yz(a). Equivalent scheme of capacitor with the working substance "insulator–partially disordered semiconductor–insulator" (b). Simplified equivalent scheme of the system (c)

Let us assume that in one part the field potential on the wafer surface is positive $\varphi(x = -d_s/2) = \varphi_s$, and in the other it is negative $\varphi(x = d_s/2) = -\varphi_s$, then the potential difference applied to the semiconductor is $U_s = \varphi(x = -d_s/2) - \varphi(x = d_s/2) = 2\varphi_s$. We will consider electrodes located parallel to the yz plane (so that the field distribution in the wafer along the y and z coordinates will be symmetric). The screening of the external electrostatic field is caused by the redistribution of electrons hopping via defects in the charge states (0, -1, and +1; in units of elementary charge e against the background of a silicon matrix), i.e. by the migration of charge states of immobile defects to a distance much greater than the average distance between them.

The capacitor with the working substance "insulator-partially disordered semiconductor-insulator" contains series-connected capacitances of insulating layers C_i and a parallel $R_s(C_s + C_g)$ -circuit of the semiconductor wafer (see Figure 1b). Here $C_i =$ $= \varepsilon_i A/d_i$ and $C_g = \varepsilon_s A/d_s$ are the geometric capacitances of insulator and semiconductor with static dielectric permittivities $\varepsilon_i = \varepsilon_{r_i} \varepsilon_0$ and $\varepsilon_s = \varepsilon_{r_s} \varepsilon_0$ (we assume that radiation defects do not contribute to the static dielectric constant of Si crystals), $\varepsilon_{r,i} = 3.5$ and $\varepsilon_{r,s} = 11.5$ are the relative permittivities of the polyimide and the silicon crystal lattice, $\varepsilon_0 = 8.85 \text{ pF/m}$ is the electric constant, $R_s = R_s(U)$ is the semiconductor resistance, $C_s = C_s(U)$ is the differential capacitance of the semiconductor, U is the voltage created by the metal plates of the capacitor.

The real part $C_{eq} = C_{eq}(U)$ of the complex electrical capacitance and the active component of the conductivity $G_{eq} = G_{eq}(U)$ of the structure in the equivalent circuit (see Figure 1*c*) is [11, 13]:

$$C_{\rm eq} = \frac{C_i}{2} \frac{1 + \omega^2 R_s^2 (C_g + C_s) (C_g + C_s + C_i/2)}{1 + \left[\omega R_s (C_g + C_s + C_i/2)\right]^2}, \quad (1)$$

$$G_{\rm eq} = \frac{\omega^2 R_s (C_i/2)^2}{1 + \left[\omega R_s (C_g + C_s + C_i/2)\right]^2},$$
(2)

where $U (= U_{\rm dc})$ is the constant voltage across the capacitor plates, ω is the angular frequency of the variable component of the measuring signal with the amplitude $|U_{\rm ac}| \ll |U|$.

From Eqs. (1) and (2) we find the total conduction (admittance) Y = Y(U) and the phase shift $\theta = \theta(U)$ between current and voltage of the capacitor with the working substance "insulator–partially disordered semiconductor–insulator":

$$Y = [G_{eq}^{2} + (\omega C_{eq})^{2}]^{1/2} =$$

= $\frac{\omega C_{i}}{2} \left(\frac{1 + [\omega R_{s} (C_{g} + C_{s})]^{2}}{1 + [\omega R_{s} (C_{g} + C_{s} + C_{i}/2)]^{2}} \right)^{1/2}$, (3)

$$\theta = \arctan\left(-\omega C_{eq}/G_{eq}\right) =$$

$$= \arctan\left(-\frac{1+\omega^2 R_s^2 (C_g + C_s)(C_g + C_s + C_i/2)}{\omega R_s C_i/2}\right), (4)$$

where $R_s = R_s(U)$ and $C_s = C_s(U)$. Note that the total resistance (impedance) Z = Z(U) is related to the admittance *Y* as follows: $Z = Y^{-1}$.



Figure 2 – Single-electron energy *E* as function of *x* coordinate in semiconductor with point two-level (triplecharged) defects of *t*-type in equilibrium (at U = 0): $E_m^{(c)}$ is the mobility edge of conduction band electrons, $E_F^{(c)} < 0$ is the Fermi level in the band gap, counted from the hole mobility edge ($E_m^{(v)} = 0$), $\Delta_t = E_2 - E_1$ is the width of energy gap between |1)- and |2)-bands, $W_{0,+1}$ and $W_{-1,0}$ are widths of |1)- and |2)-bands. Arrows show hops of single electrons e^- via |1)- and |2)-bands as well as generation [gen: 2(0) $\rightarrow \rightarrow (-1) + (+1)$] and recombination [rec: $(-1) + (+1) \rightarrow 2(0)$] electron transitions between them; |d) and |a) are states of shallow hydrogen-like donors and acceptors in the charge states (+1) and (-1), respectively

For $R_s \gg 1/\omega C_s$ from Eq. (1), the inverse equivalent capacity of the entire structure is $1/C_{eq} = 2/C_i + 1/(C_s + C_g)$. Since the capacitances of insulator layers C_i and semiconductor $C_s + C_g$ are connected in series, the charge on each of them is equal to Q. Thus, the voltage drops across insulators $U_i = Q/C_i$ and across semiconductor $U_s = Q/(C_s + C_g)$ are related to the voltage across the capacitor $U = Q/C_{eq}$ as follows: $U = 2U_i + U_s$. By substituting the charge Q on the capacitor, expressed in terms of U_s and $C_s + C_g$, into U we obtain the voltage across the semiconductor U_s the capacitor U_s and $C_s + C_g$.

$$U = U_{s} \frac{C_{s} + C_{g}}{C_{eq}} = U_{s} \frac{2(C_{s} + C_{g}) + C_{i}}{C_{i}}.$$
 (5)

A highly defective silicon crystal (*hd*-Si) contains point two-level *t*-type defects in a concentration sufficient to stabilize the Fermi level E_F in the energy gap. Defects of *t*-type in the charge states (+1) and (0) form a |1)-band with the energy levels E_1 , and the ones in the charge states (0) and (-1) form a |2)-band in the band gap (energy levels E_2), located closer to the *c*-band than |1)-band (Figure 2). Examples of *t*-defects are amphoteric impurities (Au, Cu).

Let us consider silicon under conditions of only hopping electron migration via immobile radiation defects (of *t*-type) in the charge states (-1) and (0), as well as in the charge states (0) and (+1). The total concentration of defects in the charge states (0), (-1), and (+1) is $N_t = N_0 + N_{-1} + N_{+1}$.

We assume that |d)- and |a)-centers are completely ionized and their concentrations N_d and N_a satisfy the conditions: $N_d/N_t \ll 1$ and $N_a/N_t \ll 1$. Thus, the condition of electrical neutrality of the partially disordered semiconductor has the form:

$$N_{+1} = N_{-1}, (6)$$

where $N_{+1} = N_{t,+1}$ and $N_{-1} = N_{t,-1}$.

The concentrations of ionized and neutral defects can be written as [14]:

$$N_Z = N_t f_Z,\tag{7}$$

where f_Z is the probability that the defect is in one of three possible charge states Z = -1, 0, +1.

If we neglect the excited states of radiation defects, then the inverse distribution functions $1/f_Z$ of defects in |1)- and |2)-bands over charge states are [3, 4]:

$$\begin{aligned} \frac{1}{f_{-1}} &= 1 + \beta_2 \exp\left(\frac{E_{\rm F}^{(v)} + E_2}{k_{\rm B}T}\right) + \frac{\beta_2}{\beta_1} \exp\left(\frac{E_1 + E_2 + 2E_{\rm F}^{(v)}}{k_{\rm B}T}\right),\\ \frac{1}{f_0} &= 1 + \frac{1}{\beta_1} \exp\left(\frac{E_{\rm F}^{(v)} + E_1}{k_{\rm B}T}\right) + \frac{1}{\beta_2} \exp\left(\frac{-(E_{\rm F}^{(v)} + E_2)}{k_{\rm B}T}\right),\\ \frac{1}{f_{+1}} &= 1 + \beta_1 \exp\left(\frac{-(E_{\rm F}^{(v)} + E_1)}{k_{\rm B}T}\right) + \frac{\beta_1}{k_{\rm B}T} \exp\left(\frac{-(E_1 + E_2 + 2E_{\rm F}^{(v)})}{k_{\rm B}T}\right), \end{aligned}$$

$$(8)$$

where $E_{\rm F}^{(v)} = E_v - E_{\rm F}$ is the Fermi level (chemical potential) $E_{\rm F}$, counted from the *v*-band hole mobility edge ($E_m^{(v)} = 0$) of an undoped crystal [15, 16]; $E_{\rm F}^{(v)} < 0$ for the Fermi level in the band gap; $E_1 = E_0 - E_{+1} > 0, E_2 = E_{-1} - E_0 > 0; k_{\rm B}T$ is the thermal energy. For dominant radiation defects in silicon (mainly divacancies), following the experimental data from [17–19], we assume: $E_1 = 225 \text{ meV}, E_2 = 575 \text{ meV},$ i.e. $\Delta_t = E_2 - E_1 = 350 \text{ meV}, \beta_1 = \beta_0/\beta_{+1} = 1, \beta_2 = \beta_0/\beta_{-1} = 1$, where β_Z is the number of quantum states of the defect in the charge state *Z* with energy E_Z .

With the total concentration of charged radiation defects $N_{ch} = N_{-1} + N_{+1}$ with charge $\pm e$ randomly (Poissonian) distributed over the crystal, we have equal rms fluctuations $W = W_{-1,0} = W_{0,+1}$ of the electrostatic energy, i.e. the widths of |2)- and |1)-bands are [20, 21]:

$$W_{-1,0} = W_{0,+1} = 1.637 \frac{e^2}{4\pi\varepsilon_s} \left(\frac{4\pi}{3} (N_{\rm ch})_{\rm eq}\right)_{\varphi_s=0}^{1/3}, \qquad (9)$$

where the Coulomb interaction of each charged defect only with its nearest charged defect (ion) is taken into account; *e* is the elementary charge; $(N_{ch})_{eq} = N_t/2$ is determined from the condition of maximum effective concentrations $N_{-1,0} = N_{0,+1} = N_{-1}N_0/N_t = N_0N_{+1}/N_t$ of single electrons hopping via *t*-defects in the charge states (-1), (0) and in charge states (0), (+1). Then we obtain $(N_{-1})_{max} = (N_{+1})_{max} = N_t/4$, $(N_0)_{max} = N_t/2$ and $(N_{-1,0})_{max} = (N_{0,+1})_{max} = N_t/8$ [22]. Note that $3W > \Delta_t$.

For a semiconductor with uniformly distributed point defects of the crystal lattice, the values of the function $f_Z(\varphi)$ depend on the coordinate *x* only through the potential $\varphi(x)$ and are obtained from f_Z by replacing $E_F^{(v)} < 0$ in Eq. (8) by

$$E_{\rm F}^{(v)}(\varphi(x)) = E_{\rm F}^{(v)} - e\varphi(x), \tag{10}$$

that is for $\varphi(x) < 0$ the Fermi level $E_{\rm F}^{(v)}(\varphi)$ shifts to the top of the *v*-band and for $\varphi(x) > 0$ it shifts to the band gap.

The change in the concentration of charge states Z = -1, 0, +1 of $N_Z(\varphi) - N_Z$ defects in the electric field with the potential $\varphi(x)$ is determined by Eq. (7) taking into account Eqs. (10) and (8). In this case, it is assumed that the energy gap Δ_t between |1)- and |2)-bands, as well as the width of each band W, do not depend on the potential.

Due to the symmetry of the problem with respect to reflection $x \rightarrow -x$, we consider only the region $-d_s/2 \le x \le 0$. The electrostatic potential $\varphi(x)$ inside the semiconductor at a point with coordinate *x* satisfies the Poisson equation [23, 24]:

$$\frac{d^2\varphi}{dx^2} = \frac{1}{2} \frac{d}{d\varphi} \left(\frac{d\varphi}{dx}\right)^2 = -\frac{\rho(\varphi)}{\varepsilon_s},$$
(11)

where $\rho(\varphi(x)) = e[N_{+1}(\varphi(x)) - N_{-1}(\varphi(x))]$ is the volume density of the induced charge; $N_{-1} = N_{+1}$ is the electrical neutrality condition of the semiconductor wafer at $\varphi_s = 0$.

By integrating Eq. (11) over ϕ , we obtain the electric field strength:

$$\frac{\mathrm{d}\varphi}{\mathrm{d}x} = \pm \left(-\frac{2}{\varepsilon_s} \int_0^{\varphi_s} \rho(\varphi) \,\mathrm{d}\varphi\right)^{1/2},\tag{12}$$

where for $\varphi_s > 0$ the "–" sign should be taken, while for $\varphi_s < 0$ the "+" sign should be taken.

From Eq. (11), taking into account Eq. (12), we

obtain the charge Q_s induced by the external electric field per unit area A of the flat surface of the silicon wafer:

$$\frac{Q_s}{A} = \int_{-d_s/2}^0 \rho(x) dx = \varepsilon_s \left. \frac{d\varphi}{dx} \right|_{x=-d_s/2} = \\ = \pm \left(-2\varepsilon_s \int_0^{\varphi_s} \rho(\varphi) d\varphi \right)^{1/2}, \qquad (13)$$

where for $\phi_s > 0$ the "-" sign should be taken, while for $\phi_s < 0$ the "+" sign should be taken.

The differential electrical capacitance per unit silicon surface area A, taking into account Eq. (13), is

$$\frac{C_s}{A} = -\frac{\mathrm{d}Q_s}{A\,\mathrm{d}\varphi_s} = \frac{\rho(\varphi_s)}{\mathrm{d}\varphi/\mathrm{d}x}\Big|_{x=-d_s/2} = \frac{eN_t[f_{+1}(\varphi_s) - f_{-1}(\varphi_s)]}{\pm \left(-(2/\varepsilon_s)\int_0^{\varphi_s}\rho(\varphi)\,\mathrm{d}\varphi\right)^{1/2}}.$$
(14)

The change under the action of the field effect of the hopping electrical conductivity [caused by the migration of single electrons across the wafer thickness via immobile radiation *t*-defects in the charge states (-1) and (0), as well as in the charge states (0)and (+1)] is

$$\delta\sigma(\varphi_{s}) = \frac{e}{d_{s}} \int_{\varphi_{s}}^{0} \frac{N_{-1,0}(\varphi)M_{-1,0} + N_{0,+1}(\varphi)M_{0,+1}}{d\varphi/dx} d\varphi - \frac{e}{d_{s}} \int_{\varphi_{s}}^{0} \frac{N_{-1,0}(0)M_{-1,0} + N_{0,+1}(0)M_{0,+1}}{d\varphi/dx} d\varphi + \frac{e}{d_{s}} \int_{-\varphi_{s}}^{0} \frac{N_{-1,0}(\varphi)M_{-1,0} + N_{0,+1}(\varphi)M_{0,+1}}{d\varphi/dx} d\varphi - \frac{e}{d_{s}} \int_{-\varphi_{s}}^{0} \frac{N_{-1,0}(\varphi)M_{-1,0} + N_{0,+1}(\varphi)M_{0,+1}}{d\varphi/dx} d\varphi, \quad (15)$$

where $N_{-1,0}(\phi) = N_{-1}(\phi)N_0(\phi)/N_t$ and $N_{0,+1}(\phi) = N_0(\phi) \times$ $\times N_{+1}(\varphi)/N_t$ are the effective concentrations of single electrons hopping via *t*-defects in the charge states (-1), (0) and in the charge states (0), (+1); $M_{-1,0}$ and M_{0+1} are the drift mobilities of electrons hopping via *t*-defects in the charge states (-1), (0) and in the charge states (0), (+1).

The relationship between the hopping diffusion coefficients $D_{-1,0}$ and $D_{0,+1}$ and the drift hopping mobilities $M_{-1,0}$ and $M_{0,+1}$ of electrons hopping via point t-defects of the crystal matrix is established by the Nernst-Einstein-Smoluchowski relation (see, e.g., [3, 25]):

$$\frac{D_{-1,0}}{M_{-1,0}} = \xi_{-1,0} \frac{k_{\rm B}T}{e}, \quad \frac{D_{0,+1}}{M_{0,+1}} = \xi_{0,+1} \frac{k_{\rm B}T}{e}, \tag{16}$$

where $\xi_{-1,0} \ge 1$, $\xi_{0,+1} \ge 1$ are the dimensionless parameters, which are determined by the ratio of the fluctuation spread of t-defect levels (with average values of E_1 and E_2) to the thermal energy $k_{\rm B}T$; further we assume $\xi_{-1,0} = \xi_{0,+1} = 1$.

The diffusion coefficients $D_{-1,0}$ and $D_{0,+1}$ of electrons hopping via *t*-defects in a covalent crystal matrix (see Eq. (16)) can be estimated by averaging over all probable hopping lengths r (cf. [22–27]):

$$D_{-1,0} = \frac{1}{6} \langle \Gamma_{-1,0}(r,T)r^2 \rangle, D_{0,+1} = \frac{1}{6} \langle \Gamma_{0,+1}(r,T)r^2 \rangle,$$
(17)

where $\Gamma_{-1,0}(r, T) = v_{\text{lt}} \exp[-(2r/a_{-1} + W_{-1,0}/k_{\text{B}}T)]$ and $\Gamma_{0,+1}(r,T) = v_{\text{lt}} \exp[-(2r/a_0 + W_{0,+1}/k_{\text{B}}T)]$ are frequencies of electron hopping via t-defects in charge states (-1), (0) and (0), (+1) [28]; $v_{lt} \approx 10$ THz is the characteristic frequency of crystal matrix phonons; $a_{-1} = \hbar/(2m_0E_2)^{1/2}$ and $a_0 = \hbar/(2m_0E_1)^{1/2}$ are the radii of localization of an electron at the t-defect in the charge state (-1) and (0), respectively, m_0 is the electron mass in vacuum.

From Eq. (17), taking into account the distribution of distances r between t-defects [21], we get:

$$D_{-1,0} = \frac{2\pi v_{\rm lt} N_{\rm eq}}{3} \exp\left(\frac{W_{-1,0}}{k_{\rm B}T}\right) \times \\ \times \int_{0}^{\infty} r^{4} \exp\left[-\left(\frac{2r}{a_{-1}} + \frac{4\pi r^{3}}{3}N_{\rm eq}\right)\right] dr , \\ D_{0,+1} = \frac{2\pi v_{\rm lt} N_{\rm eq}}{3} \exp\left(\frac{W_{0,+1}}{k_{\rm B}T}\right) \times \\ \times \int_{0}^{\infty} r^{4} \exp\left[-\left(\frac{2r}{a_{0}} + \frac{4\pi r^{3}}{3}N_{\rm eq}\right)\right] dr , \qquad (18)$$

where $N_{\text{eq}} = (N_{-1,0})_{\text{max}} = (N_{0,+1})_{\text{max}} = N_t/8$.

From Eq. (15), taking into account Eqs. (16)-(18), we obtain the resistance of a highly defective crystalline silicon (hd-Si) wafer due to the hopping of single electrons via *t*-defects along its thickness:

$$R_s = R_s(U(\varphi_s)) = \frac{d_s}{A\sigma},$$
(19)

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where d_s and A are the thickness and the surface area of the *hd*-Si wafer, $\sigma = \sigma(\varphi_s) = \sigma(0) + \delta\sigma(\varphi_s)$ is the electrical conductivity, and $\sigma(0) = e[N_{-1,0}(0)M_{-1,0} + N_{0,+1}(0)M_{0,+1}]$ is the conductivity at $\varphi_s = 0$. For the considered low frequencies, the electrical conductivity σ is frequency-independent [29, 30].

Note that Eqs. (14) and (19) were obtained under the assumption of quasi-stationary filling of energy levels according to Eq. (8) taking into account Eq. (10), therefore C_s and R_s are the quasistatic (low-frequency) capacitance and resistance of semiconductor. The quasi-stationarity condition is satisfied at $\omega/2\pi \ll \Gamma_{-1,0}(r, T)$ and $\omega/2\pi \ll \Gamma_{0,+1}(r, T)$. In other words, this can expressed by the inequality $\omega/2\pi \ll \sigma/\varepsilon_s$, where ε_s/σ is the Maxwell relaxation time for hopping conduction.

Calculation results and discussion

The calculations were carried out for the following parameter values: semiconductor thickness $d_s = 150 \,\mu\text{m}$, insulator thickness $d_i = 3 \,\mu\text{m}$, relative permittivities of semiconductor (*hd*-Si) $\varepsilon_{r,s} = 11.5$ and insulator (polyimide) $\varepsilon_{r,i} = 3.5$, frequency of alternating electric field $\omega/2\pi = 1 \,\text{kHz}$.

Figure 3*a* shows the results of calculating the ratio of the low-frequency admittance Y(U) to $\omega C_i/2$ according to Eq. (3) at various values of the voltage *U* created by metal electrodes on the surface of insulator interlayers, for $N_t = 3 \cdot 10^{19}$ cm⁻³ at temperatures T = 250, 300, 350 K. The values of *U* are related to U_s by Eq. (5) and $U_s = 2\varphi_s$ was chosen so that the inequality $eU_s \le \Delta_t$ is fulfilled. It is seen that for U = 0 (flat-band mode) the admittance of the capacitor with the working substance "insulator–partially disordered semiconductor–insulator" increases with temperature.

Figure 3*b* shows the results of calculating the ratio of the low-frequency admittance Y(U) to $\omega C_i/2$ according to Eq. (3) at different values of voltage *U* created by metal electrodes on the surface of insulator interlayers for temperature T = 300 K at concentrations of *t*-defects in disordered silicon $N_t = 1 \cdot 10^{19}$, $3 \cdot 10^{19}$, $1 \cdot 10^{20}$ cm⁻³. It is seen that the admittance increases with the concentration of *t*-defects.

Figure 4*a* shows the results of calculating the phase shift angle $\theta(U)$ between current and voltage according to Eq. (4) at various values of voltage *U* created by metal electrodes on the surface of insulator interlayers, for $N_t = 3 \cdot 10^{19}$ cm⁻³ at temperatures T = 250, 300, 350 K. It is seen that the absolute value of the phase shift angle decreases with temperature.



Figure 3 – Dependence of admittance $2Y/\omega C_i$ on electrode voltage *U*, calculated by Eq. (3): *a*) for $N_t = 3 \cdot 10^{19}$ cm⁻³ at temperatures *T* (K): 250 (curve *I*), 300 (2), and 350 (3); *b*) for T = 300 K at *t*-defect concentrations N_t (cm⁻³): $1 \cdot 10^{19}$ (*I*), $3 \cdot 10^{19}$ (2), and $1 \cdot 10^{20}$ (3)

Figure 4*b* shows the results of calculating the phase shift angle $\theta(U)$ according to Eq. (4) at various values of the voltage *U* created by metal electrodes for temperature T = 300 K at the concentration of *t*-defects in disordered silicon $N_t = 1 \cdot 10^{19}$, $3 \cdot 10^{19}$, $1 \cdot 10^{20}$ cm⁻³. It is seen that in the flat-band mode (at U = 0), all other conditions being equal, the phase shift angle modulus is minimum for the concentration of *t*-defects $N_t = 3 \cdot 10^{19}$ cm⁻³ and is maximum for $N_t = 1 \cdot 10^{20}$ cm⁻³.

Note that the value of the Fermi level energy $E_{\rm F}^{(v)} = 400$ meV, obtained from the electrical neutrality condition $N_{+1} = N_{-1}$, does not depend on the temperature, since $E_{\rm F}^{(v)}$ is in the middle between |1)- and |2)-band. This practically coincides with the experimental value of $E_{\rm F}^{(v)}$ in silicon [17–19], which contains a high concentration of radiation defects.

Note that the capacitor with the working substance "insulator–partially disordered semiconductor–insulator" is radiation-resistant, because radiation defects are already present in the semiconductor in large numbers. This suggests that this element is promising for use as a varactor. Also, the dependences of the electrophysical characteristics (Eqs. (1)– (4)) on the potential at the electrodes make it pos-



Figure 4 – Dependence of phase shift angle θ on electrode voltage *U*, calculated by Eq. (4): *a*) for $N_t = 3 \cdot 10^{19}$ cm⁻³ at temperatures *T* (K): 250 (curve *1*), 300 (2), and 350 (3); *b*) for *T* = 300 K at *t*-defect concentrations N_t (cm⁻³): $1 \cdot 10^{19}$ (*1*), $3 \cdot 10^{19}$ (2), and $1 \cdot 10^{20}$ (3)

sible to determine the concentration of *t*-defects in the disordered semiconductor separated by insulator interlayers from the capacitor plates [11].

Conclusion

The structure "insulator–partially disordered semiconductor–insulator" is proposed as a working substance of a capacitor. The semiconductor layer with a thickness of 150 μ m is separated from the metal electrodes of the capacitor by insulating layers of polyimide with a thickness of 3 μ m. The semiconductor layer is a highly defective silicon crystal containing radiation point two-level *t*-defects in three charge states (-1), (0), and (+1) with hopping migration of single electrons via them, i.e. defects form |1)- and |2)-bands in the band gap.

The calculation gives a nonmonotonic dependence of the low-frequency admittance and the phase angle between current and voltage on the electric potential at the metal plates. At the concentration of *t*-type radiation defects equal to $3 \cdot 10^{19}$ cm⁻³, with an increase in temperature from 250 to 350 K, the admittance increases by about 12%. With an increase in the concentration of *t*-defects from $1 \cdot 10^{19}$ to $1 \cdot 10^{20}$ cm⁻³ at temperature of 300 K, the admittance of the capacitor increases by about 13%. In the calculations, for the first time, the diffusion coefficients were averaged over all probable electron hopping lengths via *t*-defects in the charge states (-1), (0) and (0), (+1) in the covalent crystal matrix. Note that the considered element is radiation-resistant, since the semiconductor layer already contains radiation point defects in a high concentration.

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